



HAL
open science

Presentation of the dynamical core of neXtSIM, a new sea ice model

Sylvain Bouillon, Pierre Rampal

► **To cite this version:**

Sylvain Bouillon, Pierre Rampal. Presentation of the dynamical core of neXtSIM, a new sea ice model. Ocean Modelling, 2015, 91, pp.23 - 37. 10.1016/j.ocemod.2015.04.005 . hal-03405141

HAL Id: hal-03405141

<https://hal.univ-grenoble-alpes.fr/hal-03405141>

Submitted on 27 Oct 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Presentation of the dynamical core of neXtSIM, a new sea ice model

Sylvain Bouillon^{a,b}, Pierre Rampal^{a,b}

^a*Nansen Environmental and Remote Sensing Center, Bergen, Norway*

^b*Bjerknes Centre for Climate Research, Bergen, Norway*

Abstract

The dynamical core of a new sea ice model is presented. It is based on the Elasto-Brittle rheology, which is inspired by progressive damage models used for example in rock mechanics. The main idea is that each element can be damaged when the local internal stress exceeds a Mohr-Coulomb failure criterion. The model is implemented with a finite element method and a Lagrangian advection scheme. Simulations of 10 days are performed over the Arctic at a resolution of 7 km. The model, which has only a few parameters, generates discontinuous sea ice velocity fields and strongly localized deformation features that occupy a few percent of the total sea ice cover area but accommodate most of the deformation. For the first time, a sea ice model is shown to reproduce the multifractal scaling properties of sea ice deformation. The sensitivity to model parameters and initial conditions is presented, as well as the ability of the Lagrangian advection scheme at preserving discontinuous fields.

Keywords: sea ice model, Lagrangian, rheology, multifractal scaling, Arctic

Email address: sylvain.bouillon@nersc.no (Sylvain Bouillon)

1. Introduction

Sea ice dynamics, and more specifically its brittle deformation, exhibit scale invariance properties in both the temporal and spatial domains (Marsan and Weiss, 2010; Weiss, 2013). Scale invariance is a frequent characteristic of dynamical systems where energy introduced at large scale is redistributed towards smaller scales, down to the dissipation scale (e.g., the development of turbulence down to the viscous dissipation scale). In the case of sea ice, the kinetic energy is mainly coming from the wind stress, which varies over typical time and length scales $T_{wind} \approx 3 - 6$ days and $L_{wind} \approx 100 - 1000$ km, respectively. A large part of this energy is transferred to the ocean but a non-negligible part is dissipated by friction during sea ice fracturing events. These events last a few minutes (Marsan et al., 2011) and occur along faults of tens of meters (Schulson, 2004). Above this dissipation scale, sea ice drift and deformation show scaling properties over several orders of magnitude, from a few hours to a few months, and from hundreds of meters to hundreds of kilometers (Marsan et al. (2004), Rampal et al. (2008)). These properties are in fact quite universal in complex dynamical systems and are likely to emerge from the interaction of a large number of components rather than from a specific process occurring at small scales. This explains for example why simplistic models such as random fuse or random spring models are capable of reproducing complex statistical properties observed for failure in disordered materials, e.g. damage localization and power law distribution of avalanche size (Nukala et al., 2005). The external forcing is one source of scaling in the sea ice dynamics, and should become predominant as the ice cover is more fractured. However, the statistical properties of sea ice

26 dynamics differ from those of ocean and atmosphere dynamics (Rampal et al.,
27 2009). An important characteristic of sea ice dynamics is the multifractality
28 of the scale invariance of sea ice deformation (Weiss and Marsan, 2004), which
29 seems to emanate from the intrinsic properties of solid materials characterized
30 by brittle mechanical behavior (Weiss, 2013).

31 To correctly reproduce scale invariance properties of sea ice dynamics
32 may be important to better understand the exchanges of energy between
33 the ocean and the atmosphere, which are highly influenced by the opening
34 and closing of leads in the ice cover. In winter, deformation contributes to
35 about 25-40% of the ice production (Kwok, 2006) and the presence of leads,
36 which cover only a few percent of the domain, may account for more than
37 70% of the upward heat fluxes (Marcq and Weiss, 2012) and for half the salt
38 rejection (Morison and McPhee, 2001). To correctly forecast sea ice motion
39 and deformation would also give crucial information (e.g. the presence of
40 ridges) for ship operations in ice covered areas. Therefore, we think that sea
41 ice models used for forecasting and climate studies should be also evaluated
42 regarding their ability to reproduce the statistical properties of sea ice drift
43 and deformation.

44 This paper presents the dynamical core of a new sea ice model called
45 neXtSIM, which is based on an innovative mechanical modeling framework.
46 Sea ice dynamics are simulated using an adapted and optimized version of
47 the Elasto-Brittle rheology originally presented in Girard et al. (2011), which
48 initially was inspired by a progressive damage model used to simulate rock
49 mechanics (Amitrano et al., 1999). The main ingredients of this dynam-
50 ical sea ice model are detailed, and the ability of the model to generate

51 sea ice deformation fields having correct statistical and scaling properties is
52 demonstrated. An extensive sensitivity study is performed to evaluate the
53 pertinence of each key ingredient of the model. In section 2 we present the
54 main equations of the model. Section 3 describes how these equations are
55 discretized in space and time and which advection scheme the model uses.
56 Section 4 shows the results of a reference simulation of 10 days over the cen-
57 tral Arctic, for which we also present a sensitivity analysis with respect to
58 initial conditions and to some key sea ice mechanical parameters. Note that
59 for short time scale simulations, we assume the impact of thermodynamical
60 processes on the dynamics as being negligible. This study is the first step
61 towards a more complete presentation of neXtSIM, in which e.g. sea ice
62 thermodynamics should be implemented. We do not present a comparison
63 of the simulated fields to observed fields in order to keep this paper focused
64 on the description of the model and to make it accessible to a large scientific
65 audience. The evaluation of the predictive skill of the model or its impact
66 on other components of the climate system is therefore out of scope of this
67 paper.

68 **2. Model description**

69 At the present stage, the dynamical component of neXtSIM is kept as
70 simple as possible and has only five prognostic variables. h , hereafter called
71 sea ice thickness, is the volume of ice per unit area and A , hereafter called
72 sea ice concentration, is the surface of ice per unit area. \mathbf{u} is defined as the
73 horizontal sea ice velocity and $\boldsymbol{\sigma}$ is the internal stress tensor. The damage
74 d is a non-dimensional scalar variable, which is equal to 0 for undamaged

75 material and to 1 for completely damaged material.

76 One of the objectives for the model is to reproduce the failure zones
77 that are observed from satellites at a resolution of 10 km. As in Hutchings
78 et al. (2005), we assume that sea ice is heterogeneous at the scale of the
79 model, which corresponds to its resolution Δx (here about 10 km). The sea
80 ice thickness, concentration, damage, internal stress and deformation rate
81 tensors are defined for each element and could strongly vary from one element
82 to the next one. The velocities are defined at the corners of each element.
83 Our model is continuous and uses a Lagrangian approach, i.e. while the nodes
84 are moving accordingly to the ice motion the elements remain connected and
85 always cover the same domain. Eulerian approaches might also be used but
86 then one should use advection schemes that are able to transport highly
87 heterogeneous fields while conserving the extreme gradients present at the
88 scale of the elements.

89 *2.1. Evolution of sea ice thickness, concentration and velocity*

90 The evolution equations for sea ice thickness, concentration and velocity
91 are similar to those used in most sea ice models. When the thermodynamics
92 terms are neglected, the evolution of h and A are given by:

$$\frac{Dh}{Dt} = -h\nabla \cdot \mathbf{u}, \quad (1)$$

$$\frac{DA}{Dt} = -A\nabla \cdot \mathbf{u}, \quad (2)$$

93 where $\frac{D\phi}{Dt}$ is the material derivative of ϕ (being either a scalar or a vector).

94 A is limited to a maximum value of 1.

95 The evolution of sea ice velocity comes from the vertically integrated sea

96 ice momentum equation :

$$\rho_i h \frac{D\mathbf{u}}{Dt} = \nabla \cdot (\boldsymbol{\sigma}h) + A(\boldsymbol{\tau}_a + \boldsymbol{\tau}_w) - \rho_i h f \mathbf{k} \times \mathbf{u} - \rho_i h g \nabla \eta, \quad (3)$$

97 where ρ_i is the ice density, $\boldsymbol{\tau}_a$ and $\boldsymbol{\tau}_w$ are the surface wind (air) and ocean
 98 (water) stresses, respectively, f is the Coriolis parameter, \mathbf{k} is the upward
 99 pointing unit vector, g is the gravity acceleration and η is the ocean surface
 100 elevation.

101 It should be noted that in the sea ice community the term internal stress
 102 often refers to the vertically integrated (or depth-integrated) internal stress,
 103 which has units of Nm^{-1} . Such a definition may lead to confusion as in Girard
 104 et al. (2011) where the integrated internal stress (in Nm^{-1}) was compared to
 105 cohesion and tensile strength defined in Nm^{-2} (Pa). To avoid confusion, we
 106 introduce the integration of the internal stress $\boldsymbol{\sigma}$ (in Nm^{-2}) in the momen-
 107 tum equation as in Sulsky et al. (2007). The internal stress is assumed to
 108 be homogeneously distributed in the ice volume and $\boldsymbol{\sigma}h$ corresponds to the
 109 integral of the internal stress within that volume.

110 The surface wind (air) and ocean (water) stresses, $\boldsymbol{\tau}_a$ and $\boldsymbol{\tau}_w$ respectively,
 111 are both multiplied by the sea ice concentration as in Connolley et al. (2004)
 112 and Hunke and Dukowicz (2003). The air stress $\boldsymbol{\tau}_a$ is computed following
 113 the quadratic expression:

$$\boldsymbol{\tau}_a = \rho_a c_a |\mathbf{u}_a| [\mathbf{u}_a \cos \theta_a + \mathbf{k} \times \mathbf{u}_a \sin \theta_a], \quad (4)$$

114 where \mathbf{u}_a is the air velocity, ρ_a the air density, θ_a the air turning angle and
 115 c_a the air drag coefficient. The water stress $\boldsymbol{\tau}_w$ is computed following the

116 quadratic expression:

$$\boldsymbol{\tau}_w = \rho_w c_w |\mathbf{u}_w - \mathbf{u}| [(\mathbf{u}_w - \mathbf{u}) \cos \theta_w + \mathbf{k} \times (\mathbf{u}_w - \mathbf{u}) \sin \theta_w], \quad (5)$$

117 where \mathbf{u}_w is the ocean velocity, ρ_w the reference density of seawater, θ_w the
 118 water turning angle and c_w the water drag coefficient.

119 *2.2. Evolution of sea ice internal stress and damage*

120 The evolution of sea ice internal stress and damage is based on three main
 121 ingredients: the linear elasticity, the failure envelope and the link between
 122 local damage and internal stress.

