Accounting for inspection errors and change in maintenance behaviour

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Accounting for inspection errors and change in maintenance behaviour

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We propose a way to account for inspection errors in a particular framework. We consider a situation where the lifetime of a system depends essentially of a particular part. A deterioration of this part is regarded as an unacceptable state for the safety of the system and a major renewal is deemed necessary. Thus the statistical analysis of the deterioration time distribution of this part is of primary interest for the preventive maintenance of the system. In this context, we faced the following problem. In the early life of the system, unwarranted renewals of the part are decided upon, caused by overly cautious behaviour. Such unnecessary renewals make the statistical analysis of deterioration time data difficult and can induce an underestimation of the mean life of the part. To overcome this difficulty, we propose to regard the problem as an incomplete data model. We present its estimation under the maximum likelihood methodology. Numerical experiments show that this approach eliminates the pessimistic bias in the estimation of the mean life of the part. We also present a Bayesian analysis of the problem which can be useful in a small sample setting.

Keywords: preventive maintenance; left and right censored data; inspection errors; incomplete data; maximum likelihood estimation; Bayesian inference; shock model.

1. Introduction

This paper is concerned with modelling the deterioration of a component. Here, we address a particular point regarding this subject. We propose a way to account for inspection errors. This will result in a new procedure for identifying and taking into account a change in maintenance behaviour. This study originated from a problem occurring with Reactor Coolant in EDF Nuclear plants. Maintenance engineers can make unnecessary replacements in the first years of production because of a lack of knowledge in component lifetimes. However, this overly cautious behaviour is discontinued in the following years. Statistical analysis of the reliability of 900 MW and 1300 MW reactor coolant pumps

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(RCPs) concerning pump bearings was carried out. Reports were collected on this component of 900 and 1300 MW RCPs during maintenance inspections. These reports give information about the condition of the component, and its probable replacement. The maintenance policy is essentially based on the reliability of this component. In a more general setting, such a situation occurs as soon as a nonrepairable system is being studied, where a part is degradable, compared with the other parts of the system, very expensive, and fundamental for the reliability of the system. Moreover, for every maintenance inspection, due to the high costs incurred when the system is unavailable, if the degradable part is replaced, maintenance engineers prefer to proceed to a major renewal of the system. Thus, the 'ideal maintenance policy' would involve stopping the system when the deterioration of the part appears. This study deals with one degradable part of a system.

One characteristic of collected data is that they are either left- or right-censored (see for instance example 1.7 in Meeker & Escobar (1998)), i.e. all data are related to either damage that occurred before dismantling and discovered during dismantling (left-censored data), or the component discovered in good condition at the time of dismantling (survival or right-censored data). In this model, the left and right censorings are both type I censored data. In addition to this, we consider a hidden variable Z_i in order to model the errors in inspection. The occurrence of unnecessary replacements can be numerous. Thus, a statistical analysis of component lifetimes is jeopardized since the deterioration data are littered with this censored information. Unless correctly analysed, the mean life of the part before deterioration can be greatly underestimated for such corrupt data. The procedure we propose in the present paper aims to reduce this pessimistic bias dramatically.

The general framework of our study is as follows. During a periodic maintenance visit, the damaged component is replaced and thus the system is considered *as good as new* after an inspection. (Assuming minimal repair would be more realistic. However, the 'as good as new' assumption is convenient from a mathematical viewpoint and appeared to be reasonable in our application.) We assume that the part deterioration time distribution is exponential with hazard λ and the mean life without degradation of the part is $\eta = 1/\lambda$. The data are doubly (left and right) censored and we never know the true date of deterioration. Moreover, the technology of the part is relatively new, and, before a calendar date d_0, the maintenance engineers can decide to replace it in error because of a lack of knowledge about the component. In this model, we assume that, after this date d_0, the maintenance engineers will never take such an erroneous decision again. This change of maintenance behaviour induces an important bias in the maximum likelihood (ML) estimate of λ if not taken into account. This bias corresponds to a pessimistic point of view. Figure 1 gives an example of such a change in maintenance behaviour (here the calendar date d_0 is 1992).

