Embodied sequential sampling models and dynamic neural fields for decision-making: Why hesitate between two when a continuum is the answer

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Highlights

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- Sequential sampling & dynamic neural field model decisions without cross-fertilizing

- Unifying terminologies and equations are proposed to bridge this long lasting gap

- The model fits empirical mouse-tracking data relying on various response paradigms

- Cognitive & sensorimotor components combined generate plausible mouse trajectories

- Inferences on the interplay between paradigms & human decision-making are discussed

Embodied sequential sampling models and dynamic neural fields for decision-making: why hesitate between two when a continuum is the answer

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Abstract

As two alternative options in a forced choice task are separated by design, two classes of computational models of decision-making have thrived independently in the literature for nearly five decades. While sequential sampling models (SSM) focus on response times and keypresses in binary decisions in experimental paradigms, dynamic neural fields (DNF) focus on continuous sensorimotor dimensions and tasks found in perception and robotics. Recent attempts have been made to address limitations in their application to other domains, but strong similarities and compatibility between prominent models from both classes were hardly considered. This article is an attempt at bridging the gap between these classes of models, and simultaneously between disciplines and paradigms relying on binary or continuous responses. A unifying formulation of representative SSM and DNF equations is proposed, varying the number of units which interact and compete to reach a decision. The embodiment of decisions is also considered by coupling cognitive and sensorimotor processes, enabling the model to generate decision trajectories at trial level. The resulting mechanistic model is therefore able to target different paradigms (forced choices or continuous response scales) and measures (final responses or dynamics). The validity of the model is assessed statistically by fitting empirical distributions obtained from human participants in moral decision-making mouse-tracking tasks, for which both dichotomous and nuanced responses are meaningful. Comparing equations at the theoretical level, and model parametrizations at the empirical level, the implications for psychological decision-making processes, as well as the fundamental assumptions and limitations of models and paradigms are discussed.

Keywords: decision-making, sequential sampling model, leaky competing accumulator, dynamic neural field, embodied decision, mouse-tracking

1. Introduction

There has been a long tradition to explain the dynamics of decision-making by relying on mathematical and computational models based on differential equations. Nevertheless, two major branches of the associated scientific literature were spawned from two different yet almost simultaneous origins: 1) cognitivist *sequential sampling models* (SSM) fitting behavioral data from cognitive psychology (Ratcliff, 1978); 2) *dynamic neural field models* (DNF) focusing on pattern formation and stabilization in biological neural networks (Amari, 1977). Although these seminal articles root in earlier research (e.g., LaBerge, 1962; Wilson and Cowan, 1973), they are at the core of associated theories that developed throughout the following decades, and many of the derived models provided insights about decision mechanisms (e.g., by representing the way information is integrated and leads to a final decision, or the way different factors intervene in the decision process). They are both as-

sociated to sound mathematical and theoretical frameworks, including optimal decision-making and Bayesian inference (Gold and Shadlen, 2002; Bogacz et al., 2006; Gepperth and Lefort, 2016). They also received extended support from neurophysiological findings, allowing mechanistic accounts of decision-making processes and therefore endowing them with predictive capabilities (Crockett, 2016; White et al., 2010; Wijeakumar et al., 2017).

Nevertheless, these two branches of research never really cross-fertilized, as they traditionally focused on decisions of different nature: a choice among qualitatively distinct alternatives in cognitive psychology experiments for SSM, a perception or action to emerge from a continuous topology in sensory or motor neural areas for DNF. Yet, their main difference lies in the discrete vs. continuous nature of the underlying decision space, and this paper is an attempt at bridging this long lasting gap. For this purpose, we will introduce a unifying formulation of the equations underlying SSM and DNF models, and focus on the consequences of parameter choices for decision-making dynamics. The resulting dynamics will be illustrated on moral decisions, a domain which is well-adapted to both binary categorization (e.g., good vs. bad) and continuous scales (e.g., how wrong). Finally, data generated by our full model enriched with sensorimotor capabilities will be fit to human

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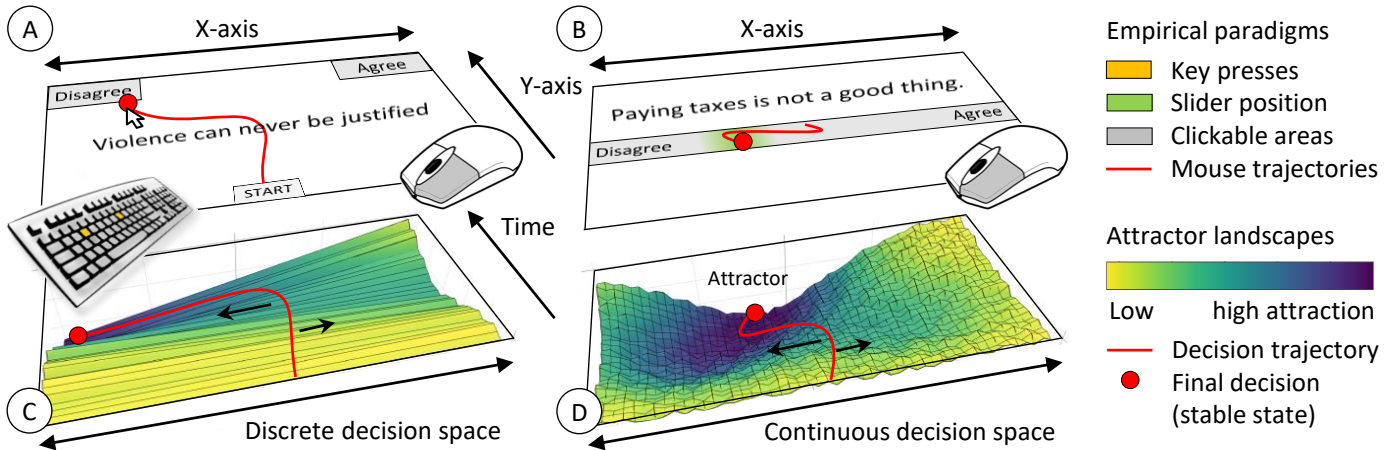[1]Institute of Engineering Univ. Grenoble Alpes

Figure 1: Illustration of decision-making in different empirical paradigms (*top*) and dynamical models (*bottom*). A) 2AFC paradigm where decision-making is either reflected by a key press corresponding to the responses displayed on top (final decision and response time only) or through mouse-tracking where the mouse starts in the bottom center area (*red trajectory* reflecting the decision process); B) Mouse-tracking enabled continuous response scale, with the mouse cursor hidden but horizontal movements starting from the center of the scale reflected by the *green slider* movements; C) Decision-making dynamics with only 2 options (here represented to the left and right of a dummy decision space). *Darker green* corresponds to lower energy and thus stronger attraction; D) With a continuous field dynamics, attractors may emerge anywhere on the decision space, with the decision trajectory following the steepest valley in the dynamical landscape. In all panels, starting from a neutral state (*yellow plateau* in panels C & D), the trajectory is first attracted to the right before converging to a response on the left side (*red dot*).

data collected with experimental paradigms that allow measuring the dynamics of decision-making, therefore also contributing to bridge another existing gap between generative models and empirical data (Stillman et al., 2018). To better understand the stakes associated to the different classes of decision-making models as well as their limits, we first turn to human decisions, focusing on experimental paradigms typically used in empirical sciences to study decision-making processes.

1.1. Empirical evidence and associated paradigms

Human decision-making has been extensively studied through two-alternative forced choice experimental paradigms (2AFC) in which the participant must choose between only two distinct alternatives (Bogacz et al., 2006; Ratcliff and McKoon, 2008; Smeding et al., 2016). In fact, such binary response mode could seem more appropriate to get a high correspondence between the operationalized binary construct and its presentation (Levine, 2001), considering the natural tendency of expressing in a dichotomous way, especially using language (with concepts such as *good* or *bad*, *close* or *far*, and so on). Notably, dichotomous thinking has been proven to be useful in human cognition for categorizing information and reducing world complexity (Berlin, 1990). Behind the apparent benefits of dichotomization, it may nevertheless distort perceptions and representations of a fundamentally continuous reality – where boundaries are often not clear cut – thereby putting constraints on the decision process. For example, studies on emotion perception showed that categorical judgment modifies perception and mental representations, in order to be consistent with the proposed categories (Satpute et al., 2016). Research indicates that the discrete or continuous nature of task properties – which include response mode – directly influences the decision-making process (Leong and Hensher, 2012; Master et al., 2012), for instance when relying on psychological constructs either defined by continuous dimensions or discrete categories (e.g., personality traits in Gonzalez et al., 2010).

Two or more alternative forced choices may not fit daily decision-making where judgments or choices call for nuance (e.g., choosing home decoration or how long to read a book) (Kvam, 2019; Master et al., 2012). Yet, 2AFC has also been a prominent paradigm in moral decision studies (Greene et al., 2001; Cushman, 2008; Gawronski et al., 2017), whereas moral decision processes are subject to rather diffuse influences and do not usually rely on a clear set of moral rules consistent across all situations (Kahane and Shackel, 2010). More specifically, we could expect binary response mode to extremize the decision process by amplifying the weight of some pieces of evidence in order to guarantee convergence towards one of the response alternatives; on the contrary, in absence of such constraint when using a continuous response mode, balancing evidence leads to a propensity for nuanced responses, a phenomenon for instance documented and coined *extremeness aversion* in consumer research (Neumann et al., 2016).

The possibility that small fluctuations during the decision process might get amplified and change the outcome could have great repercussions in the case of moral decisions (see figure 1 for an illustration of the dynamics of decision-making). Yet in this very domain, Pärnamets et al. (2015) tracked participants' gaze on a 2AFC task, and showed that simply forcing the dichotomous decision to be taken after fixating a given option was sufficient to increase its selection as the final decision. The interplay between participants' sensorimotor systems and the experimental paradigm must therefore be taken into account even when studying statements such as "Murder is sometimes justifiable".

From a dynamical and broader perspective, the embedding and relationships between continuous and discrete states, as well as their complementarity and presence at different

timescales, have been heavily reviewed in the past (Khona and Fiete, 2022; Schöner, 2016; Dale, 2008; Dale and Spivey, 2005; Bickhard and Richie, 1983; Pattee, 1974); this for instance explains how we turn continuous processes in the physical world into various forms of discrete (embodied) representations in the central nervous system. Nevertheless, this does not really address the question of how humans are able to seamlessly switch from continuous to discrete representations or behaviors, sometimes simply adjusting to minor changes in the way information is presented or behavior is constrained. One possible way is to study and model the dynamics of decision-making while directly manipulating the nature of the task (Gautheron et al., 2023).

Beyond final responses and response times, the dynamics of decision-making have been studied through tracing techniques, which include eye-tracking (e.g., in Pärnamets et al., 2015) but also mouse-tracking (Spivey and Dale, 2006; Freeman and Ambady, 2011). While eye-tracking informs on *epistemic* actions taken to collect information (in the sense of Kirsh and Maglio, 1994), mouse-tracking informs on *pragmatic* actions taken to literally reach a decision, and therefore provides complementary information (Quétard et al., 2016). By asking participants to move their mouse pointer from a fixed *START* position towards response areas on top of the screen instead of simply pressing a button when a final decision has been reached (see figure 1A), mouse-tracking provides a *window into real-time cognition* (Spivey and Dale, 2006). Specifically, mouse movements reveal underlying decision-making processes (e.g., hesitations, online preferences towards alternative responses) through deviations of the mouse trajectory or direction changes (Freeman and Ambady, 2011; Hehman et al., 2015). This is the plausible consequence of the parallel activation and competition between alternative targets, an hypothesis that has also received neuroscientific support (e.g., Cisek and Kalaska, 2005, for reaching). High temporal and spatial precisions make it possible to map decision processes onto mouse trajectories, and study early effects as well as fine-grained differences even in the social (Smeding et al., 2016) or moral domains (Koop, 2013). In the moral domain, similar results were found using either a standard 2D mouse-tracking paradigm or a 1D slider on which the participant's preference is traced throughout the decision (Gautheron et al., 2024, 2023; Gaboriaud et al., 2022).

1.2. Neurocomputational models of decision-making

Dynamic competition between alternative responses –as traced by the mouse-tracking technique in 2AFC paradigms– is traditionally well described by *Sequential Sampling Models* (SSM) (Bogacz et al., 2006) which have been integrated in theories of decision-making such as the *decision field theory* (Busemeyer and Diederich, 2002). Within this class of SSM, the prominent *Drift Diffusion Model* (DDM) represents the decision process as an accumulation of evidence toward one or the other alternative, triggering a decision when a given threshold is reached (Ratcliff and McKoon, 2008). This model has been successfully used to account for psychological results, often focusing on response times and accuracy (Ratcliff and Tuer-

Table 1: Terminology employed in the different branches of the scientific literature on dynamic decision-making (retained terms for the proposed unified model in bold), depending on the discrete or continuous nature of space and time in the models as well as the neurological or functional inspiration adopted.