123 *2.2.1. Linear elasticity*

124 Assuming planar stress and linear elasticity as in Girard et al. (2011),
 125 Hooke's law in matrix notation (see for example Bower (2011) for a reference
 126 textbook) is given by:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \frac{E(A,d)}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ 2\epsilon_{12} \end{bmatrix}, \quad (6)$$

127 where $E(A, d)$ is the effective elastic stiffness, which here is assumed to de-
 128 pend on the concentration and damage. ν is Poisson's ratio, which is set here
 129 to 0.3, which is in the range of value discussed in Mellor (1986). To simplify
 130 notation, equation 6 may also be written in tensor notation as:

$$\boldsymbol{\sigma} = \mathbf{C}(A, d) : \boldsymbol{\epsilon}, \quad (7)$$

131 and in index notation as: $\sigma_{ij} = C_{ijkl}\epsilon_{kl}$.

132 The deformation response is then controlled by the effective elastic stiff-
133 ness, which is defined as:

$$E(A, d) = Yf(A)(1 - d), \quad (8)$$

134 where Y is the sea ice elastic modulus (Young's modulus) and $f(A)$ is a
135 function equal to 1 when $A = 1$. This formulation of the effective elastic
136 stiffness is similar to the one proposed in Girard et al. (2011) except that
137 the linear dependence to h is now explicitly described in the momentum
138 equation by the integration of the internal stress, and that we use a different
139 convention for the damage d , which is equal to 0 for undamaged sea ice and
140 to 1 for completely damaged sea ice.

141 Unlike in Girard et al. (2011) where the Young's modulus was tuned (to
142 0.35 GPa) to get the right order of magnitude for the mean total deformation,
143 here we obtain realistic mean deformation when using a value of 9 GPa, i.e.
144 in the range of in-situ measurements (Schulson, 2009). The value of the
145 Young modulus does not affect the value of the cohesion, nor the failure
146 envelope. It does not impact the magnitude of the internal stress but only
147 the link between the internal stress and elastic deformation, the latter being
148 much smaller than the observed deformation. Changing the value of the
149 Young modulus modifies the elastic deformation but has no other significant
150 impacts as long as we set it to a high enough value. This was checked by
151 running a series of experiments with Y set to 9, 0.9 and 0.09 GPa respectively.

152 The impact of the concentration on the effective elastic stiffness is not
153 known and thus has to be parameterized. We assume that for low values of

154 concentration, the effective stiffness should be very low so that deformation
155 could arise without impacting the internal stress. In this study, we use the
156 same parameterization as in Girard et al. (2011):

$$f(A) = e^{\alpha(1-A)}, \quad (9)$$

157 where $\alpha \leq 0$ is a constant parameter. The sensitivity of the model to this
158 parameter is presented in section 4. This function is similar to the one used
159 in standard VP rheologies to parameterize the effect of the concentration
160 on the ice strength P , which determines the size of the plastic envelop (Hi-
161 bler, 1979). In our case, sea ice concentration has no impact on the failure
162 envelope, which is determined instead by the cohesion parameter. In the
163 future, more elaborate parameterization based on energetic considerations
164 (Thorndike et al., 1975) or on simulations with ensemble of floes (Herman,
165 2013) may be needed to increase the realism of the model results. Another
166 difference with the plastic approach is the absence of flow rule. Defining a
167 flow rule for sea ice is questionable since sea ice does not behave plastically
168 in the von Mises sense of plasticity (Weiss et al., 2007). In our case, defining
169 a flow rule is not necessary as the model assumes that the ice deforms as
170 an elastic medium (linearly with respect to the external force), whose elastic
171 stiffness evolves over time.

172 2.2.2. *Failure envelope*

173 In-situ measurements made by Richter-Menge et al. (2002) indicate that
174 sea ice internal stresses remains in an envelope, which is well represented by
175 a combination of a Mohr-Coulomb criterion, a tensile stress criterion and a

176 compressive stress criterion (see Figure 2 in Weiss et al. (2007)).

177 The Mohr-Coulomb criterion is defined by:

$$\tau \leq -\mu\sigma_N + c, \quad (10)$$

178 where μ is the friction coefficient and c is the cohesion, which is assumed
179 to be always greater than 0 in the following discussion. The shear stress
180 τ and the normal stress σ_N (also called tensile/compressive stress when it
181 is positive/negative) are two invariants of the internal stress tensor and are
182 defined by:

$$\tau = \sqrt{\left(\frac{\sigma_{11} - \sigma_{22}}{2}\right)^2 + \sigma_{12}^2}, \quad (11)$$

$$\sigma_N = \frac{\sigma_{11} + \sigma_{22}}{2}. \quad (12)$$

183 The tensile stress criterion and the compressive stress criterion are defined
184 by:

$$\sigma_N \leq \sigma_{Nmax}, \quad (13)$$

185 and

$$\sigma_N \geq \sigma_{Nmin}, \quad (14)$$

186 where $\sigma_{Nmax} > 0$ and $\sigma_{Nmin} < 0$ are the maximal tensile stress and the
187 maximal compressive stress, respectively. Of course, σ_{Nmax} has to be lower
188 than $\frac{1}{\mu}c$ to be effective.

189 As in Girard et al. (2011), the friction coefficient μ for sea ice is chosen
190 equal to 0.7, which is a common value for geo-materials (Amitrano et al.,

191 1999). This value is consistent with results from laboratory tests (Schulson
192 et al., 2006) and seems to be scale-independent (Weiss and Schulson, 2009).

193 In contrast, the value of the cohesion c depends on the spatial scale (Weiss
194 et al., 2007) according to the following relationship:

$$\frac{c_1}{c_2} \approx \left(\frac{l_2}{l_1} \right)^{0.5}, \quad (15)$$

195 where l_1 and l_2 correspond to the estimated size of the stress concentra-
196 tor at two different scales (Schulson, 2004). At the laboratory scale (a few
197 centimeters), the cohesion is estimated to be about 1 MPa, whereas in-situ
198 measurements (scale of a few meters, $l = 1$) give a value of about 40 kPa
199 (Weiss et al., 2007). By using the scaling relationship (equation 15) and as-
200 suming that the maximum size of stress concentrators "seen" by our model
201 is equal to the resolution Δx (here about 10 km), the maximum value for
202 the cohesion parameter c is set to 8 kPa. In order to study the sensitivity
203 of the model to the cohesion parameter c (see section 4), we arbitrary define
204 a set of plausible values for c (8, 4, 2, 1 and 0.5 kPa). These values for
205 c correspond to stress concentrator sizes ranging from 10 km to 25 m. It
206 should be noted that in our case all the elements have the same value for
207 the cohesion. To randomly draw the value of the cohesion from a uniform
208 distribution as done in Girard et al. (2011) does not seem to be necessary
209 for a realistic set up (i.e., complex geometry, initial conditions and forcings).
210 We tested that using a cohesion that is uniformly distributed between 0.5 c
211 and 1.5 c produces similar results than using a constant value c .

212 The maximal tensile stress and maximal compressive stress should scale

213 in the same way as c (Schulson, 2009). From in-situ measurements, Weiss
214 et al. (2007) estimated the maximal tensile stress σ_{Nmax} as equal to 50 kPa
215 and the maximal compressive stress σ_{Nmin} as at least equal to -100 kPa,
216 when the cohesion c is equal to 40 kPa (i.e., for the scale $l = 1$ m). From
217 these observations, we deduce the following relationships, $\sigma_{Nmax} = \frac{5}{4}c$ and
218 $\sigma_{Nmin} = -\frac{5}{2}c$, that are used to define the upper and lower limits on the
219 normal stress. It should be noted that in the data analyzed by Weiss et al.
220 (2007), highly biaxial compression stress states are absent, meaning that
221 σ_{Nmin} could actually be much lower. We verified that using lower values
222 for σ_{Nmin} does not significantly impact the results presented in this paper.
223 However, in longer simulations, it may affect the spatial distribution of the
224 sea ice thickness, for example, when the ice is constantly pushed towards
225 the coast. Comparing simulated sea ice thickness fields to observations could
226 help up to better determine the value of σ_{Nmin} to be used in the model.

227 *2.2.3. Internal stress and damage evolution*

228 In nature, the formation of a network of faults within a continuous sea ice
229 cover is associated with avalanches of local damage events that propagates
230 through the ice at the speed of the elastic waves. To reproduce this very
231 rapid propagation process, the model presented in Girard et al. (2011) used
232 a sub-iteration loop within each time step and a constant damage factor d_0 .
233 In our model we do not use sub-iteration and the damage factor Ψ is variable.
234 The two approaches ensure that the internal stress is within a failure envelope
235 at each time step. In our case the damage is still propagated but at a speed
236 limited by the ratio $\frac{\Delta x}{\Delta t}$. For example at a resolution of 10 km and with a
237 model time step of 800 s, it means that the damage could propagate in 3

238 days (i.e., the typical time scale at which sea ice motion is estimated from
 239 SAR-images) over 3240 km, i.e. about the size of the Arctic basin. To not use
 240 sub-iterations has no significant impact on the simulated sea ice deformation
 241 fields but has the advantage of reducing significantly the computational time.

242 In our model, the evolution of the damage is controlled by two terms, a
 243 damaging term (source) and a relaxation term (sink) corresponding to the
 244 recovery of the ice mechanical strength (i.e., healing). The evolution equation
 245 for the damage is written as:

$$\frac{Dd}{Dt} = \frac{\Delta d}{\Delta t} - \frac{d}{T_d}, \quad (16)$$

246 where Δd is the damage source term, which is defined hereafter, and T_d is
 247 the damage relaxation time, which is supposed to be much larger than the
 248 model time step Δt .

249 To obtain the evolution equation for the internal stress, we compute the
 250 time derivative of equation 7. By assuming that the healing and the variation
 251 of the concentration do not influence the internal stress but only the elastic
 252 stiffness, we get the following equation:

$$\frac{D\boldsymbol{\sigma}}{Dt} = \frac{\Delta d}{\Delta t} \frac{\partial \mathbf{C}}{\partial d} : \boldsymbol{\epsilon} + \mathbf{C}(A, d) : \dot{\boldsymbol{\epsilon}}, \quad (17)$$

253 where the deformation rate tensor is defined by $\dot{\boldsymbol{\epsilon}} = \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$. The
 254 evolution of the internal stress is computed in two steps that would corre-
 255 spond to:

$$\frac{D\boldsymbol{\sigma}}{Dt} = \frac{\boldsymbol{\sigma}^{n+1} - \boldsymbol{\sigma}'}{\Delta t} + \frac{\boldsymbol{\sigma}' - \boldsymbol{\sigma}^n}{\Delta t}. \quad (18)$$

256 A first estimate of the internal stress, $\boldsymbol{\sigma}'$, is computed without considering
 257 the damaging process:

$$\frac{\boldsymbol{\sigma}' - \boldsymbol{\sigma}^n}{\Delta t} = \mathbf{C}(A^n, d^n) : \dot{\boldsymbol{\epsilon}}. \quad (19)$$

258 With this estimate, the failure criteria are checked. For the elements for
 259 which the estimated internal stress $\boldsymbol{\sigma}'$ falls outside the failure envelope, the
 260 damage factor Ψ is set to the value for which the stress state

$$\boldsymbol{\sigma}^{n+1} = \Psi \boldsymbol{\sigma}', \quad (20)$$

261 is set back on the failure envelope following the line crossing the origin of
 262 the normal and shear stress space. For the elements for which the estimated
 263 internal stress $\boldsymbol{\sigma}'$ is inside the failure envelope, Ψ is simply set to 1.