Our approach to deal with this difficulty consists of regarding the problem as an incomplete data problem. The missing data that we consider are the indicator values that a replacement has been achieved by caution. A classical way to derive the ML estimation of an incomplete data model is the EM algorithm (McLachlan & Krishnam, 1997). But we will see that in this particular context there is no advantage in using the EM algorithm for maximizing the observed likelihood. The paper is organized as follows. In Section 2 the incomplete data model is detailed, the ML estimation of the hazard rate λ is derived and a possible use of Bayesian inference through Gibbs sampling is presented. Section 3 is devoted to the presentation of numerical experiments on both real and simulated data.

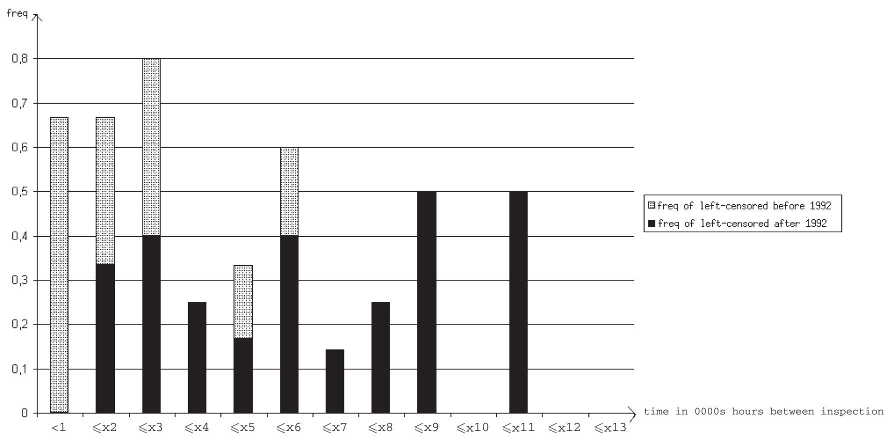

FIG. 1. Distribution of notified replacements after inspection on a 900 MW RCP component.

Simulated data are obtained with a shock model that we describe in this same section. A short discussion section ends the paper.

2. The incomplete data model

We first specify the mathematical framework of our study. Let X denote the lifetime without deterioration of the part. The distribution of the random variable X is an exponential distribution $\mathcal{E}(\eta)$ with mean value η and with cumulative distribution function F given by

$$F(t) = 1 - \exp\left(-\frac{t}{\eta}\right), \;\; \forall t \in \mathbb{R}_+, \tag{1}$$

and reliability function R

$$R(t) = \Pr(X \geqslant t) = 1 - F(t) = \exp\left(-\frac{t}{\eta}\right) \cdot \tag{2}$$

We assume that several independent systems, located in different places, are inspected at different times. After each inspection leading to diagnosis that a part is deteriorated, major renewal of the system is achieved and the system is considered *as good as new*. Let n be the total number of inspections. For inspection i, $i = 1, \ldots, n$, let t_i denote the time *since the previous inspection for the same system* or since the system installation if it is its first inspection, and let δ_i be the indicator of the inspection decision ($\delta_i = 1$ if the part is replaced, $\delta_i = 0$ otherwise). (Note that since the systems are considered as good as new and independent, there is no need to indicate the inspected system in the inspection index i.) Moreover, it happens that there exists a known calendar date d_0 from which the maintenance behaviour has changed. This date marks the end of a preventive behaviour in the sense that before d_0, the maintenance engineers can decide upon an unwarranted preventive renewal of a part caused by an overly cautious behaviour. After d_0, we assume that this overly cautious attitude does not exist any more. As a consequence, if an inspection i has occurred after d_0, we have $P(\delta_i = 1) = F(t_i)$, while if an inspection i has proceeded before d_0, we have $P(\delta_i = 1) \geqslant F(t_i)$.