Notation	Associated terms
———	**unit**, neuron, column, node, accumulator
u	**activity**, (mean field) potential, evidence, charge
$?_i$ / $?(x)$	**coordinates**, location, index
$f(...)$	**nonlinearity**, activation function
k	**decay**, leak, relaxation rate
e / s	**input**, external stimulation, slope, drift
w	**weights**, kernel, mutual (inhibitory) influence
$\xi \rightarrow W$	**noise**, random walk, Wiener / diffusion process
τ	**time constant**, time scale
h	**resting level**, resting potential
———	**attractor**, bump, peak, decision

linckx, 2002; Ditterich, 2006; Metin et al., 2013; Drugowitsch et al., 2012; Krajbich et al., 2015; Krypotos et al., 2015).

The principle idea of the DDM was enriched in the *Leaky Competing Accumulator model* (LCA) to better account for neurophysiological observations (Usher and McClelland, 2001). Although a myriad of sequential sampling models derived from the DDM have been elaborated, further specifying the decision dynamics (Bogacz et al., 2006; Ratcliff and Smith, 2004), we here opt to rely on the nonlinear-LCA equation (adapted from equation 2 in Usher and McClelland, 2001), as they will be easier to relate to other classes of models:

$$du_i = \left(e_i - ku_i - \sum_j w_{ij} f(u_j) \right) \frac{dt}{\tau} + \xi_i \sqrt{\frac{dt}{\tau}} \qquad (1)$$

Given the fluctuating notations found in the literature, renaming was performed to adopt a consistent terminology throughout this paper, and a glossary of existing denominations is provided in table 1. In equation (1), time (t) is an implicit parameter of all terms. i and j index the set of units associated to responses, usually of size 2 for models of two-alternative forced choice tasks ($i \in \{1, 2\}$). u_i corresponds to the activation of unit i which is the quantity the model aims at simulating, with the set of units being mapped to the set of response alternatives. du_i corresponds to the increase or decrease of activity u_i over dt. τ is the time constant controlling the reactivity of the unit; combined with the decay rate k associated to the leak term, it determines how fast the unit can relax to its resting level (here $u = 0$) in absence of any input, and how fast it reacts to changes in external stimulation. Given its redundancy with other parameters in the context of this paper, the decay rate will not be kept as a free parameter ($k = 1$). More generally, these two parameters are important to model temporal dynamics (e.g., response times) and deal with stimulation or environment volatility. While w_{ij} corresponds to self-excitation when $i=j$, it determines the mutual inhibition between units when $i \neq j$, i.e. how much the activation of one unit reduces the activity of the other. . e_i is a deterministic external input to unit i (e.g., evidence from current observations) which corresponds to the *drift* component

in DDM models. ξ_i is a Gaussian noise term with zero mean and variance σ^2, which corresponds to the stochastic *diffusion* component. The diffusion processes are also considered independent across units, which holds for all models introduced in the following sections. Finally, f is a nonlinear function, usually the rectified linear unit function (ReLU) which will be used throughout this article. As its presence drastically complexifies the mathematical analysis of the system, linearized versions of the equation have often been implemented by simply replacing $f(u_j)$ by u_j in equation (1). Since the lack of nonlinearity may lead to problematic and physiologically implausible positive feedback from negative activation (when multiplied by negative weights), the nonlinearity may be reintroduced outside the equation to guarantee activity u remains positive at all times (equation 4 in Usher and McClelland, 2001):

$$u_i \rightarrow f(u_i) = max(u_i, 0) \qquad (2)$$

Equation (1) is a stochastic differential equation, and the dt/τ scaling term is associated to integrating the Gaussian noise $\xi(t)$ into a Wiener process ($dW_i = \xi_i \sqrt{dt/\tau}$). When turning to numerical approximation, a Euler-Murayama integration scheme is generally used. The decision-making process is simulated using a discrete time step Δt, until activity u_i increases beyond u_{thr}, which triggers the response associated to the unit that reaches the threshold. By adjusting the time constant (τ), the amplitude of the drift and diffusion components (e and ξ) as well as the threshold, it is possible to simulate decisions with more or less time pressure and confidence in the selected response.

Noteworthily, when adopting the ReLU nonlinearity, a bounded nonlinearity (e.g., sigmoid or arctangent), or even when carefully choosing parameter values in the absence of nonlinearity, it is possible to prevent any divergence of the activity for bounded external input (in $[0, u_{max}]$). As a consequence, when guaranteeing a strong enough inhibition between units ($w_{ij} << 0$ with $i \neq j$) and in presence of sufficient evidence for the alternative responses ($e >> 0$), the system of equations and associated activities (u_1, u_2) will converge to one of the fixed-point attractors (u_{max}, ϵ) or (ϵ, u_{max}) respectively corresponding to the selection of response 1 or 2 (with $\epsilon \approx 0$). This reduces the importance of the activity threshold for triggering the response, as the activity will stabilize on the final decision and possibly even be maintained in absence of further evidence.

This dynamical system perspective on the differential equations underlying decision-making is not commonly adopted in SSM research, but is prominent in the *Dynamic Neural Field* (DNF) literature. Comparing SSM and DNF models from a psychometric perspective, their main difference lies in the underlying decision space. Indeed, SSM rely on a discrete set of units mapped to alternative responses, while the DNF models lie at the other end of the continuity spectrum, hence the *continuous field* (Amari, 1977), implemented by a theoretically infinite set of topologically organized units (e.g., cortical columns in the brain). DNF have been classically favored to model sensorimotor behaviors, for instance modeling covert (Rougier

and Vitay, 2006; Quinton and Girau, 2011) and overt visual attention as well as memory (Fix et al., 2011; Grieben et al., 2020; Carroll et al., 2014), perceptual estimation (Esnaola-Acebes et al., 2022), motor planning (Erlhagen and Schöner, 2002; Schutte and Spencer, 2007), visuo-oculomotor behaviors (Quinton and Goffart, 2018) and applying them to robotics (Erlhagen and Bicho, 2006; Schöner, 2016). The description of percepts, their location in space, their dynamics as well as motor commands all naturally fit continuous topologies. Nevertheless and through the years, DNF models have extended to higher level cognition (Schöner, 2020) and therefore integrated in computer vision (Maggiani et al., 2018) and cognitive robotics systems (Bicho et al., 2010).
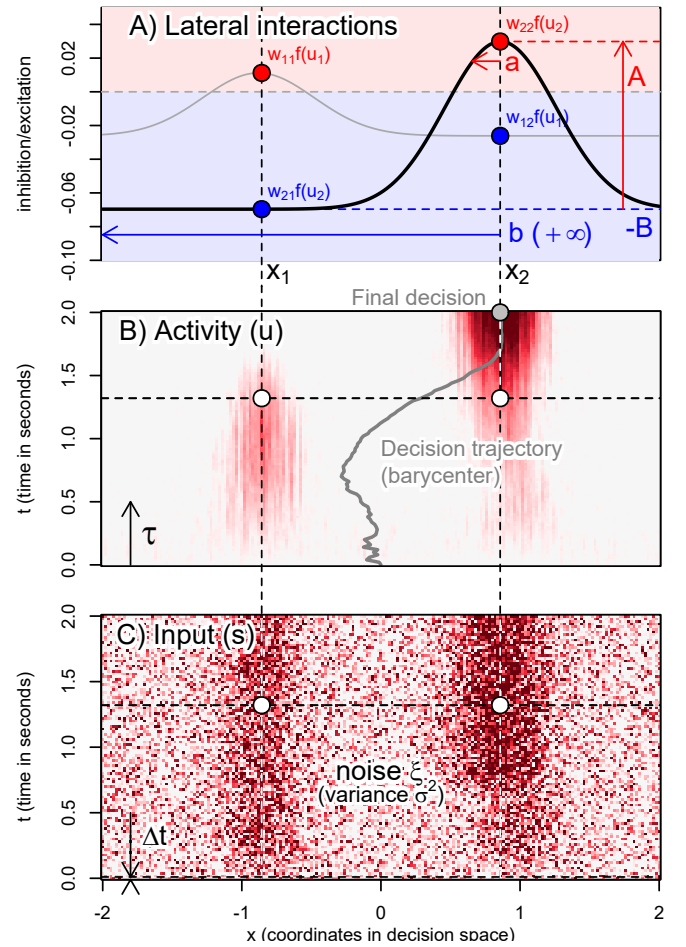

Figure 2: Representation of dynamic decision-making simulation and model parameters $(A, B, a, b, \tau, \Delta t, \sigma)$ using a near-infinite set of units for a DNF model (*curves* in panel A, *fields* in B and C) or with only 2 units for an LCA model (*dots* and *dashed vertical lines*). A) Lateral interactions $w(x, t)f(u(x, t))$ for units at x_1 and x_2 (*blue* for inhibitory, *red* for excitatory) resulting from weighted connections between units and units activity at time t (*dashed horizontal line* in panels B and C); B) Output activity $u(x, t)$ of the model as a function of space (*horizontal axis*) and time (*vertical axis*), summarized as the trajectory of the activity barycenter (*grey*); C) Noisy input dynamics $s(x, t)$ with evidence appearing at different positions and times. After an initial attraction towards the *left* (discrete option or part of the continuous decision space), the increased evidence on the *right* of the spatial dimension leads to a change in decision.

Still modeling a continuous decision process unfolding in

time, decisions are associated to *dynamic instabilities* and to the emergence of macroscopic, spatially structured and stereotyped *bumps* of activity (see figure 2B and Schöner, 2016). In presence of static stimuli and given appropriate parameter values, these bumps correspond to attractors of the associated dynamical system. Similarly to binary-scale SSM, information present in the external stimulation is integrated through time and across units, but unlike SSM, DNF rely on population coding to represent the decision (equations being usually discretized in time and space for numerical simulation). A standard one-dimensional and one-layer DNF is governed by the following (stochastic) integro-differential equation:

$$\tau \frac{\partial u(x)}{\partial t} = -u(x) + \int_X w(x - x') f(u(x')) dx' + s(x) + h \quad (3)$$

where $u(x) = u(x, t)$ is the activity at location x and time t (not indicated in the above equation as a parameter to all terms). $s(x)$ is the external stimulation at location x, possibly combining deterministic and stochastic components : $s(x) = e(x) + \xi(x)$ (see figure 2C). As a consequence, X is the 1D topology over which the external stimulation is mapped and the decision is taken. Most parameters have similar interpretations to those in equation (1), for instance f being the equivalent of the nonlinear activation function found in typical connectionist models, which prevents negative outputs and determines when units exchange information (e.g., when choosing a sigmoid rather than ReLU function). Although important to control the nonlinearity and bifurcations in the dynamical system, the resting level h can be either integrated to the stimulation s or to the activation function f, and is therefore removed in the following equations for simplicity (equivalent to $h = 0$). Yet, given the continuous nature of topology X, lateral connectivity weights w are not constant (leading to a fixed inhibition between units as in equation (1)), but defined as a function of the distance between units on X. The following Mexican hat kernel function is often used:

$$w(\Delta x) = A e^{-(\Delta x/2a)^2} - B e^{-(\Delta x/2b)^2} \quad (4)$$

The first Gaussian term allows nearby units to excite each other, leading to the emergence of spatially coherent bumps of activity. Meanwhile, the second Gaussian term controls the inhibition between distant units. Parameters A and B respectively control the intensity of the excitation and inhibition, while parameters a and b control their range. Aiming at a unique localized final decision, as it is the case in this article, guaranteeing attractors with a single bump of activity is achieved by setting b to $+\inf$, thus turning the second term to a fixed global inhibition $-B$ (similar to the inhibitory weight in SSM). Lateral weights are illustrated on figure 2A.

1.3. Sensorimotor models of decision-making

A final class of models of interest in the context of this article rather focuses on how sensorimotor interactions with data collection paradigms leads to decision trajectories that can be empirically measured. Mirroring mouse-tracking or eye-tracking techniques, these models aim at generating (realistic) mouse movements (Lepora and Pezzulo, 2015; Falandays et al., 2021),

eye movements (Quinton and Goffart, 2018) or both (Quinton et al., 2014). Even when considering the decision-making process has converged to a final decision before any physical action is implemented, time is required to initiate action and reach the target position (e.g., on the response scale); depending on the refinement of the process generating sensorimotor trajectories, it may integrate stochastic components due to motor variability or friction. Complex dynamics which are found in human trajectories can be learned and integrated in simulated controllers (Sigaud et al., 2011), but are usually not necessary to correctly approximate trajectories observed in experimental paradigms. At the core of most models used to simulate mouse trajectories lies an unbiased linear feedback control system, which is the simplest form of error-based controller (Bequette, 2003):

$$\frac{d\boldsymbol{p}}{dt} = -\kappa(\boldsymbol{p} - \boldsymbol{p_{tgt}}) \quad (5)$$

where $\boldsymbol{p} = (x, y)$ represents the current coordinates of the mouse pointer, and $\boldsymbol{p_{tgt}}$ the target coordinates on screen. The metaphor of a spring can be used to illustrate the dynamics (see figure 4, although the simulated physics does not match the spring dynamics): the further the mouse pointer is from the target coordinates the greater will be the attraction. The strength of attraction is also controlled by the proportional gain κ (equivalent to the stiffness for a spring). In addition to $\boldsymbol{p}$ depending on time, $\boldsymbol{p_{tgt}}$ may be updated during the decision process, complexifying the trajectories by simulating hesitations and changes of mind before converging towards a stable response. The same holds for κ, which may be used to either normalize the position error ($\boldsymbol{p} - \boldsymbol{p_{tgt}}$) into a unit vector to be scaled by a speed constant (i.e., only changing direction to fit spatial characteristics of mouse trajectories as in Lepora and Pezzulo, 2015) or to reflect the certainty in the current decision during the decision by increasing the speed (i.e., controlling the acceleration to also fit the dynamics as in Smeding et al., 2016).