264 To obtain the damage source term Δd of equation 16, we rewrite the
 265 damage step (equation 20) as an evolution equation :

$$\frac{\boldsymbol{\sigma}^{n+1} - \boldsymbol{\sigma}'}{\Delta t} = \frac{(\Psi - 1)}{\Delta t} \boldsymbol{\sigma}'. \quad (21)$$

266 As the left hand side of equation 21 corresponds to the first term on the right
 267 hand side of equation 18, we deduce that the right hand side of equation 21
 268 corresponds to the first term on the right hand side of equation 17:

$$\frac{\Delta d}{\Delta t} \frac{\partial \mathbf{C}}{\partial d} : \boldsymbol{\epsilon} = \frac{(\Psi - 1)}{\Delta t} \boldsymbol{\sigma}'. \quad (22)$$

269 We then derive the following expression

$$\frac{\partial \mathcal{C}}{\partial d} : \boldsymbol{\epsilon} = -\frac{1}{(1-d)} \boldsymbol{\sigma}, \quad (23)$$

270 by using the equivalence between equations 6 and 7 and the fact that $\frac{\partial E(A,d)}{\partial d} =$
271 $-Y f(A) = -\frac{1}{(1-d)} E(A, d)$. Equation 23 is introduced in equation 22 and the
272 terms are rearranged to finally obtain the equation for the damage source
273 term

$$\Delta d = (1 - \Psi)(1 - d^n). \quad (24)$$

274 The variation of the damage has exactly the same form as in Girard et al.
275 (2011), except that in our case the damage factor is not a constant chosen
276 empirically but is computed locally to bring the internal stresses back onto
277 the failure envelope in one time step. The increase of the damage induces
278 a decrease of the effective elastic stiffness. The damaged sea ice deforming
279 more easily, this may trigger new damaging events in the surrounding cells.

280 **3. Implementation**

281 The rheology generates discontinuities in the simulated fields at the scale
282 of the elements (e.g., highly localized deformation). This constrains many
283 aspects of the implementation of the model. This section describes the tem-
284 poral and spatial discretizations of the equations, as well as the Lagrangian
285 advection scheme, which is preferred to classical Eulerian schemes for its
286 natural ability at transporting highly heterogeneous fields without modify-
287 ing their spatial properties.

288 *3.1. Temporal discretization*

289 The first step consists in solving together the evolution equations for \mathbf{u}
 290 and $\boldsymbol{\sigma}$. In Girard et al. (2011), the quasi-static assumption implied that no
 291 time evolution term was present in the momentum and the internal stress
 292 equations. In our case, both equations have a time derivative and are coupled
 293 together via the elastic term. To avoid the stability constrain due to elas-
 294 tic waves, the momentum and internal stress evolution equations are solved
 295 together with an implicit scheme as follows:

$$\begin{aligned}
 \rho_i h^n \frac{\mathbf{u}^{n+1} - \mathbf{u}^n}{\Delta t} = & \nabla \cdot (h^n \boldsymbol{\sigma}') \\
 & + A^n \rho_a c_a |\mathbf{u}_a|_e (\mathbf{u}_a \cos \theta_a + \mathbf{k} \times \mathbf{u}_a \sin \theta_a) \\
 & + A^n \rho_w c_w |\mathbf{u}_w - \mathbf{u}^n|_e (\mathbf{u}_w - \mathbf{u}^{n+1}) \cos \theta_w \\
 & + A^n \rho_w c_w |\mathbf{u}_w - \mathbf{u}^n|_e \mathbf{k} \times (\mathbf{u}_w - \mathbf{u}^n) \sin \theta_w \\
 & - \rho_i h^n (f \mathbf{k} \times \mathbf{u}^* + g \nabla \eta), \tag{25}
 \end{aligned}$$

296 and

$$\frac{\boldsymbol{\sigma}' - \boldsymbol{\sigma}^n}{\Delta t} = \mathbf{C}(A^n, d^n) : \frac{1}{2} (\nabla \mathbf{u}^{n+1} + (\nabla \mathbf{u}^{n+1})^T). \tag{26}$$

297 Note that the symmetric part of the ocean drag term is treated implicitly,
 298 whereas the anti-symmetric part is treated explicitly to preserve the symme-
 299 try of the system that we need to solve. The Coriolis term is also treated
 300 explicitly to preserve the symmetry of the system. The operator $|\mathbf{u}|_e$ gives
 301 the norm of vector \mathbf{u} over an element. The sea ice velocity \mathbf{u}^* used in the

302 Coriolis term is defined as:

$$\mathbf{u}^* = \beta_0 \mathbf{u}^n + \beta_1 \mathbf{u}^{n-1} + \beta_2 \mathbf{u}^{n-2}, \quad (27)$$

303 where β_0 , β_1 and β_2 are the coefficients of the third order Adams-Bashfort
304 scheme (23/12, -16/12, 5/12), which is chosen for its stability (see Walters
305 et al. (2009) for a stability analysis of time-stepping methods for the Coriolis
306 term in a shallow water model). Using lower order schemes could be sufficient
307 in the case of sea ice but it has not been investigated in the present study. For
308 the first and second time steps, the coefficients are those of the first order (1,
309 0, 0) and second order (3/2, -1/2, 0) Adams-Bashfort schemes, respectively.

310 The second step consists in verifying the failure criteria and in computing
311 for each element the damage factor Ψ as explained in Section 2.2.3. The new
312 internal stress $\boldsymbol{\sigma}^{n+1}$ and damage d' are then given by:

$$\boldsymbol{\sigma}^{n+1} = \Psi \boldsymbol{\sigma}', \quad (28)$$

$$(d' - d^n) = (1 - \Psi)(1 - d^n). \quad (29)$$

313 The third step consists in updating the damage due to healing:

$$\frac{d^{n+1} - d'}{\Delta t} = -\frac{d'}{T_d}, \quad (30)$$

314 and the last step of the time stepping procedure consists in performing the
315 advection of the different quantities.

316 *3.2. Spatial discretization*

317 The sea ice thickness, concentration and damage are defined as scalars
 318 at the center of each triangle, whereas the velocity fields are piecewise linear
 319 with nodal values defined at triangle vertices. The internal stress tensor,
 320 whose evolution is a function of the sea ice velocity gradient, is constant
 321 within each triangle.

322 The spatial discretization of the momentum equation is not trivial since
 323 it is strongly coupled to the evolution of the internal stress. Now that the
 324 temporal discretization is defined, we can regroup the terms depending on
 325 \mathbf{u}^{n+1} , the one depending on $\boldsymbol{\sigma}'$, and the rest, so that solving the momentum
 326 equation consists in finding the solution \mathbf{u}^{n+1} of this problem:

$$k\mathbf{u}^{n+1} + \nabla \cdot (h^n \boldsymbol{\sigma}') + \mathbf{f} = 0, \quad \forall \mathbf{x} \in \Omega, \quad (31)$$

327 with $\mathbf{u}^{n+1} = 0$ on the closed boundaries and $\mathbf{n} \cdot (h^n \boldsymbol{\sigma}') = 0$ on the open
 328 boundaries. \mathbf{n} is the outward pointing normal on the open boundary. k is a
 329 scalar function that does not depend on \mathbf{u}^{n+1} . \mathbf{f} is a vector regrouping all
 330 the terms that do not depend on \mathbf{u}^{n+1} and $\boldsymbol{\sigma}'$. Hereafter \mathbf{u}^{n+1} is simply noted
 331 \mathbf{u} and equation 26 is used to replace $\boldsymbol{\sigma}'$ by a linear combination of $\boldsymbol{\sigma}^n$ and
 332 the new deformation rate tensor, which is denoted by the function $\boldsymbol{\epsilon}(\mathbf{u})$.

333 The discretization of this problem is performed by following the classical
 334 methodology of the finite element method (see for example Hughes (2012)
 335 for a reference textbook), which is composed of two steps: the definition of
 336 the variational (or weak) formulation of the problem and the approximation
 337 of the solution in a functional space that can be entirely defined with a finite
 338 number of unknowns.

339 In the present case, the equivalent variational form is to find \mathbf{u} so that

$$\langle \hat{\mathbf{u}} \cdot (k\mathbf{u} + \nabla \cdot (h^n (\boldsymbol{\sigma}^n + \Delta t \mathbf{C} : \boldsymbol{\epsilon}(\mathbf{u}))) + \mathbf{f}) \rangle = 0, \quad \forall \hat{\mathbf{u}} \in \mathcal{U}, \quad (32)$$

340 where the bracket $\langle \rangle$ refers to the integral over the domain, $\hat{\mathbf{u}}$ are the test
 341 functions and \mathcal{U} is the functional space, which is here restricted to functions
 342 that cancel on closed boundaries.

343 By applying an integration by parts, the divergence theorem and the
 344 boundary conditions, we get:

$$k \langle \hat{\mathbf{u}} \cdot \mathbf{u} \rangle - h^n \langle (\nabla \hat{\mathbf{u}}) : \boldsymbol{\sigma}^n \rangle - h^n \Delta t \langle (\nabla \hat{\mathbf{u}}) : \mathbf{C} : \boldsymbol{\epsilon}(\mathbf{u}) \rangle + \langle \hat{\mathbf{u}} \cdot \mathbf{f} \rangle = 0, \quad \forall \hat{\mathbf{u}} \in \mathcal{U}. \quad (33)$$

345 After introducing $\boldsymbol{\epsilon}(\hat{\mathbf{u}})$, using the fact that $\frac{1}{2} (\nabla \hat{\mathbf{u}} - \nabla \hat{\mathbf{u}}^T) : \mathbf{C} : \boldsymbol{\epsilon}(\mathbf{u}) = 0$
 346 as it is a product of an anti-symmetric and a symmetric tensor, and regroup-
 347 ing the unknowns on the left hand side, we get

$$k \langle \hat{\mathbf{u}} \cdot \mathbf{u} \rangle - h^n \Delta t \langle \boldsymbol{\epsilon}(\hat{\mathbf{u}}) : \mathbf{C} : \boldsymbol{\epsilon}(\mathbf{u}) \rangle = h^n \langle (\nabla \hat{\mathbf{u}}) : \boldsymbol{\sigma}^n \rangle - \langle \hat{\mathbf{u}} \cdot \mathbf{f} \rangle, \quad \forall \hat{\mathbf{u}} \in \mathcal{U}. \quad (34)$$

348 Following the finite element method, an approximate solution \mathbf{u}^h is build
 349 as a linear combination of shape functions $\theta_j(\mathbf{x})$:

$$\mathbf{u}^h = \sum_{j=1}^m \mathbf{U}_j \theta_j(\mathbf{x}), \quad (35)$$

350 where \mathbf{U}_j are the coefficients (i.e., nodal values) for the basis function $\theta_j(\mathbf{x})$
 351 and m is the number of nodes. In our case, $\theta_j(\mathbf{x})$ are piecewise linear shape

352 functions defining the discrete sub-space $\mathcal{U}^h \subset \mathcal{U}$. \mathbf{f}^h , the approximation of
 353 \mathbf{f} , is built in the same way as \mathbf{u}^h with \mathbf{F}_j as nodal values.