In order to take account of this human factor during a maintenance operation and to propose a sensible estimation of the hazard rate λ, not littered by this early cautious maintenance behaviour, we regard the problem as an incomplete structure data problem. We define, for $i = 1, \ldots, n$, z_i as the indicator of an unwarranted replacement at inspection i: $z_i = 1$ if the replacement is unwarranted and $z_i = 0$ otherwise. Before date d_0, the realizations of those latent random variables Z_i are not observed if $\delta_i = 1$, otherwise, when the inspection occurs after d_0 or if $\delta_i = 0$, we have $z_i = 0$. Note that in this case the z_i are not to be considered as data or parameters since they do not bring any information. But we introduce them to obtain simpler formulae.

Thus, the incomplete data structure is as follows:

- The observed data are the calendar date d_0, the inspection times t_i and the indicators of replacements δ_i for $i = 1, \ldots, n$, n being the number of inspections.
- The missing or hidden data are the indicators z_i of unwarranted replacements for $i = 1, \ldots, n_0$, n_0 $(1 < n_0 < n)$ being the number of inspections before d_0.

Note that n_0 is entirely defined by d_0 and should not be considered as a datum. Actually, the sample is sorted as follows: the first n_0 times correspond to inspections before date d_0 and the last $n - n_0 + 1$ ones correspond to the period after d_0.

2.1 *Maximum likelihood estimation*

We describe now how to obtain the ML estimator of η. First we derive the observed likelihood of η from the complete likelihood of η associated with the complete data. Owing to the lack of memory property of the exponential distribution, we have $R(t + t') = R(t)R(t')$ for all t, t'. Thus, the log-likelihood of η knowing the complete data $\mathbf{c} = (d_0, (t_i, \delta_i, z_i), i = 1, \ldots, n))$ is

$$\begin{aligned} L(\eta; \mathbf{c}) = &\sum_{i=n_0+1}^{n} \log[F(t_i \mid \eta)^{\delta_i} R(t_i \mid \eta)^{1-\delta_i}] \\ &+ \sum_{i=1}^{n_0} \log[\{R(t_i \mid \eta)^{z_i} F(t_i \mid \eta)^{1-z_i}\}^{\delta_i} R(t_i \mid \eta)^{1-\delta_i}]. \end{aligned} \tag{3}$$

Then, the observed likelihood of η knowing the observed data $\mathbf{o} = (d_0, (t_i, \delta_i), i = 1, \ldots, n))$ is deduced from the complete loglikelihood by summation over the possible values of the missing data z_i (see McLachlan & Krishnam (1997)), we get

$$\begin{aligned} L(\eta; \mathbf{o}) = &\sum_{i=1}^{n_0} \log R(t_i \mid \eta) + \delta_i \log(F(t_i \mid \eta)) \\ &+ \sum_{i=n_0+1}^{n} \delta_i \log(F(t_i \mid \eta)) + (1 - \delta_i) \log R(t_i \mid \eta). \end{aligned} \tag{4}$$

A popular and powerful tool to derive ML estimates for the incomplete data model is the EM algorithm (McLachlan & Krishnam, 1997). It consists of two steps. The E step is computing the conditional expectation of the 'missing' data knowing the observed data and a current value of the parameter to be estimated, and the M step is maximizing the likelihood with respect to the parameter the condition expectation of the 'missing' data.

Here it is noteworthy that, because of the presence of left censored data, the M step of EM is not closed form and does not exhibit simplification compared to a direct maximisation of the observed likelihood (4). Thus, we are in a rare situation, where there is no advantage to using EM rather than a standard method such as scoring or Newton–Raphson to maximise (4). But the analysis of the problem as an incomplete data structure model allowed us to derive the observed loglikelihood in a simple way. Moreover, this presentation appears to be useful to derive a Bayesian estimate of η, as seen in the next section.