While all derived models are similar in essence, relying on embodied frameworks of decision-making, as well as classical mechanics and control theory to simulate forces and movements, they nevertheless focus on different aspects of sensorimotor interactions. For instance, Lepora and Pezzulo (2015) study how the parallel unfolding of decision and sensorimotor processes influences mouse trajectories (e.g., simulating commitment to a response through the positive feedback of observing self-generated motion towards the target position). Quinton et al. (2014) rather focus on how the sensorimotor apparatus and active perception mechanisms impact decision-making, while Quinton and Smeding (2015) compare empirically observed and simulated mouse trajectories depending on the level at which competition between alternative responses occurs (sensorimotor, conceptual or both). These all correspond to shallower or deeper forms of embodiment, depending on whether the sensorimotor components are direct constituents or exert an indirect influence on the decision-making process. The current article contributes to this line of research by focusing on the influence of constraints imposed on human-machine interactions (e.g., computer mouse and experimental paradigm).

1.4. Covering the wide spectrum of decision-making models

Beyond their differences in origins, aims and associated paradigms, research works are found at the interface between the previously introduced classes of models. Even if most SSM have been used through the experimental paradigm of 2AFC (Ratcliff and McKoon, 2008; Bogacz et al., 2006; Smeding et al., 2016), they have been adapted to sets of discrete alternative options (Churchland et al., 2008; Krajbich and Rangel, 2011; Roxin, 2019) and to continuous scales, either approximating them using a vector of mutually inhibiting accumulators (Ratcliff, 2018) or by progressively refining the response location by accumulating evidence on a single unbounded response dimension while also modeling decision trajectories (e.g., price estimate in behavioral economics, perceived color or orientation in cognitive psychology, as in Kvam, 2019; Kvam and Busemeyer, 2020). Reversely, there have been successful attempts at applying DNF to discrete forced choice decisions (e.g., perceptual classification and motor primitives in Erlhagen and Bicho, 2006; Schutte and Spencer, 2007), sometimes relying on mouse-tracking dynamics (e.g., studying spatial relationships in Lins and Schöner, 2019) or even modeling mouse trajectories (Buss et al., 2011) and neuroimaging data (Wijeakumar et al., 2017).

While DNF models are numerically approximated using a finite set of units, usually under the form of a lattice, but also non regular topologies or adaptive sets (e.g., using Gaussian mixtures in Quinton et al., 2011), the assumed underlying topologies generally remain continuous, with a focus on sensorimotor maps and visual tasks. While such implementations of dynamic neural field dynamics maintain some of their properties in terms of selection and fusion of information (Forest et al., 2022a), assumptions on the nature of the underlying measures can be relaxed. Between SSM and DNF models, there is also a large class of models which relies on a graph of nodes, each node roughly corresponding to a (nonlinear) accumulator (or unit), the topology being this time implicitly defined by the connections between the nodes. This for instance led to the development of the *dynamic interactive theory of person construal* (Freeman and Ambady, 2011), which is at the core of the mouse-tracking developments in situated social cognition (Freeman et al., 2011; Smeding et al., 2016). A dual approach to altering the units spatial distribution is to deviate from standard isotropic and stationary assumptions, either through shifted projections (Quinton and Girau, 2011; Quinton and Goffart, 2018) or heterogenous time or location-dependent kernels (to make attractors appear at or drift towards specific parts of the field; Kilpatrick et al., 2013; Cerda and Girau, 2010). Further relaxing topological constraints (e.g., using Hopfield networks; Catenacci Volpi et al., 2014; Khona and Fiete, 2022) still allows the emergence of discrete attractor states from continuous input states, but associated networks may lack the explainability of SSM and DNF models. Instead of learning or altering underlying topologies, it is also possible to dynamically constrain attractors through preshaping (i.e., pre-activating certain areas of the neural field; Erlhagen and Schöner, 2002; Schutte and Spencer, 2007). Such extensions for both SSM and DNF models require additional degrees of freedom, and do not

necessarily guarantee the emergence of attractors under all conditions (e.g., when combining nuanced evidence with a binary response mode); we thus chose in this article to focus on core models from both literature to ease the transition and comparability, at the cost of improved capabilities.

Since compatible with a dynamic, embodied and situated approach to decision-making, models of sensorimotor interactions in experimental paradigms have been coupled with SSM (e.g., Lepora and Pezzulo, 2015; Falandays et al., 2021, for mouse movements), DNF (e.g., Quinton and Goffart, 2018, for eye movements) or intermediate graph-based models (Quinton et al., 2014; Smeding et al., 2016). Variability and stochasticity are then already embedded in the output activities of the SSM / DNF models, used as input to a sensorimotor module, in order the generate and update the target position as the decision process unfolds. Furthermore, the numerical integration of the stochastic component of the activity into the sensorimotor behavior acts as a filter, producing smooth trajectories from Wiener-like processes from equations (1) or (3). Already, the time constant τ controls how smoothly information present in the input (e) is integrated relatively to the noise (η). Whatever the combination of models and parameters, smooth trajectories as well as stable and discrete decisions are emerging from continuous and non-linear cognitive dynamics, with continuity present in both time and space (either on continuous response scales or through mouse trajectories; Spivey and Dale, 2004).

Most of the previous models also assume shared neurocircuitry for behaviors that may appear as qualitatively different, but may simply be qualified by different attractors in an underlying dynamical system; this may be the case for reaching on a discrete vs. continuous response scales (Krajbich and Rangel, 2011; Schutte and Spencer, 2007) or performing saccades vs. smooth pursuit eye movements (Quinton and Goffart, 2018; Goffart et al., 2018). Given their theoretical and practical compatibility, this article therefore aims at bridging the gap between these approaches to decision-making modeling, instead of simply opting for the class of model which best suits the needs of the task at hand (which does not necessarily imply that a single unifying model must cover all situations Dale, 2008). We therefore harness the respective strengths of SSM and DNF literatures, integrating them under a unifying formulation, while adding a sensorimotor component able to (re)produce mouse trajectories in both binary and continuous response modes (section 2). As a proof of concept and to test its validity, the proposed model is fit to empirical human mouse-tracking data from moral decision-making, a domain where hesitations are commonplace and where both extreme and nuanced responses are meaningful when judging moral assertions (section 3). To our knowledge, very few computational models have been applied in the moral psychology field with such high-level and complex stimuli (Dehghani et al., 2008; van Baar et al., 2019; Wallach et al., 2010; Yu et al., 2019), even fewer based on SSM (Falandays et al., 2021, using variations on the historical DDM). To demonstrate the flexibility of the proposed model, we simulate mouse-tracking data collected with different paradigms and response modes, then interpret and discuss model parametrizations.

As an overview of the article, the objectives of the proposed model are to: 1) bridge the gap at the theoretical level (hence also terminological) between the SSM and DNF models, as well as their associated scientific literatures; 2) account for empirical data and complex decision-making dynamics, applying the model to mouse-tracking trajectories collected during human judgment of moral assertions; 3) account for differences and generate decision-making dynamics with both binary and continuous response modes; 4) account for differences and generate continuous mouse trajectories in 1D or 2D paradigms; 5) propose explanations of the impact of paradigm constraints on the underlying decision-making mechanisms through the interpretation of fitted model parameters.

2. Model

At the core of the proposed model, we put a decision-making module which is equivalent to a LCA in the SSM class of models when only 2 units are stimulated and their activity compete for taking the final decision, but becomes equivalent to a 1D DNF when the number units increases towards infinity. Therefore, a single parameter can be used to reflect the continuous or discrete nature of the response space into the decision dynamics. Yet, information from possibly complex stimuli (e.g., written moral assertions) need to be transformed to be processed by this decision module. In turn, its outputs also need to be converted into mouse movements to interact with the empirical paradigm [e.g., reaching and clicking response alternatives, as illustrated on figure 1.

2.1. Underlying topologies

To specify design choices for the proposed model and to understand how simulated dynamics may fit human decision-making behaviors, we beforehand need to define the different topologies which are involved in the model and empirical tasks. Information indeed propagates between the computer screen (mouse pointer and input stimuli), the input device (mouse or keyboard) and the decision-making system (human or model, with their cognitive and sensorimotor components), therefore circulating across the physical world and cognitive representations of the task (as illustrated on figure 3 and 4). Since complex topologies might be needed and have been used in other works, but simple ones will be used in this article, the term *space* will be used to refer to any potential geometrical space or topology.

Stimuli space. In most sensorimotor cases, properties of interest of stimuli can be mapped to neural populations which approximate continuity (e.g., orientation, color, location in Schöner, 2016). Yet even when considering small discrete sets of units, continuity in the spatial dimension is turned into continuity of units activation by projecting input signals onto units through receptive fields (considering the barycenter of units weighted by their activity). Although not the focus of this article, it is often also possible to activate sets of units to map more abstract properties of stimuli (e.g., semantic content of linguistic stimuli, spatial relationships in Lins and Schöner,
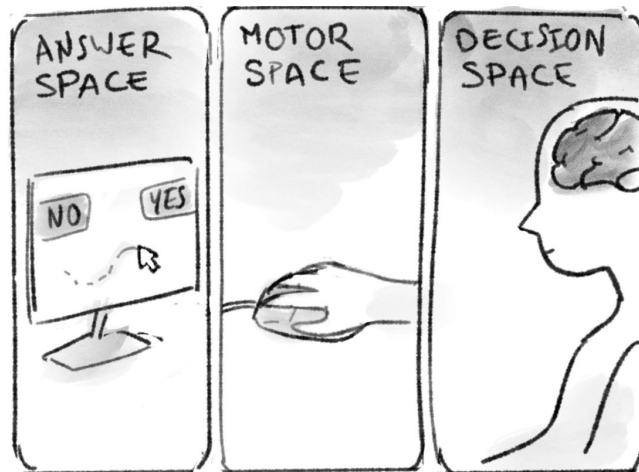

Figure 3: Representation of the different topological spaces involved during decision-making tasks on a computer using a mouse-tracking paradigm. A human participant perceives stimuli (not represented) displayed on the screen and takes a decision while controlling the mouse pointer through her hand movements, with both the decision dynamics and hand movements constrained by the available response alternatives (e.g., binary or continuous response mode).

2019). When the decision-making process operates on a single dimension, input stimuli are usually also projected on a one-dimensional topological space.

Decision space. This is where the cognitive component of the decision process is realized, for instance relying on neural dynamics. As developed in section 1, SSM (resp. DNF) assume an underlying set (resp. field) over which competition for the final decision occurs. This ranges from a set of 2 units associated to 2 alternative responses in 2AFC tasks for most SSM (Bogacz et al., 2006) to higher-dimensional manifolds for DNF operating on multidimensional stimuli (e.g., Schöner, 2016; Quinton et al., 2011). In most numerical implementations, the assumed underlying continuous manifold is approximated as a regular lattice, which therefore translates into a vector of units in the 1D case. As a consequence, both for binary and continuous response modes, the computational model manipulates a discrete set of units, usually of size 2 in equation (1) or sufficiently high to approximate infinity when numerically solving equation (3). Both sets give rise to topological spaces and we will consider the segment on X ranging from -1 to 1, locations which map extreme opposite decisions..

Motor space. This corresponds to the sensorimotor component of the decision process (e.g., by moving the hand to click on a chosen response). Precisely mimicking human hand movements on a decision task in principle requires simulating neural, muscular and physical dynamics, including inertia, elasticity and different types of friction forces. Yet for mouse movements where visual servoing can be used to control the cursor, much simpler models can be adopted as human efficiently adapts to complex dynamics. Direct and inverse models can for instance be combined to reliably reach targets (McNamee and Wolpert, 2019) despite variations in size, required precision or time constraints (here reaching towards a changing target re-

sponse). Since motor control is not the focus of the current article, and motor planning is still poorly understood at a computational level, with ongoing debates on the nature of internal representations and models (McNamee and Wolpert, 2019; Goffart et al., 2018), the basic linear feedback controller from equation (5) is adopted.

Answer space. This space is specific to the chosen empirical paradigm and maps response alternatives. In our model and simulations, we relied on either a 1D or 2D space. The 2D space corresponds to the classical MT paradigm, where participants clicks at the bottom of the screen before reaching a response at the top of the screen as illustrated on figure 1A. The alternative slider paradigm with a 1D answer space is represented on figure 1B, where movements are only effective horizontally. For both paradigms (classical MT and slider), the answer space can be adapted to binary or continuous response modes, with distinct opposite responses areas in the binary case (as illustrated on figure 1A) or a continuous response bar in the continuous case (as illustrated on figure 1B). In all cases, the two most extreme answers are located on the left and right sides of the screen.

Coupling of all spaces. Information flows between those spaces as the decision unfolds, with the participant simultaneously processing the stimulus, accumulating evidence for potential responses and controlling the mouse on screen, constrained by the human-machine interactions and response areas configuration. As a general principle, all spaces can be projected onto a single dimension corresponding to the final answer space, going from one extreme response alternative (e.g., *Disagree* on figure 1) to the other (e.g., *Agree*). In the case of a SSM model, the 2 units directly map to the extreme decisions, while in the case of a DNF, the field maps the continuum between them. Basically, activity built on the left half of the field is evidence for responses associated to the left of the screen, which in turn trigger movements towards the left. Yet transformations to align the different spaces might be required when propagating information. For instance, a person could think that torture is sometimes acceptable while confronted to binary options such as *never acceptable* and *always acceptable* which would not correspond to her spontaneous nuanced opinion, forcing a dichotomization of her response. On the contrary, someone deeply convinced that torture never is acceptable would saturate a scale going from *usually not acceptable* to *always acceptable*, her deontological decision not being reflected by the permitted response range.