354 As all possible approximated solutions \mathbf{u}^h and approximated test func-
 355 tions $\hat{\mathbf{u}}^h$ are built as linear combinations of the elements of the following
 356 base:

$$\left\{ \begin{bmatrix} \theta_1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \theta_1 \end{bmatrix}, \dots, \begin{bmatrix} \theta_m \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \theta_m \end{bmatrix} \right\}, \quad (36)$$

357 solving the discrete problem is equivalent to solving the linear system:

$$\sum_{j=1}^m \mathbf{A}_{ij} \mathbf{U}_j = \mathbf{B}_i \quad i = 1, \dots, m, \quad (37)$$

358 where \mathbf{A}_{ij} and \mathbf{B}_i are assembled by summing the contributions of each element
 359 to the integral over the domain as explained in the Appendix. The system
 360 is currently solved with CHOLMOD, which is based on supernodal sparse
 361 Cholesky factorization (Chen et al., 2008).

362 It should be noted that the finite element method does not require to
 363 define shape functions $\theta_j(\mathbf{x})$ in a unique global coordinate system. One can
 364 define a nodal coordinates system to avoid pole singularities and to solve the
 365 equations on any smooth surface as in Comblen et al. (2009). This approach
 366 is however not yet implemented in our model, and so we use for this study a
 367 polar stereographic projection to define the spatial coordinates $\mathbf{x} = (x, y)$.

368 3.3. Advection scheme

369 Most sea ice models use an Eulerian approach for the advection. However,
 370 we believe that a purely Lagrangian approach as in Wang and Ikeda (2004)

371 may be more appropriate. Purely Lagrangian schemes necessitate unstruc-
 372 tured meshes and a procedure for the mesh adaptation. Nowadays, efficient li-
 373 braries based on local mesh adaptation are available (see for example BAMG,
 374 <http://www.ann.jussieu.fr/hecht/ftp/bamg/bamg.pdf> or MAdLib, [http://](http://http://sites.uclouvain.be/madlib/)
 375 <http://sites.uclouvain.be/madlib/>). Local mesh modifications can
 376 be done in parallel and introduce very low numerical dissipation (Compère
 377 et al., 2009). It also verifies local conservation (Compère et al., 2008).

378 In the purely Lagrangian approach, the vertices of the element (i.e., the
 379 nodes of the grid) move with the sea ice velocity \mathbf{u} . The material derivative
 380 is then simply equal to the temporal derivative $\frac{\partial \phi}{\partial t}|_X$ relative to the moving
 381 mesh so that the quantities are naturally transported with the ice. The sea
 382 ice thickness and concentration are simply updated by:

$$h^{n+1} = h^n \frac{S^n}{S^{n+1}}, \quad (38)$$

383 and

$$A^{n+1} = \min \left(A^n \frac{S^n}{S^{n+1}}, 1 \right), \quad (39)$$

384 where S^n and S^{n+1} are the surface of the element at time steps n and $n + 1$.

385 In the Eulerian approach, the mesh is fixed and the transport of the
 386 different quantities from one cell to the others is computed by specific ad-
 387 vection schemes. High-order advection schemes have been widely developed
 388 for structured meshes (e.g. Prather, 1986). When using the finite element
 389 method in the Eulerian approach, the choice of the advection scheme de-
 390 pends on the order of the spatial discretization. In our case, the quantities

391 to be transported are represented by a scalar for each element. We could ei-
 392 ther choose an upwind scheme, which is highly diffusive or a centred scheme,
 393 which generates unrealistic oscillations (Hanert et al., 2004).

394 To illustrate the problem of numerical diffusion, we show an example when
 395 using an upwind Eulerian advection scheme for h and A . In this Eulerian
 396 approach, the mesh is fixed and the material derivative is defined as

$$\frac{D\phi}{Dt} = \frac{\partial\phi}{\partial t}\Big|_x + \mathbf{u} \cdot (\nabla\phi), \quad (40)$$

397 where $\frac{\partial\phi}{\partial t}\Big|_x$ is the temporal derivative of the variable relative to a fixed refer-
 398 ential. The sea ice thickness and concentration evolution within each element
 399 are computed by the budget of the upwind fluxes through its boundaries:

$$\frac{(h^{n+1} - h^n)}{\Delta t} = \frac{1}{S} \sum_{b=1}^3 h_b^n (\mathbf{u}_b^{n+1} \cdot \mathbf{n}_b) L_b, \quad (41)$$

400 and

$$\frac{(A^{n+1} - A^n)}{\Delta t} = \frac{1}{S} \sum_{b=1}^3 A_b^n (\mathbf{u}_b^{n+1} \cdot \mathbf{n}_b) L_b, \quad (42)$$

401 where L_b and \mathbf{n}_b correspond to the length and the outward normal of the edge
 402 b , \mathbf{u}_b is the sea ice velocity vector evaluated at the middle of the edge and h_b^n
 403 and A_b^n are the upwind sea ice thickness and concentration, respectively.

404 4. Sensitivity analysis

405 To analyze the sensitivity of the model, an Arctic configuration is set up
 406 on a triangular mesh having a mean resolution $\Delta x = 7$ km (i.e., each triangle

407 of the mesh has a surface S of about 50 km^2). We use a polar stereographic
408 projection centred on the North Pole and with the negative y-axis aligned
409 with the 45W meridian. The domain extends from Bering Strait to Den-
410 mark Strait and to the shortest line linking Iceland and Norway across the
411 Norwegian Sea. The northern gates of the Canadian Arctic Archipelago are
412 closed, except Nares Strait. The coasts are defined at the resolution Δx by a
413 B-spline interpolation of a coarsened set of the Global Self-consistent, Hier-
414 archical, High-resolution Shoreline database (GSHHS) following the method
415 explained by Lambrechts et al. (2008). All these operations are performed
416 with the Gmsh mesh generator (Geuzaine and Remacle, 2009). The bound-
417 ary conditions are no slip everywhere except at the open boundaries (Nares
418 Strait, Bering Strait, Denmark Strait and in the Norwegian Sea) that have
419 a zero stress condition.

420 A consequence of using a Lagrangian advection scheme is that a remeshing
421 scheme is needed to adapt the mesh when it becomes too distorted. The use of
422 a remeshing scheme is not necessary here since the simulations we performed
423 are relatively short (i.e. 10 days). Over such period of time the deformation
424 of the mesh is not generating badly shaped triangles that require a remeshing
425 procedure. In addition of not calling any remeshing method, the forcings and
426 the shape coefficients used for the spatial discretization are defined relative
427 to the initial position of the mesh. This approach is only valid for short
428 simulations though (a few days) as it progressively introduces errors in the
429 position of the ice relative to the geometry and the forcing. On time scales of
430 few days the mechanical recovery due to the healing of the sea ice is supposed
431 to be negligible. The healing term is therefore deactivated in the simulations

432 we present here by setting the damage relaxation time to a very large value
433 ($T_d = 10^{20}$ seconds).

434 The atmospheric forcing fields consist in the 3-hourly 10-meter wind ve-
435 locities coming from the Arctic System Reanalysis (ASR) distributed at
436 30 km spatial resolution (<http://rda.ucar.edu/datasets/ds631.0/>, Byrd
437 Polar Research Center/The Ohio State University (2012). Accessed 01 Jan
438 2014). The oceanic forcing fields consist in the daily elevation and 30-meter
439 depth velocities of the ocean coming from the TOPAZ reanalysis at an av-
440 erage spatial resolution of 12.5 km in the Arctic (Sakov et al., 2012). The
441 simulations presented hereafter all ran over the same 10-day period, 5-15
442 March 2008. The forcings are progressively applied during a spin-up period
443 of one day.

444 To keep the presentation of this model concise, we follow the classical
445 formulation where the values for the turning angles and drag coefficients are
446 constant in time and over the whole domain. This approach is an approx-
447 imation that does not reflect the status of the knowledge concerning the
448 boundary layers above and below the ice (McPhee, 2012) and the recent ef-
449 forts to build new parameterizations of the drag (Tsamados et al., 2014).
450 The water turning angle is fixed at 25° , which is a common value for sea ice
451 (Leppäranta, 2005), whereas there is no turning angle applied to the surface
452 wind stress computed from ASR wind velocities. The values of the drag co-
453 efficients c_a and c_w , and more specifically the ratio between the two, may
454 depend on the forcing and could be tuned along with other parameters to
455 improve the simulations made with a given model. Different tuning experi-
456 ments have led to different optimized ratio for the Arctic. Massonnet et al.

457 (2014) estimated that the optimal value for c_a/c_w was equal to 0.48 for the
458 NEMO-LIM3 model forced by NCEP/NCAR winds (analyzed period: winter
459 seasons 2007 and 2012). Kreyscher and Flato (2000) found $c_a/c_w = 0.5$ for
460 the VP model forced by ECMWF winds (analyzed period: 1979-1994). Miller
461 et al. (2006) found for the CICE sea ice model forced by ECMWF (ERA-40)
462 winds different ratios ranging from 0.11 to 0.3 (analyzed period: 1994-2001).
463 All these analysis compared the simulated and observed sea ice drift over the
464 whole Arctic basin, leading to interdependence between the optimization of
465 the mechanical parameters and the optimization of the drag coefficients. We
466 here propose a tuning approach that has the advantage to differentiate the
467 choice of the drag coefficients from the mechanical parameters. Instead of
468 tuning the drag coefficients over the whole domain, we select a region South
469 of Fram Strait, where sea ice generally moves in a free drift mode. For this
470 region, we tuned the air and water drag coefficients by comparing the sim-
471 ulated and SAR-derived sea ice velocities over the period 18-28 Feb 2008.
472 From the following pairs of $c_a, c_w = [0.003, 0.003; 0.003, 0.004; 0.003, 0.0055;$
473 $0.004, 0.004; 0.004, 0.0055]$, we found that the lowest error (defined in terms
474 of the norm of the difference between the simulated and observed velocity
475 vector) is obtained with $c_a = 0.003$ and $c_w = 0.004$. Here, the value used for
476 c_a/c_w is found to be higher than the classical values. This is consistent with
477 the fact that ASR surface winds are weaker than the geostrophic winds and
478 than the surface wind of ERA-INTERIM produced by ECMWF (Bromwich
479 et al., 2015), which are frequently used to force large scale sea ice models.