2.2 *Bayesian estimation via Gibbs sampling*

In a small-sample setting with a few notified replacements, it can happen that the ML estimator gives unreliable results (typically as soon as $n < 20$). In such cases a Bayesian approach can be desirable to provide a regularized estimate of η. In this section, we consider Bayesian inference for our problem and we work on the parameter $\lambda = 1/\eta$ rather than η to present the equations in a simpler way. From the choice of a prior distribution $\pi(\lambda)$ for λ which is considered as a random vector, Bayesian inference consists of deriving parameter estimates from features of the posterior distribution $\pi(\lambda|\mathbf{o})$ which, using Bayes' theorem, is defined as

$$\pi(\lambda|\mathbf{o}) = \frac{\pi(\lambda)P(\mathbf{o}|\lambda)}{\int \pi(\lambda)P(\mathbf{o}|\lambda)\,\mathrm{d}\lambda},$$

where $P(\mathbf{o}|\lambda)$ denotes the likelihood of the parameter λ for the data $\mathbf{o} = (d_0, (t_i, \delta_i), i = 1, \ldots, n))$. Then, the Bayesian estimate of λ for the quadratic loss function is the posterior expectation of λ (see Robert (1994))

$$E(\lambda|\mathbf{o}) = \frac{\int \lambda\pi(\lambda)P(\mathbf{o}|\lambda)\,\mathrm{d}\lambda}{\int \pi(\lambda)P(\mathbf{o}|\lambda)\,\mathrm{d}\lambda}.$$

In our setting, calculating this posterior expectation is a difficult task. Markov chain Monte Carlo methods aim to evaluate posterior expectations by Monte Carlo integration using simulated Markov chains (Casella & George, 1992; Gilks *et al.*, 1996). More precisely, we make use of Gibbs sampling to approximate the posterior distribution of λ. Gibbs sampling consists of sampling from the full conditional distributions involved in the model to get a Markov chain whose stationary distribution is precisely the desired unconditional posterior distribution.

Here, we make use of a conjugate prior distribution for λ and we consider that λ is a random variable following a Gamma distribution $\mathcal{G}(a, b)$ with parameters a and b defined such that the mean of this distribution is a/b and its variance is a/b^2 (Robert, 1994). The posterior distribution of λ is approximated through Gibbs sampling which consists of repeating the following steps from an initial value η_0:

◇ **Repeat G times**

• **For each i in** $[1, n]$

draw z_i in the following way:

∘ *If* $i \leqslant n_0$

$* \text{ If } \delta_i = 1, \begin{cases} z_i = 1 & \text{with probability} \quad R(t_i \mid \eta), \\ z_i = 0 & \text{with probability} \quad F(t_i \mid \eta)\cdot \end{cases}$

$*$ If $\delta_i = 0$, $z_i = 0$.

∘ *If* $i > n_0$, $z_i = 0$.

The degradation times are drawn in the following way:

$*$ If $\delta_i = 1$ and $z_i = 0$, then draw $\tilde{t}_i$ from $\mathcal{E}\left(\eta = \dfrac{1}{\lambda}\right)$ until $\tilde{t}_i < t_i$.

$*$ If $\delta_i = 0$ or if ($\delta_i = 1$ and $z_i = 1$), draw $\tilde{t}_i$ from $\mathcal{E}\left(\eta = \dfrac{1}{\lambda}\right)$ until $\tilde{t}_i > t_i$.

Draw $\left(\lambda \mid (\tilde{t}_i)_{1 \leqslant i \leqslant n}\right) \sim \mathcal{G}(a + n, \sum_i \tilde{t}_i + b)$.