Such issues need to be accounted for in the model, especially when switching between binary and continuous response modes, while limiting the number of model parameters to prevent overfitting. These conceptual issues stack with boundary problems encountered with DNF models; indeed, when approaching the sides of the simulated discretized space, asymmetries appear as the excitatory neighborhood reduces relatively to the global inhibition. This results in an absence of extreme answers which does not reflect the psychological reality. The usual and simplest technical solution to boundary problems is

to rely on a circular topology (Schöner, 2016), but connecting and equating the right and left ends of the response scale would be meaningless in the current context. Hence, we rather relied on the idea of a decision space as border-free as neurobiological space (Huth et al., 2012, and see top of figure 4), extending it beyond the paradigm answer space (from $x \in [-1, 1]$ to $[-2, 2]$).

2.2. Cognitive part - simulation of decision dynamics

At the core of the decision-making part of the model, we adopt a discrete formulation compatible with both SSM and DNF models. We hence propose to define the activity of a vector of units $\boldsymbol{u} = (u_i)_{i \in [\![1,n]\!]}$ as follows:

$$\tau \frac{du_i}{dt} = -u_i + \frac{2}{n} \sum_j w_{ij} f(u_j) + s_i \qquad (6)$$

where the different terms fit earlier descriptions provided in section 1.2. When only considering 2 units as in the LCA model ($n = 2$), a factoring of constant k and reorganization of terms lead to a merged version of equations (1) and (2). As the lateral kernel function is usually symmetric and in the special case of only 2 units, weight $w_{12} = w_{21} = -w$ from equation (1) (also see points on figure 2A). On the contrary, as n increases towards infinity, the discretized version approximates a DNF dynamics over a continuous decision space X from equation (3); $x \in [-1, 1]$ is approximated by indices $i \in [\![1, n]\!]$ associated to a regular 1D-lattice on the decision space with $x_i = 2(i - 1)/(n - 1) - 1$. The same applies when relying on an extended decision space to allow extreme decisions, simply mapping the extended interval with n units (e.g., $[-2, 2]$ as illustrated on figure 4). The mutual influence between units is normalized (scaled down by $n/2$), which corresponds to a spatial discretization step $\Delta x'$ substituted to dx' from equation (3) when approximating the neural continuum by a vector of units (integration space $[-1, 1]$ partitioned in n sub-intervals). As the number of units grows ($n \to \infty$), $\Delta x'$ becomes smaller, its impact on the dynamics reduces, simply improving the Riemann sum approximation of the DNF integral. It also guarantees the equivalence with equation (1) (since $n/2 = 1$ for a LCA model) and makes parameter values comparable between model instances relying on different numbers of units. Alternatively, to keep the equation closer to a classical neural network formulation, the scaling factor could be integrated in parameters A and B which determine weights amplitude.

For efficient numerical resolution, equation (6) is temporally discretized using Euler-Murayama integration scheme, and the deterministic (e_i) and stochastic parts (dW_i) of input (s_i) can be separated and vectorized. Also, using stationary weights as defined in equation (4), the entire system of equations can be written using matrix algebra with the product of vector $\boldsymbol{u}$ with Toeplitz matrix $\boldsymbol{w} = (w_{ij})$ with $(i, j) \in [\![0, n]\!]^2$ implementing a discrete convolution. Finally, to guarantee activity remains numerically bounded despite the integration scheme (e.g., kept positive to represent probability-like distributions), non-linearity f is applied to the activity vector as a final operator, approximating the nonlinear dynamics through linearization followed by truncation, therefore leading to the following

8

algorithmic formulation:

$$\boldsymbol{u} = f\left(\boldsymbol{u} + \frac{\Delta t}{\tau}\left(-\boldsymbol{u} + \frac{2}{n}\boldsymbol{w}\boldsymbol{u} + \boldsymbol{e}\right) + \boldsymbol{\Delta W}\right) \qquad (7)$$

Once input at time t has been projected onto the decision space, evidence for alternative responses is accumulated by building up activity at corresponding locations. The activation of each unit therefore depends on the amount of evidence in the input for this specific location, but also on its interactions with other units. For an increasing number of units (n), evidence can therefore be considered as accumulating more for a region of the decision space rather than a specific point or response alternative. The model then shifts from qualitative to quantitative decisions-making (from binary to continuous response mode), in both cases representing the final decision as the attractor on the underlying decision space. The inner nonlinearity f in equation (6) allows the network to act as a linear integrator when u remains below threshold (e.g., if threshold increased above 0 or when integrating a negative resting potential h into the stimulation s), with lateral interactions only becoming effective above threshold. While this is a behavior oftentimes expected in DNF models, the outer nonlinearity in (7) derived from SSM linearized and truncated formulations generates lateral interactions at all times. For a nonlinearity f which prevents negative activity (e.g., ReLU), the summation under the excitatory part of the kernel added to the input must compensate the inhibitory influence from further units for u to reach strictly positive values. In presence of noise, the convergence to an attractor reflecting a decision therefore requires a sufficiently spatiotemporally coherent and strong stimulation. While this behavior was sufficient for the tasks at hand in the current article, the nonlinearity should be integrated to guarantee other documented DNF-like regimes are attainable.

The non deterministic part of the decision process modeled by the Wiener process (integration of $\boldsymbol{dW}$ over time) not exhaustively encompasses noise when processing stimuli (e.g., considering $\Delta t/\tau \times \boldsymbol{e} + \boldsymbol{\Delta W}$) as well as noise in the neural system and decision process (akin to the diffusion component of SSM).

2.3. Sensorimotor part - simulation of mouse trajectories

When the decision model is simulated and converges to an attractor, the decision is represented as a distribution of activity over the vector of units. Intuitively, this can be paralleled to a probability distribution over the decision space, i.e. of giving the corresponding response if made possible given paradigm constraints. Yet for complications and limitations of direct probabilistic interpretations of DNF activity without taking accounts its dynamics, please refer to Gepperth and Lefort (2016).

While the decision may be represented in a distributed manner in the cognitive part of the model, pressing a key or controlling the mouse on the screen require generating a single physical command at any time. For this reason, and given the stereotyped attractors of the dynamical system (as a single symmetric bump of activity), a barycenter of the DNF activity equivalent to a maximum aposteriori estimate on probability distributions

is computed (unit coordinates x_i in the decision space weighted by their activity u_i; Rougier and Vitay, 2006). Selecting the location associated to the maximal activity instead would generate more noise at the begining of the decision trajectories, but could be filtered out through motor planning and control (see figure 5). Both maximum (threshold-based or parametric) and barycenter (summation-based or population-coding) approaches have received supporting empirical evidence (Goffart et al., 2018; Cisek and Kalaska, 2005) and our objective was not to discriminate between motor projections or control schemes in this article. In all cases, this process reduces the activity vector $\boldsymbol{u}(t)$ to a scalar value at each time step, then mapped to a $x_{tgt}(t)$ coordinate in the answer space:

$$x_{tgt} = \frac{\sum u_i x_i}{\sum u_i} \qquad (8)$$

This value corresponds to the current preference in the answer space, given accumulated evidence and convergence of the cognitive part of the model. As additional parameters for projecting the decision space to the answer space were not required for the current application of the model, we simply assumed here that both map the $[-1, 1]$ interval. For simulating trajectories on a standard mouse-tracking paradigm, a 2D position must be specified, simply combining the result from equation (8) with the constant Y-axis coordinate where the answer areas lie on screen: $\boldsymbol{p_{tgt}} = (x_{tgt}, y_{tgt})$. This target position can then be injected in the standard error-based controller described by equation (5), attracting the mouse pointer in order to reach a match between the current mouse position and the targeted answer. The time step Δt to simulate mouse trajectories is not a free parameter of the model, since only used in the computational implementation for the temporal discretization of a theoretically continuous process. As a consequence, and since chosen small enough to correctly approximate the nonlinear dynamics of DNF in equation (7), the same parameter value is used in equation (5) ($\Delta t = 0.005$ seconds).

The proportional gain κ makes it possible to control the speed at which the mouse pointer is attracted by the target position, with an adaptative gain introduced as follows:

$$\kappa = \frac{\lambda}{n} \sum u_i \qquad (9)$$

Complex trajectories may emerge as κ, $\boldsymbol{p_{tgt}}$ and $\boldsymbol{p}$ evolve through time until convergence of both the cognitive and sensorimotor parts of the model (both illustrated on figure 4). κ is also directly proportional to the total activation of all units in order to model the acceleration of mouse movements, since the higher the activation, the stronger the preference and the lower the uncertainty for the current decision (location in decision space). Following equation (9), λ is a scaling constant used to adjust the maximal speed that can be attained by the simulated mouse pointer, assuming the activity vector $\boldsymbol{u}$ is bounded (guaranteed within reasonable parameter value ranges in equation (6) and (7)). Normalized time is generally used to study decision dynamics through trajectories (e.g., for mouse-tracking Hehman et al., 2015; Freeman and Ambady, 2010), λ is therefore not included in the set of free parameters of the model, as the time
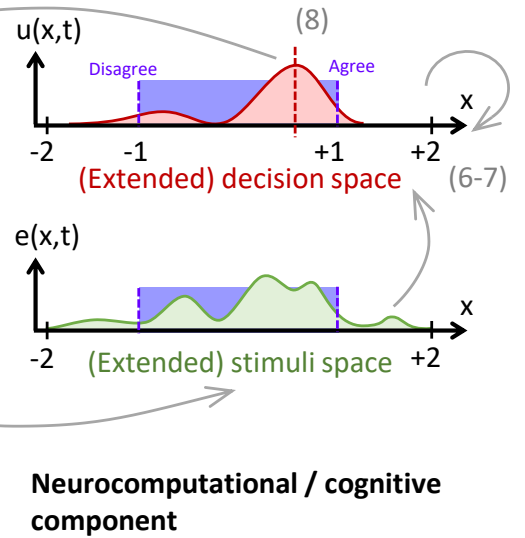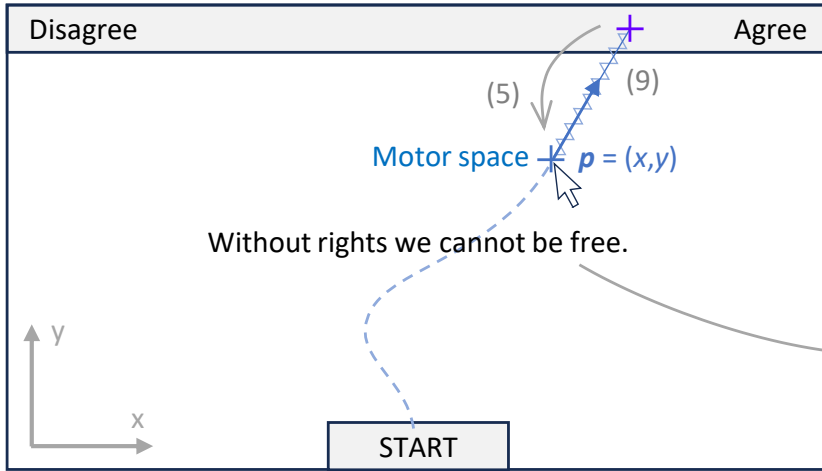
Figure 4: Components of the decision-making model coupling cognitive processing (*right*) and sensorimotor interactions (*left*). Stimuli are processed and projected as an input (*green*) to the integro-differential equation (with numbered references in *grey*) controlling the decision-making process (*red*). Activity on both the stimuli and decision spaces reflect evidence for the associated and possibly nuanced response. By aggregating the activity over the decision space, the dynamic decision is then mapped and rescaled if necessary onto the answer space (*purple*, with shaded background limited within the [-1,1] interval on the decision space), in turn attracting the mouse cursor to generate a response trajectory (*blue*), until a mouse click on the response scale (*grey shaded area at the top*) ends each trial.

constant τ already partially accounts for the temporal dynamics of the system, directly impacting $\boldsymbol{u}$ and indirectly κ through activity summation.

2.4. Final decision - stop criteria

In most decision-making empirical paradigms, the participant must press a key or reach and click on a response area on screen to give her response. In both cases, the action ends the current trial, determining the final response, response time, and ending the trajectory (in the case of mouse-tracking) with no possibility to turn back time and correct the provided response. Empirical paradigms therefore introduce a convergence constraint on the decision process, with the participant only pressing a key or clicking on a response area when the decision reached a high enough level of certainty. We therefore assume participants take the required time to select a final response that matches their decision, and that such decision can be considered stable and definitive. To simulate this process in the special case of mouse-tracking paradigms, we consider that the decision is reached when the three following conditions are met:

1. The activity of at least one unit passes a fixed elevated threshold. This usually corresponds to the formation of a stable bump of activity, and the associated attractor being attained in the cognitive part of the model. This can be guaranteed by exploiting the nonlinear dynamics of equation (7) and choosing the threshold in adequacy with f (in combination with other parameters that control the feedback loops across units). Opting for a sigmoid instead of a ReLU for f would simply turn this threshold into a tolerance (distance to the function maximal value of 1).