480 In the following cases, the initial sea ice damage, velocities and internal
481 stresses are set to zero. For the sea ice concentration, two different initial

482 conditions are used: either the sea ice concentration A_{topaz} from the TOPAZ
 483 reanalysis or the sea ice concentration A_{obs} from observations. A_{obs} is defined
 484 as a combination of the sea ice concentration and lead area fraction fields com-
 485 ing respectively from two different datasets: the AMSR-E/ASI sea ice con-
 486 centration, here denoted A_{tot} (<http://www.iup.uni-bremen.de/seaice/amsr/>,
 487 University of Bremen, Bremen, Germany, October 2011) and the AMSR-E
 488 lead area fraction, here denoted A_{lead} (<http://icdc.zmaw.de/>, Integrated Cli-
 489 mate Date Center, University of Hamburg, Hamburg, Germany, May 2014).
 490 Both datasets are given at 6.5km horizontal resolution (see Spreen et al.
 491 (2008) and Röhrs and Kaleschke (2012) for the description of the methodolo-
 492 gies). A_{lead} and A_{tot} provide different information. A_{lead} identifies the narrow
 493 leads in high concentration areas from anomalies in the brightness tempera-
 494 ture ratio whereas A_{tot} provides the smooth background concentration fields
 495 and may also identify large open water areas such as polynias. To study the
 496 impact of having information on the leads in the initial condition, we define
 497 the ice concentration A_{obs} as:

$$A_{obs} = A_{tot} (1 - A_{lead}), \quad (43)$$

498 The two sets of initial conditions, A_{topaz} and A_{obs} , are very similar in term
 499 of sea ice extent but differ significantly in terms of sea ice concentration
 500 distribution (not shown here) because of the representation of the leads.
 501 A_{topaz} is relatively smooth, whereas A_{obs} already contains localized linear
 502 features (Figure 1). From the mechanical point of view, initialization with
 503 A_{obs} is preferred as the ice in the leads is generally the weakest, which in turn
 504 potentially impacts the results of the simulation. The impact of starting from

505 one dataset or the other is analyzed in section 4.3. For the sea ice thickness,
 506 two different initial conditions are also defined. When A_{topaz} is used, the
 507 initial sea ice thickness h_{topaz} is directly taken from the TOPAZ reanalysis.
 508 When A_{obs} is used, the initial sea ice thickness h_{obs} is also derived from the
 509 TOPAZ reanalysis but is corrected to be consistent with A_{obs} by defining h_{obs}
 510 as:

$$h_{obs} = \frac{h_{topaz}}{A_{topaz}} A_{obs}, \quad (44)$$

511 The reference simulation runs with the Lagrangian scheme, is initialized
 512 with A_{topaz} and h_{topaz} and uses the following set of parameters: $c = 4$ kPa,
 513 $\alpha = -20$, $\Delta t = 800$ s and $Y = 9$ GPa. Simulations with smaller time steps (Δt
 514 set to 100, 200 and 400 s) produce similar results than the one with $\Delta t = 800$
 515 s but simulations with larger time steps (Δt set to 2400, 7200 and 21600 s)
 516 do not, presumably because the time step is not small enough compared to
 517 the forcing time scale.

518 The sea ice velocity fields, simulated over the last 3 days of the 10-day
 519 simulations, exhibit spatial discontinuities, which are located along quasi
 520 linear features spanning almost the entire Arctic basin (Figure 2, for the
 521 reference simulation). In the following sections, we discuss the realism of
 522 the simulated dynamics by analysing the deformation fields and we present
 523 the sensitivity of the model to the type of advection scheme, to the initial
 524 conditions and to the value of the cohesion parameter c and compactness
 525 parameter α . The sensitivity to Y has been discussed in section 2.

526 *4.1. Sensitivity to the advection scheme*

527 To preserve discontinuities in the ice concentration and thickness fields
528 when sea ice moves requires a particular attention to the choice of the ad-
529 vection scheme. Starting from the same initial conditions, A_{obs} and h_{obs} , and
530 with the same forcing fields and parameters, the simulations with the La-
531 grangian scheme and the Eulerian upwind scheme give radically different sea
532 ice concentration and thickness fields after a period as short as 10 days (Fig-
533 ure 3). With the Lagrangian scheme, the distribution of sea ice concentration
534 remains similar to the observations, whereas the distribution obtained for the
535 Eulerian upwind scheme is greatly affected by numerical diffusion. However,
536 one should note that Eulerian upwind schemes are known to be much more
537 diffusive than other Eulerian schemes. This example is only presented as an
538 illustration and is meant to show that the Lagrangian approach at least can
539 naturally conserve discontinuities even when they are located at the native
540 resolution of the model.

541 *4.2. Statistical analysis of the simulated sea ice deformation*

542 The simulated ice deformation fields shown in Figure 4 (i.e. shear and
543 divergence) exhibit obvious localization properties expressed by the presence
544 of linear features (the so-called linear kinematics features, Kwok (2000)).
545 However, to evaluate the realism of the deformation fields requires a thor-
546 ough statistical analysis. We performed such analysis using the deformation
547 derived from the sea ice displacement field simulated over the last 3 days of
548 10-day simulations and on a domain restricted to the elements of the Arctic
549 basin being at least 150 km away from the coast. Several statistical diag-
550 nostics are used for the analysis, i.e. the cumulative distribution of sea ice

551 deformation, the total shearing, opening and closing rates, and the charac-
552 teristics of the spatial scaling of sea ice deformation. These diagnostics can
553 be computed for SAR-derived drift and deformation datasets, and compared
554 to the values obtained with the model. Such comparison has been routinely
555 done during the development of the present model for a large set of simula-
556 tions, and showed very good agreement. These results will be presented in a
557 dedicated paper.

558 The cumulative distributions (i.e., the probability of exceedance) for the
559 shear and divergence rates are computed as in Marsan et al. (2004) and are
560 plotted in semi-log scale to highlight the differences between the simulations
561 (Figure 4). The same results plotted in logarithmic scales (not shown here)
562 show similar power law tails as in Marsan et al. (2004). One should note that
563 detection and characterization of power law tails in statistical distributions
564 are very sensitive to the method of analysis and therefore require a proper
565 quality check (Clauset et al., 2009).

566 The total opening $\langle \dot{\mathcal{O}} \rangle$, closing $\langle \dot{\mathcal{C}} \rangle$ and shearing $\langle \dot{\mathcal{S}} \rangle$ rates are
567 computed by integrating over the domain of analysis the positive divergence,
568 negative divergence and shear rates respectively (Table 3). For the reference
569 simulation the total opening rate $\langle \dot{\mathcal{O}} \rangle$ is equal to $15000 \text{ km}^2\text{day}^{-1}$ and
570 the total closing rate $\langle \dot{\mathcal{C}} \rangle$ is equal to $-24000 \text{ km}^2\text{day}^{-1}$. These quantities
571 are more interesting than the total divergence rate as they are related to
572 the opening and closing of leads and to the formation of ridges. These inte-
573 grated values but also the ratio between opening and closing vary drastically
574 at the typical time scale of the wind forcing and should be analyzed in a
575 statistical sense over a month or a season, and not just from one snapshot.

576 However, snapshot analyses remain useful for estimating the sensitivity to
 577 model parameters (see the following subsections).

578 The heterogeneity of the deformation fields is estimated by computing
 579 the area that accommodates the largest 50% of the deformation as in Girard
 580 et al. (2011). $\delta_{\dot{\mathcal{O}}_{50\%}}$, $\delta_{\dot{\mathcal{C}}_{50\%}}$ and $\delta_{\dot{\mathcal{S}}_{50\%}}$ are defined as the minimum fractions of
 581 the total area needed to accommodate 50% of the total opening, closing and
 582 shearing rate, respectively (see Stern and Lindsay (2009) for a more complete
 583 description of the method). A homogeneous deformation field would give
 584 $\delta_{\dot{\mathcal{S}}_{50\%}} = 50\%$. The deformation fields simulated by our model are highly
 585 heterogeneous. For the reference simulation, 50% of the largest shear are
 586 accommodated by only 8% of the domain area. In Girard et al. (2011),
 587 $\delta_{\dot{\mathcal{S}}_{50\%}}$ was equal to 4% for a simulation with the EB rheology and to 6% for
 588 observations (model and observations being at the same resolution of 10 km
 589 and computed for the same date). The value of these diagnostics for the
 590 different simulations are given in Table 3.

591 The spatial scaling of the deformation is determined as in Marsan et al.
 592 (2004). A coarse-graining procedure is performed to compute the shear and
 593 divergence rate fields at different spatial scales L (Figure 5). The spatial
 594 scaling of the total deformation is similar to the one of the shear and is not
 595 presented here. The mean of the distribution (i.e., first order moment) of the
 596 simulated shear and absolute divergence rates computed at different scales
 597 can be described by a power law:

$$\langle \dot{\epsilon} \rangle_L \sim L^{-\beta(1)}, \quad (45)$$

598 with different scaling exponent $\beta_{shear}(1)$ and $\beta_{div}(1)$ (Table 4). For the ref-

599 erence simulation, we found $\beta_{shear}(1) = 0.04$ and $\beta_{div}(1) = 0.15$. Previous
600 studies have analysed the scaling properties of sea ice deformation. Stern and
601 Lindsay (2009), for example, found a mean scaling exponent of 0.18 with a
602 standard deviation of 0.10 for the total deformation computed from satellite-
603 derived observations covering several winter seasons. However, as shown in
604 Bouillon and Rampal (2014), the artificial noise present in the deformation
605 dataset used for these studies could lead to an overestimation of these spa-
606 tial scaling exponents of about 60% for the shear and total deformation, and
607 100% for the absolute divergence. Moreover, due to the high variability in
608 time of these exponents, one should not compare the values obtained over a
609 unique example but rather the distributions of the simulated and observed
610 scaling exponents over a season.

611 How the distribution of the deformation varies with the scale of compu-
612 tation can be fully described by performing a multifractal analysis, which
613 consists in looking at the different moment orders q of the distribution. Sim-
614 ilarly to observed sea ice deformation fields, model data follow the power law
615 scaling $\langle \dot{\epsilon}^q \rangle_L \sim L^{-\beta(q)}$ with $\beta(q)$ being a quadratic function of $0 \leq q \leq 3$.
616 The structure function $\beta(q)$ (Figure 6) characterizes how the moments of the
617 distribution evolve as a function of the spatial scale (i.e., $\beta(1)$ for the mean,
618 $\beta(2)$ for the standard deviation, $\beta(3)$ for the skewness). The curvature of
619 the structure functions $\beta(q)$ indicates that our simulated sea ice deformation
620 fields show a multifractal spatial scaling. The curvature is described by the
621 coefficient a of the quadratic fit $\beta(q) = aq^2 + bq$, and its value gives the de-
622 gree of multifractality of the scaling. For the reference simulation, we found
623 $a_{shear} = 0.18$ and $a_{div} = 0.23$. For comparison, Marsan et al. (2004) found

624 $a = 0.13$ for a total deformation field derived from observations. The values
 625 of this diagnostic for the different simulations are recapped in Table 4.

626 4.3. Sensitivity to the initial conditions

627 To study the impact of the initial conditions, a simulation is started with
 628 A_{obs} and h_{obs} and compared to the reference simulation. We found similar
 629 distributions of deformation (Figure 4), and almost identical values for the
 630 total opening, closing and shearing rate (16 000, -25 000, 103 000 km²day⁻¹,
 631 respectively) and for $\delta_{\dot{O}_{50\%}}$, $\delta_{\dot{C}_{50\%}}$ and $\delta_{\dot{S}_{50\%}}$ (0.06, 0.005, 0.08, respectively).
 632 The structure function $\beta(q)$ is also very close to the one obtained for the
 633 reference simulation (Figure 6). The only significant difference concerns the
 634 bars on $\beta(q)$ function that quantifies the deviation from the power law scaling
 635 (see caption of Figure 6 for more details). These are larger compared to the
 636 reference run, especially for the absolute divergence rate. In both cases,
 637 results exhibit a strong spatial localization and similar statistical properties,
 638 meaning that this characteristic of the model is not inherited from initial
 639 conditions but rather generated by the model itself. For the rest of the
 640 sensitivity study, we only use simulations initialized with A_{topaz} and h_{topaz} so
 641 that there are no discontinuities in the initial fields.