End repeat

At the end of the procedure, the sequence $\{\lambda^g, g = g_0, \ldots, G\}$ can be regarded as a sample from the posterior distribution of λ. The integer g_0 defines the length of the burn-in. It must be chosen large enough to ensure that the chain has ‘forgotten’ its starting position. The integer G has to be chosen large enough to ensure that the posterior distribution of λ is well approximated. The theoretical specification of g_0 and G is a difficult open problem (see Gilks *et al.*, 1996). In the experiments reported hereafter we chose g_0 and G on an empirical ground. It appears that $g_0 = 1000$ and $G = 10\,000$ provide good estimates of the posterior mean of η by the formula

$$\hat{\eta} = \frac{1}{G - g_0} \sum_{g=g_0+1}^{G} \eta^g.$$

3. Numerical experiments

In this section, we present the failure time ML estimates derived from our model. The ML estimate of the parameter η of the exponential distribution $\mathcal{E}(\eta)$ is derived by maximizing (4) by a scoring method. We first present an illustration on a real data set. Then we assess the ability of our procedure to reduce the pessimistic bias caused by the unwarranted replacements through Monte Carlo experiments. In this section we also illustrate the possible interest of the Bayesian approach in some circumstances.

3.1 *A real data set*

The results on the data of Fig. 1 are presented in Table 1. For this example, concerning a component of 900 MWRCPs, we have $n = 68$ and before $d_0 = 1992$, each inspection leads to a replacement. In this table, η_m represents the pessimistic ML estimate of η where all the replacements are considered warranted, η_M represents the optimistic ML estimate of η

TABLE 1 *Estimation of η for data corresponding to Fig. 1*

d_0	% repl. before d_0	% repl. after d_0	η_m	η_M	η_{ml}
1992	100	20	168 000	351 700	178 200

where all the replacements before d_0 are considered unwarranted, and η_{ml} represents the ML estimate of η derived from our model. In this case, the pessimistic bias does not seem to be important. The reason for that is that there are only $n_0 = 8$ inspections before 1992, and even if all those inspections lead to a replacement, there are not enough to influence greatly the η estimate. But, since we do not know its true value, it is difficult to assess the ability of our procedure to eliminate the pessimistic bias from such a real example. For this reason, we now turn to Monte Carlo experiments.

3.2 *Simulation*

The shock model defined and simulated according to our 'change maintenance behaviour' model is as follows. Assume that a system receives shocks at random. A comprehensive reference on shock models is Gaudoin & Soler (1997). The distribution of the time between two shocks is a Gamma distribution $\mathcal{G}(1/s, \eta)$ and shocks are independent events. We suppose that after s shocks there is the need for a replacement of the system. Hence, the distribution of time between two replacements is an exponential distribution $\mathcal{E}(\eta)$. We suppose the existence of over-cautious behaviour for the first n_0 inspections among n: maintenance engineers decide to replace the system after $s' < s$ shocks. But for the remaining $n - n_0$ inspections, a replacement is decided after s shocks. The statistical problem is to estimate η. Finally, this shock model depends of the following quantities:

- n is the total number of inspections,
- n_0 is the number of inspections for which a unwarranted replacement is possible (as already noted, it is equivalent to give n_0 or the date d_0 of change of maintenance behaviour),
- s the number of shocks producing a warranted replacement,
- s' the number of shocks producing an unwarranted replacement,
- η the parameter to be estimated of the exponential distribution.

We simulated this shock model for different parameter values given in Table 2 and with $\eta = 20\,000$ and $s = 10$. Each simulated situation was replicated 100 times. This table provides the mean values and in parentheses the standard error of the pessimistic ML estimate η_m, the optimistic ML estimate η_M and the ML estimate η_{ml} derived from the model presented in Section 2. The results are satisfactory. Our procedure provides reasonable estimates of η and the pessimistic bias is essentially eliminated. As expected, results are more variable for the small sample size $n = 20$, and the ML estimate η_{ml} seems to be less reliable when $n = 20$ and $s' = 7$ and appears to be somewhat optimistic.