2. The sensorimotor part has converged to match the cognitive part. This means that the distance between the current mouse position ($\boldsymbol{p}$ from equation (5)) should be sufficiently close to the current decision location in the answer space ($\boldsymbol{p_{tgt}}$ derived from equation (8)). While having a low enough tolerance on the x-axis may be important as it determines the precision of nuanced final response, the y-axis is generally less relevant for mouse-tracking paradigms relying on binary or continuous response mode; less so when more than two alternatives are used and positioned in 2D, albeit seldom (Freeman and Ambady, 2010, and associated online software documentation).

3. The mouse pointer lies within a clickable response area. This is a strict criterion to simulated the actual graphical user interface used in human experiments, where the current mouse position ($\boldsymbol{p}$) should lie within available response areas for the click event to be correctly processed. These areas correspond to rectangles usually positioned in the top-left and top-right corners of the screen for binary response mode, or to a full width rectangle for continuous response mode(see figures 1A and 1B).

Aforementioned thresholds and tolerances are considered as hyperparameters and are not optimized in the current article, as they should have little impact on the dynamics under reasonable assumptions and when leaving other model parameters free (e.g., not expecting reaching a decision or position with near infinite precision when there is a lot of noise in the decision process).

Although they may seem partially redundant, these criteria are actually complementary. Indeed, complex decision dynamics in both human and simulations may lead to situations where the mouse hovers on a clickable response area corresponding to

the current preference (*2 & 3* satisfied) but a stable decision has not yet been reached (*1* violated); or the mouse is on a clickable response area and a stable decision has been reached (*1 & 3* satisfied), but the participant changed her mind and the decision process bifurcated toward another response location (especially with a continuous response mode, *2* violated); or the cognitive and sensorimotor part converged (*1 & 2* satisfied) to a response that is not available (e.g., nuanced decision on a binary response modeparadigm). As a consequence of this third situation where criterion *3* may remain violated (e.g., if the computational model never converges to an acceptable response), an arbitrary time constraint is introduced, simply to prevent infinite simulations and to discard the associated trials. This parameter should have no impact when adequate parameters are chosen for the computational model, and is therefore simply set to a large enough value that should guarantee convergence, given other parameters that control simulation time (e.g., τ and λ). Such constraint is therefore optional in the computational model, as it is for humans, participants not wanting to spend an infinite amount of time doing the experiment. Conversely, while introducing temporal constraints may heavily influence human decision dynamics (e.g., speed-accuracy trade-off when imposing a maximal trial duration), such influences or biases are not implemented in the model. Simulated trajectories satisfying or violating the different criteria are illustrated on figure 5 using non-optimal parameters.

2.5. Versions to account for different empirical conditions

The full model composed of the cognitive and sensorimotor parts can adapt to different empirical configurations. Firstly, while designed to fit 2D trajectories obtained using standard mouse-tracking paradigms (see Figure 1A; Freeman and Ambady, 2010), it can be easily adapted to alternative paradigms. This includes novel slider-based designs applied to moral stimuli (see Figure 1B; Gautheron et al., 2023; Gaboriaud et al., 2022) that have been developed to address mouse-tracking related limitations (e.g., when presenting complex stimuli, as pointed in Stillman et al., 2018). Such designs rely on a mouse-tracking enabled continuous response scale, with the mouse cursor hidden but horizontal (resp. vertical) movements starting from the center of the scale reflected by a *green slider* movements, hence generating 1D trajectories $x(t)$ (resp. $y(t)$). Since the decision model already only generates a single target coordinate (x_{tgt} from equation (8)), the mouse coordinates vectors (***p*** and ***p**_{tgt}*) are reduced to scalars. When considering standard computed mouse interactions, it is equivalent to consider that $y(t) = y_{tgt}$ at all times, since the mouse starts on the slider and that controlling the y-coordinate of an hidden cursor is both difficult and irrelevant for the task.

Secondly, focusing on the integration between the SSM and DNF literature aimed in this article, our generative model is able to produce trajectories for both binary and continuous response modes. However, this capability highly depends on parameter values, conditioned by the number of units (n). The difficulties are better illustrated when considering extreme configurations, so that we will here only consider LCA-like ($n = 2$) and DNF-like models ($n \rightarrow \infty$). Crossing paradigm response
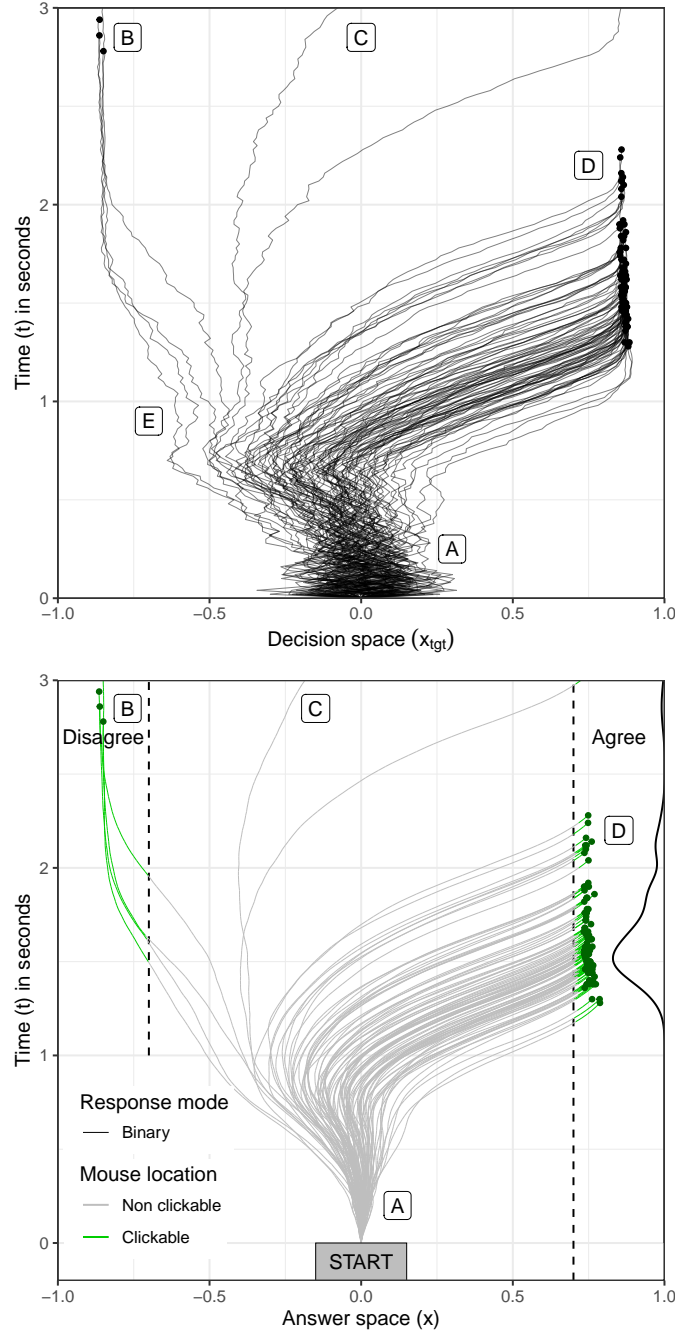

Figure 5: Sample of simulated decisions (x_{tgt}) and mouse trajectories (x) as functions of time, replicating the exact configuration from figure 2, illustrating prototypical dynamics on the binary response mode design from empirical Study 2. All trajectories correspond to a single stimulation scenario using a DNF-like model, with only noise varying across trials. Since this configuration corresponds to a ∞ unit model combined with binary response mode, evidence must be accumulated near locations corresponding to the extremes of the response scale to guarantee convergence. Equations (5) and (9) smooth out trajectories (label E), with lateral movements initially prevented by the low average unit activity despite a large variability in center of mass (label A). Color indicates areas where the simulated mouse can be clicked in order to select a final decision, reflecting that criterion 3 may be satisfied before criteria 1 & 2 (label B). Most decisions converge on the right (due to stronger evidence later during the trial) after deviating more or less towards the opposite side, generating classical decision paradigms and mouse-tracking RT distribution (label D). However with this specific design, input scenario and parameter configuration, a few of the simulations might take more time to converge (label C) or accumulate enough evidence and bifurcate towards the opposite attractor (label B).

Table 2: Possible combinations of empirical constraints (binary or continuous response scale) and parametrizations of the model (2 or ∞ number of units).

Units	Response mode	
	Binary	Continuous
2	**OK**	Partial competition
→ ∞	Lack dichotomization	**OK** (with rescaling)

modes and the number of units in the model gives rise to the following 2×2 configurations:

- Binary response mode with $n = 2$ is the natural fit between classical SSM models and 2AFC paradigms. In presence of sufficiently strong inhibition between the two distant units (determined by parameter B), the vector of activities (u_1, u_2) either converges to $(u_{max}, 0)$ or $(0, u_{max})$ as for the original LCA. This precludes the simultaneous activation of opposite units and prevents stable nuanced responses.

- Continuous response mode with $n = 2$ requires generating nuanced decisions from only two units. Nuanced decisions can either be the consequence of a shared activity across distant units (see process illustrated on figure 1C) or of a bump of strongly active units at intermediate positions on the decision space, inhibiting units closer to the extremes (see final state illustrated on figure 1D or 2B). With only two units available, shared activity is therefore required. To this end, stop criteria can be relaxed to allow answers in absence of full convergence of the cognitive part, but this solution is unsatisfactory from a human decision modeling perspective (since the answer corresponds to a non final decision); or the nonlinear competition between units must be dropped from the model in order for stable attractors (u_1, u_2) with $u_1 \neq 0$ and $u_2 \neq 0$ to exist, which conflicts with most of the SSM literature.

- Continuous response mode with $n \to \infty$ is the other natural fit when dealing with decisions on continuous topologies (such as those usually encountered in sensorimotor systems, e.g., object location or direction of movement). For near-continuous configurations ($n \gg 2$), strong global inhibition (B) guarantees the convergence to a localized attractor and thus well-defined decision as a single bump of activity. While parameter A in equation (4) controls the reciprocal excitation between nearby units, parameter a defines what *nearby* or *localized* mean, thus the scale of the activity bumps and granularity of decisions (i.e., the number of neighboring units that can be strongly activated at the same time). A system forced to take a unique decision should sooner of later be more likely to hesitate between nearby responses (e.g., *Not at all* vs. *Not that much*) rather opposites (e.g., *Not at all* vs. *Completely*).

- Binary response mode with $n \to \infty$ may be simply implemented by dichotomizing target coordinates, but generated trajectories cannot match empirical trajectories (as developed in Lepora and Pezzulo, 2015); a more satisfying solution is to generate attractors at the borders of the decision

space. Border effects preventing extreme answers in DNF-like models (e.g., due to weaker self-excitation, unstable or asymmetric attractors) are easily dealt with by extending the decision space or rescaling coordinates from the decision to the answer space (as illustrated on figures 2 and 5). Yet, preventing nuanced answers is less trivial and requires additional model parameters (e.g., tweaking of weights w to alter the distribution of possible attractor locations).

Among these 4 versions, only those where the number of units matches the number of available answers (in *bold* in table 2) are able to produce the type of continuous decision-making dynamics expected on binary and continuous response mode mouse-tracking paradigms, at least without further complexifying the model.

In practice, we used $n = 50$ for the DNF-like version of the model, as a trade-off between the accurate approximation of a continuum neural field ($n \to \infty$) and computational complexity. Even though the algorithmic complexity in $O(n^2)$ for the 1D convolution can be reduced (e.g., through kernel restriction, singular value decomposition and Fourier transform in Quinton et al., 2011), simulating a single trial by solving the equations remains sequential. Also, optimizing parameters for different versions of the model over many trials can be partially parallelized, but remains intensive in terms of computational time. As for the temporal resolution used when numerically solving the differential equations, increasing the number of units beyond the chosen value has no qualitative impact and a negligible quantitative impact on dynamics or final decisions.

3. Results

Beyond theoretical capabilities of the model characterized in the previous section, we performed a quantitative fit to empirical data relying on different mouse-tracking paradigms. We could then qualitatively compare empirical and generated mouse trajectories, but also parameters across paradigm settings, using both $n = 2$ and $n = 50$ versions of the model.

3.1. Empirical data

We exploited the empirical data from Gautheron et al. (2023), made freely available on OSF (https://osf.io/hqgfd/). The mouse-tracking paradigm (classical 2D screen or 1D slider) varied across two moral decision-making studies. These studies relied on a the same set of 60 moral assertions adapted from (Pärnamets et al., 2015), including sentence stimuli as "Murder is sometimes justifiable" or "It is our duty to protect things of beauty" (also see stimuli in figures 1 and 4). Within each study and participant, each assertion was presented with the two response modes (binary or continuous), with extremes responses corresponding to "Disagree" and "Agree".