642 4.4. Sensitivity to the cohesion parameter c

643 Simulations with the cohesion parameter equal to 8, 4, 2, 1 and 0.5 kPa
 644 are performed and their results summarized in Tables 3 and 4 and in Figure
 645 7. Decreasing the value of the cohesion parameter induces higher but less
 646 localized deformation (i.e., significant increase of the total opening $\langle \dot{O} \rangle$,
 647 closing $\langle \dot{C} \rangle$ and shearing $\langle \dot{S} \rangle$ rates and almost a doubling of $\delta_{\dot{O}_{50\%}}$,

648 $\delta_{\dot{\epsilon}_{50\%}}$ and $\delta_{\dot{\epsilon}_{50\%}}$). Changing the value of the cohesion also changes the visual
 649 appearance of the deformation fields. For example, the simulation with $c=0.5$
 650 kPa (Figure 7) shows many more features than in the reference simulation
 651 (Figure 4). For the shear rate distribution, decreasing the cohesion induces
 652 a gradual shift to higher values. For the divergence rate distribution, such
 653 a gradual shift is not observed, all the distributions are similar except for
 654 the simulation with $c=8$ kPa. The cohesion has also a clear impact on the
 655 structure functions $\beta(q)$ (Figure 7) as the curvatures a_{shear} and a_{div} decrease
 656 by almost a factor of 2 between the reference simulation (with $c=4$ kPa) and
 657 the simulation with the lowest cohesion ($c=0.5$ kPa)(Table 4). Overall we
 658 note that the cohesion parameter is the one having the highest impact on the
 659 degree of multifractality of the spatial scaling of the simulated deformation
 660 fields.

661 4.5. Sensitivity to the compactness parameter α

662 Simulations with the compactness parameter ranging -40, -20, -10 and 0
 663 are performed (see Tables 5 and 6 and Figure 8). Increasing α from -40 to 0
 664 leads to higher values for the effective elastic stiffness and then induces lower
 665 deformation, especially for the opening and closing rates. It also significantly
 666 decreases the heterogeneity of the shear deformation fields (i.e., increase of
 667 $\delta_{\dot{\epsilon}_{50\%}}$) and induces lower scaling exponent (estimated by $\beta(1)$). However,
 668 it has only small impacts on the degree of multifractality (estimated by a).
 669 Moving α towards 0 leads to a more symmetrical distribution of divergence
 670 (Figure 8), whereas the distribution of the shear rate is almost unchanged.
 671 Symmetry of the distribution of divergence has been reported in Girard et al.
 672 (2009) and shown to depend on the spatial scale. This suggests that the best

673 estimate of the compactness parameter for being used in our model could be
674 established by performing a thorough comparison against observations.

675 **5. Conclusions**

676 The dynamical core of this new sea ice model neXtSIM is presented and
677 outputs from 10-days sea ice standalone simulations are analyzed to evaluate
678 the sensitivity of simulated sea ice dynamics to model parameters. neXtSIM
679 is a Lagrangian model running on an unstructured finite element mesh. The
680 introduction of the sea ice damage variable produces discontinuities at the
681 scale of the elements in the simulated fields. We propose a specific implemen-
682 tation for the temporal and spatial discretization as well as for the advection
683 scheme in order to preserve as much as possible these discontinuities over
684 time. The model produces sea ice deformation fields showing similar statis-
685 tical signatures as those found for the Arctic sea ice cover, and especially
686 a multifractal spatial scaling invariance. These statistical properties do not
687 rely on the realism of the initial concentration and thickness fields but rather
688 emerge from the sea ice rheological model. The sensitivity analysis shows
689 that the degree of multifractality of the sea ice deformation scaling invari-
690 ance is mainly controlled by the cohesion parameter c . The compactness
691 parameter α mainly impacts the total opening and closing rate with minor
692 impact on the total shear rate. An extensive validation of the current model
693 based on the comparison of the simulated fields against SAR-derived drift
694 and deformation fields has been performed but will be presented in a dedi-
695 cated study for more clarity. It would be important to evaluate the impact
696 of using such sea ice model over longer time scales by looking at the seasonal

697 cycle, spatial distribution and inter-annual variability of the sea ice concen-
698 tration, thickness and velocities. However, further model developments are
699 required to perform long simulations. First, sea ice thermodynamics has to
700 be implemented and taken into account to parameterize the recovery of the
701 ice mechanical strength (i.e. "healing") due to thermal forcing. Second, a
702 remeshing procedure has to be implemented to adapt the mesh when it be-
703 comes too deformed. Finally, the coupling with an interacting ocean and
704 atmosphere could also be necessary to asses the impact of better resolving
705 sea ice dynamics on the other components of the Arctic system.

706 **Appendix: Assembly of the finite element matrices**

707 To compute the values of \mathbf{A}_{ij} and \mathbf{B}_i , a transformation is applied to work
 708 in a parametric space defined by $\boldsymbol{\xi} = (\xi, \eta)$ instead of $\mathbf{x} = (x, y)$. All the
 709 elements are related to a unique parent element thanks to the transformation:

$$\mathbf{x}(\boldsymbol{\xi}) = \phi_1(\xi, \eta)\mathbf{X}_1^e + \phi_2(\xi, \eta)\mathbf{X}_2^e + \phi_3(\xi, \eta)\mathbf{X}_3^e, \quad (46)$$

710 where \mathbf{X}_1^e , \mathbf{X}_2^e and \mathbf{X}_3^e are the coordinates of the first, second and third
 711 vertices of the element e and

$$\begin{aligned} \phi_1(\xi, \eta) &= 1 - \xi - \eta, \\ \phi_2(\xi, \eta) &= \xi, \\ \phi_3(\xi, \eta) &= \eta. \end{aligned} \quad (47)$$

712 The sum over the nodes is then replaced by a sum over the three vertices
 713 of each element and \mathbf{u}^h is then built as

$$\mathbf{u}^h = \sum_{i=1}^3 \mathbf{U}_i^e \phi_i^e(\mathbf{x}), \quad (48)$$

714 where $\phi_j(\mathbf{x}) = \phi_j(\boldsymbol{\xi}(\mathbf{x}))$. The local base is then defined as:

$$\left\{ \left[\begin{array}{c} \phi_1^e \\ 0 \end{array} \right], \left[\begin{array}{c} 0 \\ \phi_1^e \end{array} \right], \left[\begin{array}{c} \phi_2^e \\ 0 \end{array} \right], \left[\begin{array}{c} 0 \\ \phi_2^e \end{array} \right], \left[\begin{array}{c} \phi_3^e \\ 0 \end{array} \right], \left[\begin{array}{c} 0 \\ \phi_3^e \end{array} \right] \right\}. \quad (49)$$

715 With this representation, the values of the deformation rate tensor are

716 then computed by

$$\boldsymbol{\epsilon}(\mathbf{u}) = \mathbf{G}^e \mathbf{U}^e, \quad (50)$$

717 where

$$\mathbf{U}^e = \begin{bmatrix} U_{x1}^e \\ U_{y1}^e \\ U_{x2}^e \\ U_{y2}^e \\ U_{x3}^e \\ U_{y3}^e \end{bmatrix}, \quad (51)$$

718 and

$$\mathbf{G}^e = \begin{bmatrix} \phi_{1,x}^e & 0 & \phi_{2,x}^e & 0 & \phi_{3,x}^e & 0 \\ 0 & \phi_{1,y}^e & 0 & \phi_{2,y}^e & 0 & \phi_{3,y}^e \\ \phi_{1,y}^e & \phi_{1,x}^e & \phi_{2,y}^e & \phi_{2,x}^e & \phi_{3,y}^e & \phi_{3,x}^e \end{bmatrix}. \quad (52)$$

719 The derivatives of the shape functions, also called shape coefficients, are com-

720 puted by:

$$\begin{aligned} \phi_{1,x}^e &= (Y_2^e - Y_3^e)/J^e, & \phi_{1,y}^e &= (X_2^e - X_3^e)/J^e, \\ \phi_{2,x}^e &= (Y_3^e - Y_1^e)/J^e, & \phi_{2,y}^e &= (X_3^e - X_1^e)/J^e, \\ \phi_{3,x}^e &= (Y_1^e - Y_2^e)/J^e, & \phi_{3,y}^e &= (X_1^e - X_2^e)/J^e, \end{aligned} \quad (53)$$

721 where J^e , the Jacobian of the transformation (i.e., $\det(\frac{\partial \mathbf{x}}{\partial \xi})$), is computed as

$$J^e = X_2^e Y_3^e + X_3^e Y_1^e + X_1^e Y_2^e - X_2^e Y_1^e - X_3^e Y_2^e - X_1^e Y_3^e. \quad (54)$$

722 The local contributions of \mathbf{A}_{ij} and \mathbf{B}_i are then given by:

$$\mathbf{A}_{ij}^e = k^e \mathbf{M}^e - h^e \Delta t \mathbf{S}^e \mathbf{E}^e (\mathbf{G}^e)^T \mathbf{D} \mathbf{G}^e, \quad (55)$$

723 and

$$\mathbf{B}_i^e = h^e \mathbf{S}^e (\mathbf{G}^e)^T \boldsymbol{\sigma}^e - \mathbf{M}^e \mathbf{F}^e, \quad (56)$$

724 where \mathbf{S}^e is the surface of the element, which is actually equal to $J^e/2$. \mathbf{M}^e is
 725 the local mass matrix, h^e is the value of h for the element e , \mathbf{E}^e is the value
 726 of $E(A, d)$ for the element e , \mathbf{F}^e contains the values of the vector \mathbf{f} at the 3
 727 nodes of the element e , and

$$\boldsymbol{\sigma}^e = \begin{bmatrix} \sigma_{11}^e \\ \sigma_{22}^e \\ \sigma_{12}^e \end{bmatrix}. \quad (57)$$

728 The local mass matrix is equal to:

$$\mathbf{M}^e = \begin{bmatrix} \langle \phi_1^e \phi_1^e \rangle & 0 & \langle \phi_1^e \phi_2^e \rangle & 0 & \langle \phi_1^e \phi_3^e \rangle & 0 \\ 0 & \langle \phi_1^e \phi_1^e \rangle & 0 & \langle \phi_1^e \phi_2^e \rangle & 0 & \langle \phi_1^e \phi_3^e \rangle \\ \langle \phi_2^e \phi_1^e \rangle & 0 & \langle \phi_2^e \phi_2^e \rangle & 0 & \langle \phi_2^e \phi_3^e \rangle & 0 \\ 0 & \langle \phi_2^e \phi_1^e \rangle & 0 & \langle \phi_2^e \phi_2^e \rangle & 0 & \langle \phi_2^e \phi_3^e \rangle \\ \langle \phi_3^e \phi_1^e \rangle & 0 & \langle \phi_3^e \phi_2^e \rangle & 0 & \langle \phi_3^e \phi_3^e \rangle & 0 \\ 0 & \langle \phi_3^e \phi_1^e \rangle & 0 & \langle \phi_3^e \phi_2^e \rangle & 0 & \langle \phi_3^e \phi_3^e \rangle \end{bmatrix}, \quad (58)$$

729 where $\langle \phi_i^e \phi_j^e \rangle = \frac{1}{12} S^e$ when $i \neq j$ and $\langle \phi_i^e \phi_j^e \rangle = \frac{2}{12} S^e$ when $i = j$.