Thus for this particular case ($n = 20, n_0 = 10, s = 10, s' = 7$), we run the Gibbs sampler defined in Section 2.2 with the prior distribution $G(1/10, 200\,000)$. It corresponds

TABLE 2 *Estimation of η for simulated data from the shock model*

n	n_0	s'	η_m	η_M	η_{ml}
100	50	5	13 120 (*1801*)	46 828 (*11 053*)	19 637 (*2006*)
100	30	5	15 505 (*2229*)	32 055 (*5 770*)	19 935 (*2399*)
100	70	5	11 209 (*1450*)	87 423 (*26 100*)	19 598 (*1557*)
20	10	5	14 297 (*4683*)	$>$ 1000 000	20 913 (*49 34*)
20	10	3	11 125 (*3133*)	$>$ 1000 000	18 275 (*3614*)
20	10	7	18 147 (*9630*)	$>$ 1000 000	24 190 (*10 051*)
100	50	7	15 833 (*2493*)	48 511 (*11 475*)	21 911 (*2651*)

TABLE 3 *ML and Bayesian estimates of η for three samples with a few replacements*

no. of repl.	η_m	η_M	η_{ml}	η_{ba}
7	23 214	44 814	28 999	18 801
9	16 727	34 761	22 407	18 169
9	16 727	44 814	23 536	18 423

to a good prior information since its mean value is 20 000 but it is a weak prior distribution since its variance is 4×10^9. We aim to see if Bayesian inference can regularize the ML estimate in a small-sample setting. Since running 10 000 iterations of the Gibbs sampler is rather slow, we do not perform Monte Carlo experiments, but we run it for ten different samples. From those experiments, it appears that when the number of replacements is over 20 possible replacements, the ML and the Bayesian estimates are not really different and are both satisfactory. But when the number of replacements is less than 10, it appears that the Bayesian estimate can be better than the ML estimate as illustrated in Table 3 for the three following samples for which we get a number of replacements smaller than 10. In this table, the Bayesian estimate is denoted η_{ba}.

4. Discussion

We have proposed a model to tackle an estimation problem related to a human factor, namely a tendency to replace a system too early in the first years of its life. Regarding this problem as an incomplete structure model, we propose ML estimation of the parameter of the failure time distribution, eliminating the pessimistic bias induced by this cautious behaviour. Numerical experiments show that our procedure applied to an exponential distribution works well. We also describe a Bayesian analysis of our incomplete structure model. This Bayesian approach makes use of Gibbs sampling and is much more time consuming than the ML approach. Also, the need to specify a prior distribution for the failure time parameter is not an easy task. For an exponential distribution, it does not seem that the Bayesian analysis of our model is very useful except in a small-sample setting with a few replacements available and good prior knowledge. But here again, the Bayesian approach can be more reliable than the ML estimation method in a more complex setting involving the Weibull distribution with small sample size (see Bacha *et al.*, 1998).

REFERENCES

BACHA, M., CELEUX, G., IDÉE, E., LANNOY, A. & VASSEUR, D. (1998) *Estimation de durées de vie fortement censurées*. Eyrolles.

CASELLA, G. & GEORGE, E. I. (1992) An introduction to Gibbs Sampling. *The American Statistician*, **46**, 167–174.

GAUDOIN, O. & SOLER, J. L. (1997) Failure rate behavior of components subjected to random stresses. *Reliability Engineering and System Safety*, **58**, 19–30.

GILKS, W. R., RICHARDSON, S. & SPIEGELHALTER, D. I. (1996) *Markov Chains Monte Carlo in Practice*. Chapman and Hall.

MCLACHLAN, G. & KRISHNAM, T (1997) *The EM Algorithm and Extensions*. New York: Wiley.

MEEKER, W. Q. & ESCOBAR, L. A. (1998) *Statistical Methods for Reliability Data*. New York: Wiley.

ROBERT, C. P. (1994) *The Bayesian Choice*. Berlin: Springer.