Study 1 (N=103 participants) relied on the standard mouse-tracking screen configuration. In the binary response mode, the two extreme response options were presented in the top-left and top-right corners of the computer screen (see figure 1A). The mouse 2D-trajectory $(x, y)(t)$ was recorded for each trial from

the time when participants clicked on the "START" button (positioned at the bottom-center of the screen), triggering the onset of the stimulus, until they clicked on their chosen response at the top of the screen. An online written feedback was given when participants did not move quickly or regularly enough, so that participants' mouse movements would cover the whole decision process after a quick learning phase. In the continuous response mode (see figure 4), the only change was that extreme options were connected by a single full-width response area, allowing participants to click on any intermediate position (thus nuanced answer). Moral assertions were presented in random order within two blocks, each block associated to a response mode, counter-balanced across participants. As responses were displayed before the participant initiated trials, response mode was expected to exert an influence from the very beginning of mouse-trajectories, at a stage where a final decision has not yet been made.

Study 2 (*N*=65) relied on a novel mouse-tracking design addressing limitations of mouse-tracking in presence of complex written stimuli. Participants answered using an horizontal slider with a "START" area placed in the middle. Once clicked upon, the indication as well as the mouse cursor were replaced by a color gradient of varying width which was controlled by the participant (see figure 1B). In absence of movement, the gradient width was progressively reduced to induce time pressure on the decision process (the trial being considered failed if the gradient disappeared). Gradient color was used to indicate whether the current slider location could be clicked to provide a final response. If the gradient was green whatever the location of the mouse, it signaled a continuous response mode; in binary response mode, it turned from gray to green only when hovering extreme response options written within the slider bar. As a consequence and contrary to Study 1, participants could not guess the condition before moving the mouse due to randomization of conditions across trials. The 1D-trajectory *x(t)* along the slider axis was recorded for each trial (see figure 6 for a sample of trajectories produced by human participants). While the model inherently generates trajectories reflecting ongoing preferences, a dynamically shrinking slider was empirically required to elicit continuous mouse trajectories that reflect the entire decision process.

In both studies, an early difference between the two response modes was confirmed in mouse-tracking trajectories, much before the final decision is made, reflecting the constraint induced on the decision process. This could be attributed to a trivial difference in final response distributions (i.e. bimodal in binary response mode, with nuanced responses that cannot be selected), yet results were confirmed when analyses were conditioned on final responses (Gautheron et al., 2023). Simply knowing whether nuanced responses are available therefore seems to impact both the cognitive and sensorimotor components of human decision-making.

Psychologically, this translates into participants dichotomizing a potential response continuum into hermetic categories when integrating and accumulating evidence throughout the decision process in binary response mode. Given the SSM and DNF properties, as well as their integration in our model, such
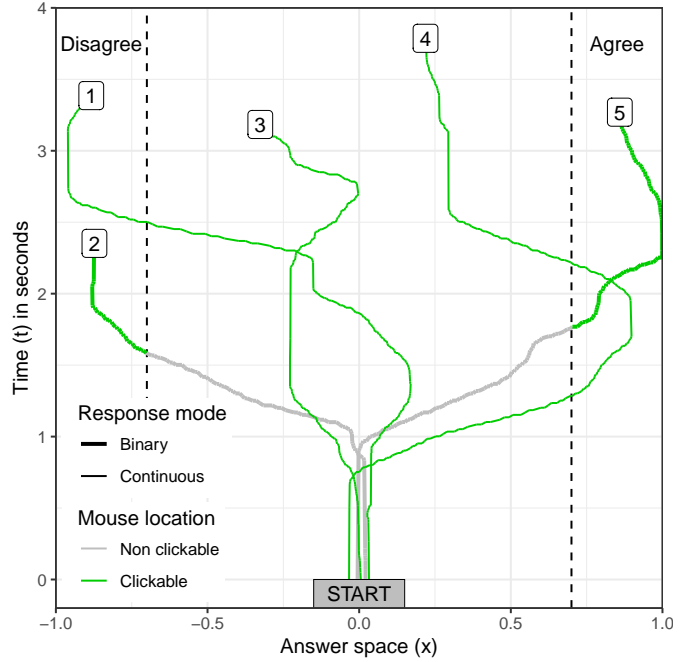

Figure 6: Sample of mouse trajectories from Study 2. Participants respond and move at different times (y-axis), with hesitations and changes of mind reflected by lateral movements (x-axis), mouse coordinates bounded by screen limits (e.g., trajectory with final response indicated by label 5). While the slider remains *green* at any location in continuous response mode (1, 3 & 4), the slider (varying width not represented here) is *gray*ed out for nuanced response locations in binary response mode (2 & 5).

differences could be explained by paradigm-driven changes to the number of active units or to kernel parameters. With only two units or a narrow excitatory part of the kernel (low *a* parameter), mainly competition between units occurs. On the contrary, with more units and a wider excitation among units, merging of localized evidence may occur, leading to nuanced decisions.

3.2. Quantitative fit to empirical data

Data obtained in Study 1 were used to provide informative a priori input parameters for model simulations while minimizing dependence relative to evaluation; model evaluation was indeed performed by fitting generated mouse trajectories to Study 2 data (thus on an adapted empirical paradigm and different participants). To specify the input vector *e* in equation (7) for each trial (e.g., given stimulus and response mode), a Gaussian profile activity was generated to account for uncertainty and provide evidence for an extended region of the decision space. It was centered on the response position on corresponding trials in Study 1, back-transformed into the decision space. Given the optimization performed on other parameters, the amplitude of the Gaussian profiles was kept equal to 1. Beyond its position, the shape of the input distribution may impact results, but the standard deviation in case of a unimodal input did not and was therefore kept constant for all conditions and stimuli (equal to 0.1). Indeed, the differential equation integrate and filter out the input signal to conform to the kernel shape.

To assess the effectiveness of the model to reproduce decision dynamics, we fitted empirical mouse trajectories using a method combining the averaging of trajectories from Hehman et al. (2015) with an optimization criterion and procedure inspired by Ratcliff and Tuerlinckx (2002). It consisted in time normalizing trajectories (vertical scaling of individual trajectories on figure 6 so they all end at the same time) before averaging x-coordinates within spatio-temporal bins (equivalent to grid cells on figure), making it compatible with the 1D slider design used in Study 2. The same procedure was applied for each stimulus and response mode with human and simulated trajectories, which could then be compared using a χ^2 goodness of fit statistic. The χ^2 was used as a fit criterion to minimize (since equal to 0 for a perfect match between binned distributions) as classically used for fitting response times distributions with SSM despite its limitations (Ratcliff and McKoon, 2008; Ratcliff and Tuerlinckx, 2002). This approach led to an acceptable bias-variance trade-off, balancing observations across cells, limiting the influence of outliers (e.g., extreme response times) while reducing the large inter-trial variance through averaging (present in both human and simulated data) as well as the number of parameters. The number of intervals in the spatiotemporal histogram (5×10 grid aggregating millions of points) was also chosen to maintain a good enough resolution while smoothing out the stochasticity in the optimization process. Due to the time normalization step, any similarity between empirical and simulated response time distributions (mean, variance or shape) cannot be attributed to the model fitting procedure, and may support the model validity. An illustrative density distribution is provided on the right of figure 5, but should not be directly compared to classical SSM results. Indeed, response times here depend on parameters in both the cognitive and sensorimotor parts of our model, therefore benefiting from additional degrees of freedom but also encompassing other constraints (further developed in Gautheron et al., 2024).

Reparametrization was performed to prevent singularities and unrealistic settings (e.g., instantaneous integration of information when $\tau \to 0$), by introducing lower or upper bounds for some parameters (Quinton, 2010). Manual initialization was performed to start from a region of the parameter space producing plausible trajectories given empirical data. Estimated optimal parameters for both versions of the model are reported in table 3, where χ^2 optimal values (criterion to minimize) were converted into Cramér's V effect sizes for easier interpretation. Indeed, while the χ^2 statistic is rather inferential and has no upper bound, V remains between 0 (perfect fit) and 1 (maximal divergence between relative frequency distributions) independently of the sample size. Values of a and σ close to 0.05 reflect the optimizer at some point being constrained by a priori parameter boundaries, which were nevertheless required to prevent singularities. Nevertheless, the low V values obtained for both the binary and continuous response modes indicate a satisfactory model fit.

To assess the robustness of results, we conducted a local sensitivity analysis in order to quantify the influence of parameter variations on model fit (±10% around optimal values reported

in table 3). While overfitting is unlikely given the ratio between the 5 free parameters and $60 \times 5 \times 10$ empirical mean coordinates in each condition, asserting the stability of the model behavior is key to establish its validity and generalizability potential. Simulating thousands of datasets, we obtained Cramér's $V \in [.033, .036]$ for binary and $[.098, .100]$ for continuous response modes, thus only slightly degrading performance in the worst case. Values obtained below the V statistics reported in table 3) reflect the difficulty of the optimization process, due to both the non-linearity and stochasticity of the model.

Resp. mode	n	τ	A	B	a	σ	V
Binary	2	3.00	75.3	25.3	0.06	0.05	.035
Continuous	50	2.67	88.5	23.5	0.05	2.62	.098

Table 3: Optimal parameter values (τ, A, B, a, σ) for $n = 2$ units (resp. $n = 50$) following optimization of the fit of model generated trajectories to empirical data in binary response mode (resp. continuous). The criterion to minimize is reported as Cramér's V effect size $\in [0, 1]$.

3.3. Qualitative analysis of generated trajectories

For a wide range of parameter values including the optimal ones reported above, the model was able to generate valid mouse trajectories for each simulated trial. This means not leading to singularities, divergence or failing on a single trial, despite varying inputs (depending on stimulus and noise) and paradigm settings (binary and continuous response modes, but also classical 2D MT or novel 1D slider). This was achieved by mainly changing a single parameter (from $n = 2$ to $n = 50$), while relying on the same unifying equations and criteria. The number of units, whatever they may represent in the brain, body or mind of simulated participants, therefore seems critical in this class of models to match paradigm constraints. Although we made sure to safeguard against most singularities, this is no small feat given the highly nonlinear dynamics and stochasticity of the coupled differential equations. For the cognitive part, activity could indeed either diverge (or at least saturate the neural field), converge to zero (resting level), or converge to a decision incompatible with response mode. For the sensorimotor part, the simulated mouse coordinates could diverge (leading to the cursor getting stuck in screen corners), oscillate without stabilizing on a response (violating the stop criterion) or converge to an inadequate location (e.g., middle of the screen for both response modes, or top-middle of the screen for binary response mode).

Reversely to fitting the model to empirical data to find adequate parameter values, we can exploit the unifying and generative nature of the model to propose mechanistic explanations about differences between response modes based on differences in parameters (Wilson and Collins, 2019). Sets of parameter values are here comparable thanks to the normalization factors integrated in equations (6) and (7). First of all, combinations of n, A, τ and σ lead to different average convergence speeds, with significant differences found in empirical data depending on the design (Gautheron et al., 2023). A smaller time constant (τ) may result in quicker response, but can easily be compensated by an increased amplitude of the lateral interaction kernel

(higher A) or by an increase number of units (e.g., $n = 50$ compared to $n = 2$) in absence of normalization in equations (4), (6) or (7). Indeed, both of these latter parameters increase the amount of information exchanged by units, helping convergence. The amount of noise (controlled by σ) may have contradictory effects on the system, either facilitating bifurcations in presence of balanced evidence for different response alternatives, or slowing down convergence to a stable attractor (acting as a perturbation relative to the integration of signal over decision space and time). Despite a large variability in simulated decision dynamics, average response times for binary response mode were shorter than for continuous response mode when relying on optimal parameters. Since fit criteria were based on normalized time, such differences are by-product of the model, adding to its predictive capabilities.

The optimal values for σ constitute the most striking difference between the binary and continuous versions. For $n = 2$, the fit of simulated to empirical trajectory distributions is possible in the near absence of noise. On the contrary, optimal parameters lead to a low signal-to-noise ratio for $n = 50$ which induces more hesitations, later changes during the decision process, and of course reflect the larger variability in x-coordinates given the continuous answer space. Given the fit criteria on the distributions of x-coordinates in normalized timebins, these differences naturally match empirical observations made on the human participants' datasets. Put differently, for similar kernel parameters (especially A and B), a weaker noise for the binary response mode means that each piece of evidence will be given more weight in the decision process compared to the continuous response mode. Psychologically, it could be interpreted as a stronger need to converge towards one of the alternatives in binary response mode; while with nuanced answers available, participants would be more demanding regarding the evidence, in order to refine their decision and increase the precision of their final answer location.

4. Discussion and limitations

As for any optimization procedure, modeling choices and numerical constraints impose limitations to the inferences that can be drawn from parameters and simulations. For instance, parameter a is nearly identical for both versions of the model, approaching the lower bound imposed to prevent singularities. This means that lateral interactions among units are almost limited to inhibition (hence competition between options), without the need for local excitation (absence of interpolation or mutual reinforcement of coherent spatialized inputs). This is the classical functioning for $n = 2$, as opposite responses are expected to compete for decision and action, much less for $n = 50$. However, to avoid introducing additional parameters and only absorb the large between-stimuli variability (here between moral assertions), we provided a stationary and static unimodal input to the system for each simulated trial. Relying on DNF properties, a larger number of units in interaction act as a spatiotemporal filter, especially in presence of white noise which lacks spatial and temporal structure (Quinton and Girau, 2011;

Quinton, 2010). Doing so, we did not really put pieces of evidence in competition, and rather relied on noise (with a large σ value) to simply generate hesitations and deviations that might actually result from fine-grained processing of assertions (from orthographic elements to their moral semantics and implications). As hypothesized in the moral decision-making literature, it may well be that moral words are processed earlier or faster (Gantman and Van Bavel, 2014), leading to a complex interplay between complex sentence processing and decision dynamics. While the proposed unified model is perfectly adapted to dynamic input (as illustrated on figure 2), introducing word-level parameters (e.g., valence and timing) would have drastically reduced model parsimony without adding much value to this article contributions.