730 Acknowledgements

731 Sylvain Bouillon is supported by the Research Council of Norway through
 732 the post-doc project SIMech, Sea Ice Mechanics: from satellites to numerical
 733 models (No. 231179/F20, 2014-2016). We would like to thank Total E&P
 734 for their support and the reviewers for their useful comments.

735 Amitrano, D., Grasso, J. R., Hantz, D., 1999. From diffuse to localised dam-
736 age through elastic interaction. *Geophys. Res. Lett.* 26 (14), 2109–2112.

737 Bouillon, S., Rampal, P., 2014. On producing sea ice deformation dataset
738 from sar-derived sea ice motion. *The Cryosphere Discussions* 8 (5), 5105–
739 5135.
740 URL <http://www.the-cryosphere-discuss.net/8/5105/2014/>

741 Bower, A. F., 2011. *Applied mechanics of solids*. CRC press.
742 URL <http://solidmechanics.org>

743 Bromwich, D. H., Wilson, A. B., Bai, L.-S., Moore, G. W. K., Bauer, P.,
744 2015. A comparison of the regional arctic system reanalysis and the global
745 era-interim reanalysis for the arctic. *Quarterly Journal of the Royal Mete-*
746 *orological Society*, n/a–n/a.
747 URL <http://dx.doi.org/10.1002/qj.2527>

748 Chen, Y., Davis, Timothy, A., Hager, William, W., Rajamanickam, S., 2008.
749 Algorithm 887: CHOLMOD, Supernodal Sparse Cholesky Factorization
750 and Update/Downdate. *ACM Transactions on Mathematical Software* 35,
751 22:1–22–14.

752 Clauset, A., Shalizi, C., Newman, M., 2009. Power-law distributions in em-
753 pirical data. *SIAM Review* 51 (4), 661–703.
754 URL <http://dx.doi.org/10.1137/070710111>

755 Comblen, R., Legrand, S., Deleersnijder, E., Legat, V., 2009. A finite element
756 method for solving the shallow water equations on the sphere. *Ocean Mod.*
757 28, 12–23.

- 758 Compère, G., Remacle, J.-F., Jansson, J., Hoffman, J., 2009. A mesh adap-
759 tation framework for dealing with large deforming meshes. *Int. J. Numer.*
760 *Meth. Engng.* 82, 843–867.
- 761 Compère, G., Remacle, J. F., Marchandise, E., 2008. Transient mesh adap-
762 tivity with large rigid-body displacements. In: Garimella, R. (Ed.), *Pro-*
763 *ceedings of the 17th International Meshing Roundtable*. Springer, Berlin,
764 pp. 213–230.
- 765 Connolley, W. M., Gregory, J. M., Hunke, E. C., McLaren, A. J., 2004. On
766 the consistent scaling of terms in the sea-ice dynamics equation. *J. Phys.*
767 *Oceanogr.* 34, 1776–1780.
- 768 Geuzaine, C., Remacle, J.-F. c. o., 2009. Gmsh: a three-dimensional finite
769 element mesh generator with built-in pre- and post-processing facilities.
770 *International Journal For Numerical Methods In Engineering* 79, 1309–
771 1331.
- 772 Girard, L., Bouillon, S., Weiss, J., Amitrano, D., Fichefet, T., Legat, V.,
773 2011. A new modelling framework for sea ice mechanics based on elasto-
774 brittle rheology. *Ann. Glaciol.* 52(57), 123–132.
- 775 Girard, L., Weiss, J., Molines, J.-M., Barnier, B., Bouillon, S., Aug. 2009.
776 Evaluation of high-resolution sea ice models on the basis of statistical and
777 scaling properties of Arctic sea ice drift and deformation. *J. Geophys. Res.*
778 114 (C8).
- 779 Hanert, E., Le Roux, D. Y., Legat, V., Deleersnijder, E., 2004. Advection
780 schemes for unstructured grid ocean modelling. *Ocean Model.* 7, 39–58.

- 781 Herman, A., 2013. Numerical modeling of force and contact networks in
782 fragmented sea ice. *Ann. Glaciol.* 54 (62), 114–120.
- 783 Hibler, W. D., 1979. A dynamic thermodynamic sea ice model. *J. Phys.*
784 *Ocean.* 9, 817–846.
- 785 Hughes, T. J. R., 2012. *The Finite Element Method: Linear Static and Dy-*
786 *namic Finite Element Analysis.* Dover Civil and Mechanical Engineering.
787 Dover Publications.
- 788 Hunke, E. C., Dukowicz, J. K., 2003. The sea ice momentum equation in
789 the free drift regime. Tech. rep., T-3 Fluid Dynamics Group, Los Alamos
790 National Laboratory.
- 791 Hutchings, J. K., Heil, P., Hibler, W. D., 2005. Modeling linear kinematic
792 features in sea ice. *Month. Weath. Rev.* 133 (12), 3481–3497.
- 793 Kreyscher, M., Flato, G. M., 2000. Results of the Sea Ice Model Intercompar-
794 ison Project: Evaluation of sea ice rheology schemes for use in simulations
795 Project (SIMIP). Four different sea ice rheology schemes. *Journal of Geo-*
796 *physical Research* 105 (C5), 112299–11320.
- 797 Kwok, R., 2000. Deformation of the Arctic ocean sea ice cover between
798 November 1996 and April 1997: A survey. *IUTAM Symposium on Scaling*
799 *Laws in Ice Mechanics and Ice Dynamics*, 1–12.
- 800 Kwok, R., 2006. Contrasts in sea ice deformation and production in the Arctic
801 seasonal and perennial ice zones. *J. Geophys. Res.* 111 (C11).

- 802 Lambrechts, J., Comblen, R., Legat, V., Geuzaine, C., Remacle, J. F., 2008.
803 Multiscale mesh generation on the sphere. *Ocean Dyn.* 58 (5), 461–473.
- 804 Leppäranta, M., 2005. *The Drift of Sea Ice*. Springer, Helsinki.
- 805 Marcq, S., Weiss, J., 2012. Influence of sea ice lead-width distribution
806 on turbulent heat transfer between the ocean and the atmosphere. *The*
807 *Cryosphere*, Volume 6, Issue 1, 2012, pp. 143-156 6, 143–156.
- 808 Marsan, D., Stern, H. L., Lindsay, R., Weiss, J., Oct. 2004. Scale dependence
809 and localization of the deformation of Arctic sea ice. *Phys. Rev. Lett.*
810 93 (17), 178501.
- 811 Marsan, D., Weiss, J., Aug. 2010. Space/time coupling in brittle deformation
812 at geophysical scales. *Earth Planet. Sci. Lett.* 296 (3-4), 353–359.
- 813 Marsan, D., Weiss, J., Metaxian, J.-P., Grangeon, J., Roux, P.-F., Haapala,
814 J., 2011. Low-frequency bursts of horizontally polarized waves in the Arctic
815 sea-ice cover. *J. Glaciol.* 57 (202), 231–237.
- 816 Massonnet, F., Goose, H., Fichet, T., Counillon, F., 2014. Calibration of
817 sea ice dynamic parameters in an ocean-sea ice model using an ensemble
818 Kalman filter. *Journal of Geophysical Research: Oceans* 119 (7), 4168–
819 4184.
- 820 McPhee, M. G., 2012. Advances in understanding ice-ocean stress during and
821 since AIDJEX. *Cold Regions Science and Technology* 76-77, 24–36.
- 822 Mellor, M., 1986. Mechanical Behavior of Sea Ice. In: Untersteiner, N. (Ed.),

823 The Geophysics of Sea Ice. NATO ASI Series. Springer US, Ch. 2, pp.
824 165–282.

825 Miller, P. A., Laxon, S. W., Feltham, D. L., 2006. Optimization of a sea ice
826 model using basinwide observations of Arctic sea ice thickness, extent, and
827 velocity. *J. Climate* 19, 1089–1108.

828 Morison, J. H., McPhee, M., 2001. Ice-ocean interaction.

829 Nukala, P. K. V. V., Zapperi, S., Simunovic, S., 2005. Statistical properties
830 of fracture in a random spring model. *Phys. Rev. E* 71 (6), 066106.

831 Prather, M. J., 1986. Numerical advection by conservation of second-order
832 moments. *J. Geophys. Res.* 91, 6671–6681.

833 Rampal, P., Weiss, J., Marsan, D., Bourgoin, M., Oct. 2009. Arctic sea
834 ice velocity field: General circulation and turbulent-like fluctuations. *J.*
835 *Geophys. Res.* 114 (C10), C10014.

836 Rampal, P., Weiss, J., Marsan, D., Lindsay, R., Stern, H. L., Dec. 2008.
837 Scaling properties of sea ice deformation from buoy dispersion analysis. *J.*
838 *Geophys. Res.* 113 (C3), C03002.

839 Richter-Menge, J. A., McNutt, S. L., Overland, J. E., Kwok, R., 2002. Re-
840 lating Arctic pack ice stress and deformation under winter conditions. *J.*
841 *Geophys. Res.* 107 (C10), 8040.

842 Röhrs, J., Kaleschke, L., 2012. An algorithm to detect sea ice leads by using
843 AMSR-E passive microwave imagery. *The Cryosphere* 6 (2), 343–352.

- 844 Sakov, P., Counillon, F., Bertino, L., Lisæter, K. A., Oke, P., Korablev, A.,
845 2012. TOPAZ4: An ocean sea ice data assimilation system for the North
846 Atlantic and Arctic. *Ocean Sci.* 8, 633–662.
- 847 Schulson, E. M., 2004. Compressive shear faults within Arctic sea ice: Frac-
848 ture on scales large and small. *J. Geophys. Res.* 109 (C7).
- 849 Schulson, E. M., 2009. Fracture of ice and other coulombic materials. *Me-*
850 *chanics of Natural Solids*, 177.
- 851 Schulson, E. M., Fortt, A. L., Iliescu, D., Renshaw, C. E., 2006. Failure
852 envelope of first-year Arctic sea ice: The role of friction in compressive
853 fracture. *J. Geophys. Res.* 111, C11S25.
- 854 Spreen, G., Kaleschke, L., Heygster, G., 2008. Sea ice remote sensing us-
855 ing AMSR-E 89-GHz channels. *Journal of Geophysical Research: Oceans*
856 113 (C2).
- 857 Stern, H. L., Lindsay, R., Oct. 2009. Spatial scaling of Arctic sea ice defor-
858 mation. *J. Geophys. Res.* 114 (C10).
- 859 Sulsky, D., Schreyer, H., Peterson, K., Kwok, R., Coon, M., Feb. 2007. Using
860 the material-point method to model sea ice dynamics. *J. Geophys. Res.*
861 112 (C2).
- 862 Thorndike, A. S., Rothrock, D. A., Maykut, G. A., Colony, R., 1975. The
863 thickness distribution of sea ice. *J. Geophys. Res.* 80, 4501–4513.
- 864 Tsamados, M., Feltham, D. L., Schroeder, D., Flocco, D., Farrell, S. L.,
865 Kurtz, N., Laxon, S. W., Bacon, S., Jan. 2014. Impact of variable atmo-

- 866 spheric and oceanic form drag on simulations of arctic sea ice. *Journal of*
867 *Physical Oceanography* 44 (5), 1329–1353.
- 868 Walters, R. A., Lane, E. M., Hanert, E., 2009. Useful time-stepping methods
869 for the Coriolis term in a shallow water model . *Ocean Modelling* 28 (1–3),
870 66–74.
- 871 Wang, L. R., Ikeda, M., 2004. A Lagrangian description of sea ice dynamics
872 using the finite element method. *Ocean Mod.* 7, 21–38.
- 873 Weiss, J., 2013. *Drift, Deformation, and Fracture of Sea Ice: A Perspective*
874 *Across Scales*. Springer.
- 875 Weiss, J., Marsan, D., 2004. Scale properties of sea ice deformation and
876 fracturing. *CR Phys.* 5 (7), 735–751.
- 877 Weiss, J., Schulson, E. M., Oct. 2009. Coulombic faulting from the grain
878 scale to the geophysical scale: Lessons from ice. *J. Phys. D: Appl. Phys.*
879 42 (21), 214017.
- 880 Weiss, J., Stern, H. L., Schulson, E. M., 2007. Sea ice rheology from in-situ,
881 satellite and laboratory observations: Fracture and friction. *Earth Planet.*
882 *Sci. Lett.* 255 (1-2), 1–8.