In absence of fine-grained theory-driven models of how sensory and linguistic information is processed in the context of moral assertions, data were heavily aggregated in the statistical evaluation of the model. Indeed, mouse coordinates were spatially and temporally binned, and histograms formed over participants for each moral assertion. This process was needed to make the hard problem of optimizing parameters in stochastic integro-differential equations less hard; it led to an accurate fit since the very large trajectory variability was reduced through aggregation, following the central limit theorem. We therefore minimally accounted for between-stimuli variability by providing empirically derived inputs, but neglected between-participant variability. As a consequence, most sources of trajectory variability were integrated in the noise amplitude (through parameter σ), therefore matching noise modeling in the statistical model for which the fit to aggregated empirical data was optimized. While the proposed model is able to generate hesitation and simulated changes of mind during the decision process (e.g., on figure 2C), there is no guarantee that such phenomena appear under the same conditions and for the same stimuli as in empirical data. Put differently, due to the nonlinearity and stochasticity of the model, parameters leading to an optimal fit at the aggregated trajectories level do not necessarily produce the best matching trajectories at the individual trial level (e.g., lacking late bifurcations and changes in decision).

Nevertheless and beyond the fit to empirical distributions, we proposed a generative model of decision-making dynamics and mouse trajectories. The model is indeed able to generate plausible trajectories at trial level, from which distributions can be constructed through repeated simulations of trials (possibly in different conditions or with different inputs/stimuli), paralleling repeated measure designs in empirical studies. Although SSM and DNF models are also generative, they do not usually integrate a sensorimotor component to simulate mouse trajectories; reversely, models that focus on movement generation generally do no account for decision-making dynamics at trial level. These are also to be differentiated from models which only focus on the description of distributions across trials, such as the LATER model when remaining close to the SSM literature (Noorani and Carpenter, 2016), or most Bayesian models used to model empirical decision-making data (as discussed in Forest et al., 2022b).

Under its current formulation, the model does not encom-

pass all mechanisms known to significantly impact decision-making dynamics. While the decision process is represented as a closed loop across spaces on figure 3, several reciprocal arrows are missing. For instance, the sensorimotor component of the model could be improved to better account for physical constraints, both in the motor apparatus of the agent and in the environment (e.g., friction forces of the computer mouse). These would for instance be required to account for Fitts' law which applies to most pointing devices (Fitts, 1954). Similarly, while the slower convergence observed for continuous response scales matches the logic of Hick's law (Hick, 1952), it does not account for all its variations and subtlety. Albeit the progressive commitment towards response alternatives is reflected in the emergence of basins of attraction (as developed in Lepora and Pezzulo, 2015), other sources of nonlinearity and bifurcations are missing. Through visual perception and interaction with the paradigm, the agent may integrate the motor cost of reaching and clicking on a response (e.g., *m*DDM in Falandays et al., 2021) or focus on a specific response alternative through active perception and active sampling of information (e.g., *a*DDM in Falandays et al., 2021; Quinton et al., 2014). As the aforementioned models rely on SSM or DNF and are therefore clearly compatible with our proposed model, the associated mechanisms were not embedded for parsimony purpose. These mechanisms would indeed not have much contributed to our focus on the continuum between models and decision spaces. Research work is therefore still required to develop a fully integrated model and to collect adequate empirical data to test its predictions.

5. Conclusions

We started by showing how SSM and DNF models evolved in parallel in different (sub)disciplines and targeting different objectives, focusing on continuous vs. discrete spaces and tasks. Several decades of development reduced the gap, with a core feature of DNF being the emergence and maintenance of localized bumps of activity, and SSM progressively extending towards tasks with multiple alternatives or continuous response scales. Yet, the proposed generative model is a first attempt at explicitly bridging the gap by unifying associated equations, and integrating a sensorimotor component flexible enough to fit different paradigms and response modes, simulating individual decision and response trajectories. By relying on a spatially vectorized form of the equations (i.e. simulating the activity of a set of units), we diverge from the continuum field characteristic of DNF, but generalize and extend representative instances of the SSM class.

To demonstrate the potential of the model in terms of applicability, we fitted empirical trajectory distributions collected in moral decision tasks which are known to generate hesitations, changes of mind, while remaining meaningful with both binary and continuous response scales. To illustrate the potential of our mechanistic model in terms of explainability (contrary to most deep neural network models) and inference to be drawn on human decision-making, despite limitations resulting from the nonlinearity and stochasticity of the equations, parameters

were compared and interpreted across conditions. Results suggest that relaxing response scale constraints (i.e. allowing nuanced responses) may impact the whole dynamical system involved in the decision, and not only the sensorimotor control of the mouse. Delayed bifurcations and late adjustments are possible in continuous response mode, while the presence of wider basins of attraction in binary response mode may allow the early processing of stimuli to have a stronger and irreversible impact on decisions (e.g., principles, emotions or intuitions evoked by moral stimuli, see Koop, 2013; Greene et al., 2008).

Drawing back from its specific application to moral decisions using sentence-based stimuli, the proposed model is generic enough to be applied to many empirical paradigms. This goes beyond applying it to 1D sliders or 2D classical mouse-tracking by having movement on the *y*-axis impact the dynamics. It also goes beyond answering on a binary or continuous response scale by adjusting clickable areas, or presenting the scale as a chromatic slider or rectangles on top the screen. Instead of relying on mouse movements, finger trajectories on touch screen may be produced and collected, leading to similar response patterns and dynamics Dotan et al. (2019). Even key presses could be simulated in the same way, simply trading the sensorimotor component of the model for a simpler effector implementation (e.g., simulating delays between commands and key presses).

Researchers often consider that the cognitive system simply adapts to empirical constraints, with two alternative assumptions: psychometric choices on response scales hide the richness and nuanced nature of a continuous mind; or conversely mouse trajectories are artifacts of the paradigm, only reflecting the shallow embodiment of decisions and hiding the discrete nature of cognitive representations. We would rather focus on the interplay between all the components that are involved and may in the end constitute or allow the emergence of a decision-making system adapted to the current situation. Stimulus, decision, motor and answer topologies all participate in the definition of the decision system, with some combinations more effective than others. The resulting system in turn constrains decision trajectories which emerge from the interactions of the neural and sensorimotor systems with human-machine-interfaces and paradigms.

Even a simple visual task can combine a myriad of continuous and discrete spaces, with distributed processes unfolding in overlapping periods, at different rates and with different delays (Goffart et al., 2018), while also being considered continuous or discrete in time. For instance in a visual search task, these processes range from linguistic stimulus and visual features processing, saccadic motor control and object localization, context and object categorization, to generating a response trajectory to indicate whether a target object was found or not (Quétard et al., 2016). The model proposed in this article is therefore a step to discriminate and study the complementary roles of the many spaces in which decision processes are dynamically shaped, and to study the emergence of discrete representations from continuous activity (joining research tackling the symbol grounding problem since Harnad, 1990).

At a more theoretical level, adopting socially situated (Smith and Semin, 2004), dynamical system (Spivey and Dale, 2004)

or interactivist perspectives on cognition (Buisson and Quinton, 2010), it comes as no surprise that the task context (including the empirical paradigm) impacts decision dynamics. This research work nevertheless contributes to questioning the definition of a decision and its boundaries: 1) in time, through the study of the dynamics which determines and precedes the final answer; 2) in space, beyond the limits of a constrained response scale; but also 3) at the interface between the neural and the physical, involving sensorimotor interactions with the environment.

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References

Amari, S.-i. (1977). Dynamics of pattern formation in lateral-inhibition type neural fields. *Biological cybernetics*, 27(2):77–87.

Bequette, B. W. (2003). *Process control: modeling, design, and simulation*. Prentice Hall Professional.

Berlin, S. B. (1990). Dichotomous and complex thinking. *Social Service Review*, 64(1):46–59.

Bicho, E., Louro, L., and Erlhagen, W. (2010). Integrating verbal and nonverbal communication in a dynamic neural field architecture for human-robot interaction. *Frontiers in neurorobotics*, 4:5.

Bickhard, M. H. and Richie, D. M. (1983). *On The Nature Of Representation: A Case Study Of James Gibson's Theory Of Perception*. Praeger, New York.

Bogacz, R., Brown, E., Moehlis, J., Holmes, P., and Cohen, J. D. (2006). The physics of optimal decision making: a formal analysis of models of performance in two-alternative forced-choice tasks. *Psychological review*, 113(4):700.

Buisson, J.-C. and Quinton, J.-C. (2010). Internalized activities. *New Ideas in Psychology*, 28(3):312–323.

Busemeyer, J. R. and Diederich, A. (2002). Survey of decision field theory. *Mathematical Social Sciences*, 43(3):345–370.

Buss, A. T., Wifall, T., Schoener, G., Hazeltine, E., and Spencer, J. P. (2011). Integrating mind and body in a response-selection task: From neural decisions to mouse trajectories... and back again. In *52nd Annual Meeting of the Psychonomic Society*.

Carroll, S., Josić, K., and Kilpatrick, Z. P. (2014). Encoding certainty in bump attractors. *Journal of computational neuroscience*, 37:29–48.

Catenacci Volpi, N., Quinton, J. C., and Pezzulo, G. (2014). How active perception and attractor dynamics shape perceptual categorization: a computational model. *Neural Networks*, 60:1–16.

Cerda, M. and Girau, B. (2010). Bio-inspired visual sequences classification. In *Brain Inspired Cognitive Systems 2010-BICS 2010*, pages pp–20.

Churchland, A. K., Kiani, R., and Shadlen, M. N. (2008). Decision-making with multiple alternatives. *Nature neuroscience*, 11(6):693.

Cisek, P. and Kalaska, J. F. (2005). Neural correlates of reaching decisions in dorsal premotor cortex: specification of multiple direction choices and final selection of action. *Neuron*, 45(5):801–814.

Crockett, M. J. (2016). How formal models can illuminate mechanisms of moral judgment and decision making. *Current directions in psychological science*, 25(2):85–90.

Cushman, F. (2008). Crime and punishment: Distinguishing the roles of causal and intentional analyses in moral judgment. *Cognition*, 108(2):353–380.

Dale, R. (2008). The possibility of a pluralist cognitive science. *Journal of Experimental and Theoretical Artificial Intelligence*, 20(3):155–179.

Dale, R. and Spivey, M. J. (2005). From apples and oranges to symbolic dynamics: A framework for conciliating notions of cognitive representation. *Journal of Experimental & Theoretical Artificial Intelligence*, 17(4):317–342.

Dehghani, M., Tomai, E., Forbus, K. D., and Klenk, M. (2008). An Integrated Reasoning Approach to Moral Decision-Making. pages 1280–1286.

Ditterich, J. (2006). Stochastic models of decisions about motion direction: behavior and physiology. *Neural networks*, 19(8):981–1012.

Dotan, D., Pinheiro-Chagas, P., Al Roumi, F., and Dehaene, S. (2019). Track it to crack it: dissecting processing stages with finger tracking. *Trends in cognitive sciences*, 23(12):1058–1070.

Drugowitsch, J., Moreno-Bote, R., Churchland, A. K., Shadlen, M. N., and Pouget, A. (2012). The cost of accumulating evidence in perceptual decision making. *Journal of Neuroscience*, 32(11):3612–3628.

Erlhagen, W. and Bicho, E. (2006). The dynamic neural field approach to cognitive robotics. *Journal of neural engineering*, 3(3):R36.

Erlhagen, W. and Schöner, G. (2002). Dynamic field theory of movement preparation. *Psychological review*, 109(3):545.

Esnaola-Acebes, J. M., Roxin, A., and Wimmer, K. (2022). Flexible integration of continuous sensory evidence in perceptual estimation tasks. *Proceedings of the National Academy of Sciences*, 119(45):e2214441119.

Falandays, J. B., Spevack, S., Pärnamets, P., and Spivey, M. (2021). Decision-making in the human-machine interface. *Frontiers in Psychology*, 12:99.

Fitts, P. M. (1954). The information capacity of the human motor system in controlling the amplitude of movement. *Journal of experimental psychology*, 47(6):381.

Fix, J., Rougier, N., and Alexandre, F. (2011). A dynamic neural field approach to the covert and overt deployment of spatial attention. *Cognitive Computation*, 3:279–293.

Forest, S., Lefort, M., and Quinton, J.-C. (2022a). Combining manifold learning and neural field dynamics for multimodal fusion. In *International Joint Conference on Neural Networks (IEEE WCCI / IJCNN)*.

Forest, S., Quinton, J.-C., and Lefort, M. (2022b). A dynamic neural field model of multimodal merging: application to the ventriloquist effect. *Neural Computation*, 34(8):1701–1726.

Freeman, J., Dale, R., and Farmer, T. (2011). Hand in motion reveals mind in motion. *Frontiers in Psychology*, 2:59.

Freeman, J. B. and Ambady, N. (2010). MouseTracker: Software for studying real-time mental processing using a computer mouse-tracking method. *Behavior research methods*, 42(1):226–241.

Freeman, J. B. and Ambady, N. (2011). A dynamic interactive theory of person construal. *Psychological review*, 118(2):247.

Gaboriaud, A., Gautheron, F., Quinton, J.-C., and Smeding, A. (2022). The effects of intent, outcome, and causality on moral judgments and decision processes. *Psychologica Belgica*, 62(1):218–229.

Gantman, A. P. and Van Bavel, J. J. (2014). The moral pop-out effect: Enhanced perceptual awareness of morally relevant stimuli. *Cognition*, 132(1):22–29.

Gautheron, F., Quinton, J.-C., Muller, D., and Smeding, A. (2023). Paradigm constraints on moral decision-making dynamics. *Journal of Behavioral Decision Making*, page e2324.

Gautheron, F., Quinton, J.-C., and Smeding, A. (2024). Conflict in moral and nonmoral decision making: an empirical study coupled with a computational model. *Cognitive Processing*, 25(2):281–303.

Gawronski, B., Armstrong, J., Conway, P., Friesdorf, R., and Hütter, M. (2017). Consequences, norms, and generalized inaction in moral dilemmas: The cni model of moral decision-making. *Journal of Personality and Social Psychology*, 113(3):343.

Gepperth, A. and Lefort, M. (2016). Learning to be attractive: Probabilistic computation with dynamic attractor networks. In *2016 Joint IEEE International Conference on Development and Learning and Epigenetic Robotics (ICDL-EpiRob)*, pages 270–277.

Goffart, L., Bourrelly, C., and Quinton, J.-C. (2018). Neurophysiology of visually guided eye movements: critical review and alternative viewpoint. *Journal of Neurophysiology*, 120(6):3234–3245.

Gold, J. I. and Shadlen, M. N. (2002). Banburismus and the brain: decoding the relationship between sensory stimuli, decisions, and reward. *Neuron*, 36(2):299–308.

Gonzalez, C. M., Zosuls, K. M., and Ruble, D. N. (2010). Traits as dimensions or categories? developmental change in the understanding of trait terms.

Developmental Psychology, 46(5):1078.

Greene, J. D., Morelli, S. A., Lowenberg, K., Nystrom, L. E., and Cohen, J. D. (2008). Cognitive load selectively interferes with utilitarian moral judgment. *Cognition*, 107(3):1144–1154.

Greene, J. D., Sommerville, R. B., Nystrom, L. E., Darley, J. M., and Cohen, J. D. (2001). An fMRI investigation of emotional engagement in moral judgment. *Science*, 293(5537):2105–2108.

Grieben, R., Tekülve, J., Zibner, S. K., Lins, J., Schneegans, S., and Schöner, G. (2020). Scene memory and spatial inhibition in visual search: A neural dynamic process model and new experimental evidence. *Attention, Perception, & Psychophysics*.

Harnad, S. (1990). The symbol grounding problem. *Physica D: Nonlinear Phenomena*, 42(1-3):335–346.

Hehman, E., Stolier, R. M., and Freeman, J. B. (2015). Advanced mouse-tracking analytic techniques for enhancing psychological science. *Group Processes & Intergroup Relations*, 18(3):384–401.

Hick, W. E. (1952). On the rate of gain of information. *Quarterly Journal of experimental psychology*, 4(1):11–26.

Huth, A. G., Nishimoto, S., Vu, A. T., and Gallant, J. L. (2012). A continuous semantic space describes the representation of thousands of object and action categories across the human brain. *Neuron*, 76(6):1210–1224.

Kahane, G. and Shackel, N. (2010). Methodological issues in the neuroscience of moral judgement. *Mind & language*, 25(5):561–582.

Khona, M. and Fiete, I. R. (2022). Attractor and integrator networks in the brain. *Nature Reviews Neuroscience*, 23(12):744–766.

Kilpatrick, Z. P., Ermentrout, B., and Doiron, B. (2013). Optimizing working memory with heterogeneity of recurrent cortical excitation. *Journal of neuroscience*, 33(48):18999–19011.

Kirsh, D. and Maglio, P. (1994). On distinguishing epistemic from pragmatic action. *Cognitive science*, 18(4):513–549.

Koop, G. J. (2013). An assessment of the temporal dynamics of moral decisions. *Judgment and decision making*, 8(5):527.

Krajbich, I., Hare, T., Bartling, B., Morishima, Y., and Fehr, E. (2015). A common mechanism underlying food choice and social decisions. *PLoS Comput Biol*, 11(10):e1004371.

Krajbich, I. and Rangel, A. (2011). Multialternative drift-diffusion model predicts the relationship between visual fixations and choice in value-based decisions. *Proceedings of the National Academy of Sciences*, 108(33):13852–13857.

Krypotos, A.-M., Beckers, T., Kindt, M., and Wagenmakers, E.-J. (2015). A Bayesian hierarchical diffusion model decomposition of performance in Approach–Avoidance Tasks. *Cognition and Emotion*, 29(8):1424–1444.

Kvam, P. D. (2019). Modeling accuracy, response time, and bias in continuous orientation judgments. *Journal of experimental psychology: human perception and performance*, 45(3):301.

Kvam, P. D. and Busemeyer, J. R. (2020). A distributional and dynamic theory of pricing and preference. *Psychological review*, 127(6):1053.

LaBerge, D. (1962). A recruitment theory of simple behavior. *Psychometrika*, 27(4):375–396.

Leong, W. and Hensher, D. A. (2012). Embedding decision heuristics in discrete choice models: A review. *Transport Reviews*, 32(3):313–331.

Lepora, N. F. and Pezzulo, G. (2015). Embodied choice: how action influences perceptual decision making. *PLoS computational biology*, 11(4):e1004110.

Levine, T. R. (2001). Dichotomous and continuous views of deception: A re-examination of deception ratings in information manipulation theory. *Communication Research Reports*, 18(3):230–240.

Lins, J. and Schöner, G. (2019). Computer mouse tracking reveals motor signatures in a cognitive task of spatial language grounding. 81(7):2424–2460.

Maggiani, L., Bourrasset, C., Quinton, J.-C., Berry, F., and Sérot, J. (2018). Bio-inspired heterogeneous architecture for real-time pedestrian detection applications. *Journal of Real-Time Image Processing*, 14:535–548.

Master, A., Markman, E. M., and Dweck, C. S. (2012). Thinking in categories or along a continuum: Consequences for children's social judgments. *Child development*, 83(4):1145–1163.

McNamee, D. and Wolpert, D. M. (2019). Internal models in biological control. *Annual review of control, robotics, and autonomous systems*, 2:339–364.

Metin, B., Roeyers, H., Wiersema, J. R., van der Meere, J. J., Thompson, M., and Sonuga-Barke, E. (2013). ADHD performance reflects inefficient but not impulsive information processing: A diffusion model analysis. *Neuropsychology*, 27(2):193.

Neumann, N., Böckenholt, U., and Sinha, A. (2016). A meta-analysis of extremeness aversion. *Journal of Consumer Psychology*, 26(2):193–212.

Noorani, I. and Carpenter, R. (2016). The later model of reaction time and decision. *Neuroscience & Biobehavioral Reviews*, 64:229–251.

Pattee, H. H. (1974). Discrete and continuous processes in computers and brains. In Conrad, M., Güttinger, W., and Dal Cin, M., editors, *Physics and Mathematics of the Nervous System*, pages 128–148. Springer Berlin Heidelberg.

Pärnamets, P., Johansson, P., Hall, L., Balkenius, C., Spivey, M. J., and Richardson, D. C. (2015). Biasing moral decisions by exploiting the dynamics of eye gaze. *Proceedings of the National Academy of Sciences*, 112(13):4170–4175.

Quinton, J.-C. (2010). Exploring and optimizing dynamic neural fields parameters using genetic algorithms. In *The 2010 International Joint Conference on Neural Networks (IJCNN)*, pages 1–7. IEEE.

Quinton, J.-C. and Girau, B. (2011). Predictive neural fields for improved tracking and attentional properties. pages 1629–1636. IEEE.

Quinton, J.-C., Girau, B., and Lefort, M. (2011). *Competition in high dimensional spaces using a sparse approximation of neural fields*, volume 718 of *Advances in Experimental Medicine and Biology*, pages 123–138. Springer.

Quinton, J.-C. and Goffart, L. (2018). A unified dynamic neural field model of goal directed eye movements. *Connection Science*, 30(1):20–52.

Quinton, J.-C. and Smeding, A. (2015). Dynamic competition and binding of concepts through time and space. *Cognitive processing*, 16(1):349–353.

Quinton, J.-C., Volpi, N. C., Barca, L., and Pezzulo, G. (2014). The cat is on the mat. Or is it a dog? Dynamic competition in perceptual decision making. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 44(5):539–551.

Quétard, B., Quinton, J.-C., Mermillod, M., Barca, L., Pezzulo, G., Colomb, M., and Izaute, M. (2016). Differential effects of visual uncertainty and contextual guidance on perceptual decisions: Evidence from eye and mouse tracking in visual search. *Journal of Vision*, 16(11):28.

Ratcliff, R. (1978). A theory of memory retrieval. *Psychological review*, 85(2):59.

Ratcliff, R. (2018). Decision making on spatially continuous scales. *Psychological review*, 125(6):888.

Ratcliff, R. and McKoon, G. (2008). The diffusion decision model: theory and data for two-choice decision tasks. *Neural computation*, 20(4):873–922.

Ratcliff, R. and Smith, P. L. (2004). A comparison of sequential sampling models for two-choice reaction time. *Psychological review*, 111(2):333.

Ratcliff, R. and Tuerlinckx, F. (2002). Estimating parameters of the diffusion model: Approaches to dealing with contaminant reaction times and parameter variability. *Psychonomic bulletin & review*, 9(3):438–481.

Rougier, N. P. and Vitay, J. (2006). Emergence of attention within a neural population. *Neural Networks*, 19(5):573–581.

Roxin, A. (2019). Drift–diffusion models for multiple-alternative forced-choice decision making. *The Journal of Mathematical Neuroscience*, 9(1):1–23.

Satpute, A. B., Nook, E. C., Narayanan, S., Shu, J., Weber, J., and Ochsner, K. N. (2016). Emotions in "black and white" or shades of gray? how we think about emotion shapes our perception and neural representation of emotion. *Psychological science*, 27(11):1428–1442.

Schöner, G. (2020). The dynamics of neural populations capture the laws of the mind. *Topics in cognitive science*, 12(4):1257–1271.

Schutte, A. R. and Spencer, J. P. (2007). Planning "discrete" movements using a continuous system: insights from a dynamic field theory of movement preparation. *Motor Control*, 11(2):166–208.

Schöner, G. (2016). *Dynamic thinking: A primer on dynamic field theory*. Oxford University Press.

Sigaud, O., Salaün, C., and Padois, V. (2011). On-line regression algorithms for learning mechanical models of robots: a survey. *Robotics and Autonomous Systems*, 59(12):1115–1129.

Smeding, A., Quinton, J.-C., Lauer, K., Barca, L., and Pezzulo, G. (2016). Tracking and simulating dynamics of implicit stereotypes: A situated social cognition perspective. *Journal of personality and social psychology*, 111(6):817.

Smith, E. R. and Semin, G. R. (2004). Socially situated cognition: Cognition in its social context. *Advances in experimental social psychology*, 36:57–121.

Spivey, M. J. and Dale, R. (2004). On the continuity of mind: Toward a dynamical account of cognition. *The psychology of learning and motivation: Advances in research and theory*, 45:87–142.

Spivey, M. J. and Dale, R. (2006). Continuous dynamics in real-time cognition. *Current Directions in Psychological Science*, 15(5):207–211.

Stillman, P. E., Shen, X., and Ferguson, M. J. (2018). How mouse-tracking can advance social cognitive theory. *Trends in cognitive sciences*, 22(6):531–543.

Usher, M. and McClelland, J. L. (2001). The time course of perceptual choice: the leaky, competing accumulator model. *Psychological review*, 108(3):550.

van Baar, J. M., Chang, L. J., and Sanfey, A. G. (2019). The computational and neural substrates of moral strategies in social decision-making. *Nature communications*, 10(1):1–14.

Wallach, W., Franklin, S., and Allen, C. (2010). A conceptual and computational model of moral decision making in human and artificial agents. *Topics in cognitive science*, 2(3):454–485.

White, C. N., Ratcliff, R., Vasey, M. W., and McKoon, G. (2010). Using diffusion models to understand clinical disorders. *Journal of Mathematical Psychology*, 54(1):39–52.

Wijeakumar, S., Ambrose, J. P., Spencer, J. P., and Curtu, R. (2017). Model-based functional neuroimaging using dynamic neural fields: An integrative cognitive neuroscience approach. *Journal of Mathematical Psychology*, 76:212–235.

Wilson, H. R. and Cowan, J. D. (1973). A mathematical theory of the functional dynamics of cortical and thalamic nervous tissue. *Kybernetik*, 13(2):55–80.

Wilson, R. C. and Collins, A. G. (2019). Ten simple rules for the computational modeling of behavioral data. *Elife*, 8:e49547.

Yu, H., Siegel, J. Z., and Crockett, M. J. (2019). Modeling morality in 3-d: Decision-making, judgment, and inference. *Topics in cognitive science*, 11(2):409–432.