Symbol	Meaning	units
h	sea ice thickness	m
A	sea ice concentration	-
d	sea ice damage	-
\mathbf{u}	sea ice velocity	m s^{-1}
$\boldsymbol{\sigma}$	sea ice internal stress	N m^{-2}

Table 1: Variables used in the model

Symbol	Meaning	Values	Units
ρ_a	air density	1.3	kg m^{-3}
c_a	air drag coefficient	0.003	-
θ_a	air turning angle	0	degree
ρ_w	water density	1025	kg m^{-3}
c_w	water drag coefficient	0.004	-
θ_w	water turning angle	25	degrees
ρ_i	ice density	917	kg m^{-3}
ν	Poisson coefficient	0.3	-
μ	internal friction coefficient	0.7	-
Y	elastic modulus	9	GPa
Δx	mean resolution of the mesh	7	km
Δt	time step	800	s
T_d	damage relaxation time	10^{20}	s
c	cohesion parameter	[8, <u>4</u> , 2, 1, 0.5]	kPa
α	compactness parameter	-[40, <u>20</u> , 10, 0]	-

Table 2: Parameters used in the model with their values for the simulations presented in this study. Underlined values are for the reference simulation.

c [kPa]	$\langle \dot{O} \rangle$ [km ² day ⁻¹]	$\langle \dot{C} \rangle$ [km ² day ⁻¹]	$\langle \dot{S} \rangle$ [km ² day ⁻¹]	$\delta_{\dot{O}_{50\%}}$	$\delta_{\dot{C}_{50\%}}$	$\delta_{\dot{S}_{50\%}}$
8	15 000	-17 000	86 000	0.06	0.01	0.07
4	15 000	-24 000	106 000	0.06	0.01	0.08
2	15 000	-27 000	117 000	0.09	0.01	0.10
1	17 000	-28 000	129 000	0.10	0.01	0.11
0.5	19 000	-28 000	141 000	0.11	0.01	0.14

Table 3: Values of the total opening, closing and shearing rates computed at the scale of the elements over the last 3 days of simulations using different values of the cohesion parameter c . $\delta_{\dot{O}_{50\%}}$, $\delta_{\dot{C}_{50\%}}$ and $\delta_{\dot{S}_{50\%}}$ are the minimum fractions of the total area needed to accommodate 50% of the total opening, closing and shearing rates, respectively.

c [kPa]	$\beta_{shear}(1)$	a_{shear}	$\beta_{div}(1)$	a_{div}
8	0.04	0.21	0.09	0.22
4	0.04	0.18	0.15	0.23
2	0.05	0.14	0.18	0.18
1	0.07	0.13	0.21	0.11
0.5	0.09	0.11	0.24	0.14

Table 4: Impact of using different values of the cohesion parameter c on the structure function $\beta(q)$, which is well fitted by a quadratic function $\beta(q) = aq^2 + bq$. $\beta(1)$ and a are given for the shear and divergence rates, respectively.

α	$\langle \dot{O} \rangle$ [km ² day ⁻¹]	$\langle \dot{C} \rangle$ [km ² day ⁻¹]	$\langle \dot{S} \rangle$ [km ² day ⁻¹]	$\delta_{\dot{O}_{50\%}}$	$\delta_{\dot{C}_{50\%}}$	$\delta_{\dot{S}_{50\%}}$
-40	22 000	-29 000	109 000	0.05	0.01	0.05
-20	15 000	-24 000	106 000	0.06	0.01	0.08
-10	12 000	-22 000	105 000	0.05	0.01	0.10
0	11 000	-19 000	102 000	0.06	0.02	0.12

Table 5: Values of the total opening, closing and shearing rates computed at the scale of the elements over the last 3 days of simulations using different values of the compactness parameter α . $\delta_{\dot{O}_{50\%}}$, $\delta_{\dot{C}_{50\%}}$ and $\delta_{\dot{S}_{50\%}}$ are the minimum fractions of the total area needed to accommodate 50% of the total opening, closing and shearing rate, respectively.

α	$\beta_{shear}(1)$	a_{shear}	$\beta_{div}(1)$	a_{div}
-40	0.06	0.18	0.18	0.21
-20	0.04	0.18	0.15	0.23
-10	0.04	0.17	0.15	0.21
0	0.04	0.14	0.14	0.20

Table 6: Impact of using different values of the compactness parameter α on the structure function $\beta(q)$, which is well fitted by a quadratic function $\beta(q) = aq^2 + bq$. $\beta(1)$ and a are given for the shear rate and divergence rates, respectively.

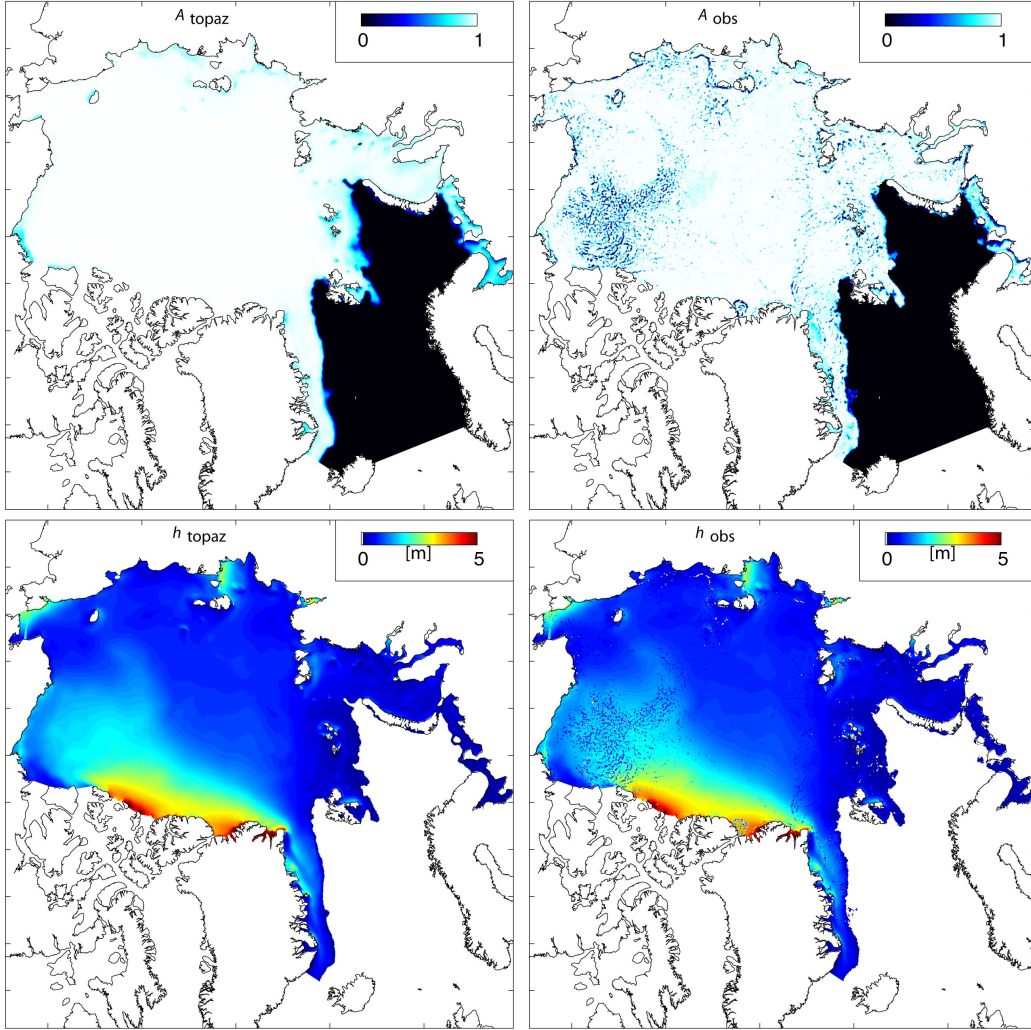


Figure 1: Two sets of sea ice conditions are used to initialize the model (here for the 5th March 2008): either the sea ice concentration (A_{topaz}) and thickness (h_{topaz}) from the TOPAZ reanalysis (left panel), or a combination of the sea ice concentrations A_{tot} and lead area fraction A_{lead} derived from AMSR-E (right panel). The initial sea ice concentration is then defined as $A_{obs} = A_{tot} (1 - A_{lead})$ and the initial sea ice thickness is defined as $h_{obs} = h_{topaz} / A_{topaz} A_{obs}$.

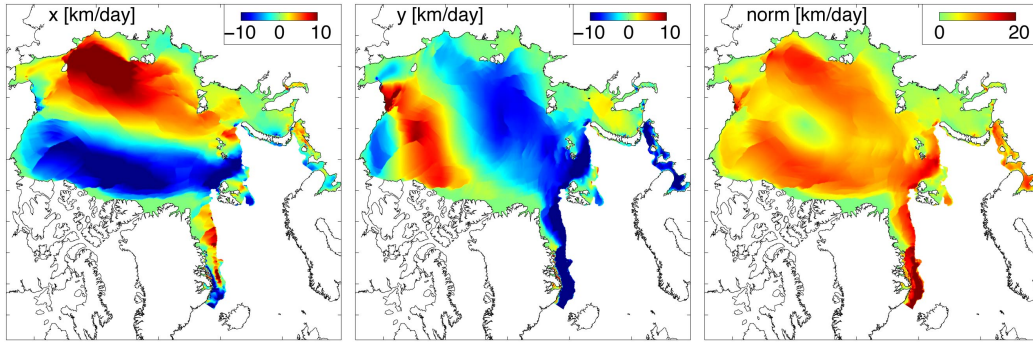


Figure 2: x-component, y-component and norm of the sea ice velocity (in km/day) computed over the last 3 days of the reference simulation (from 12 to 15 March 2008).

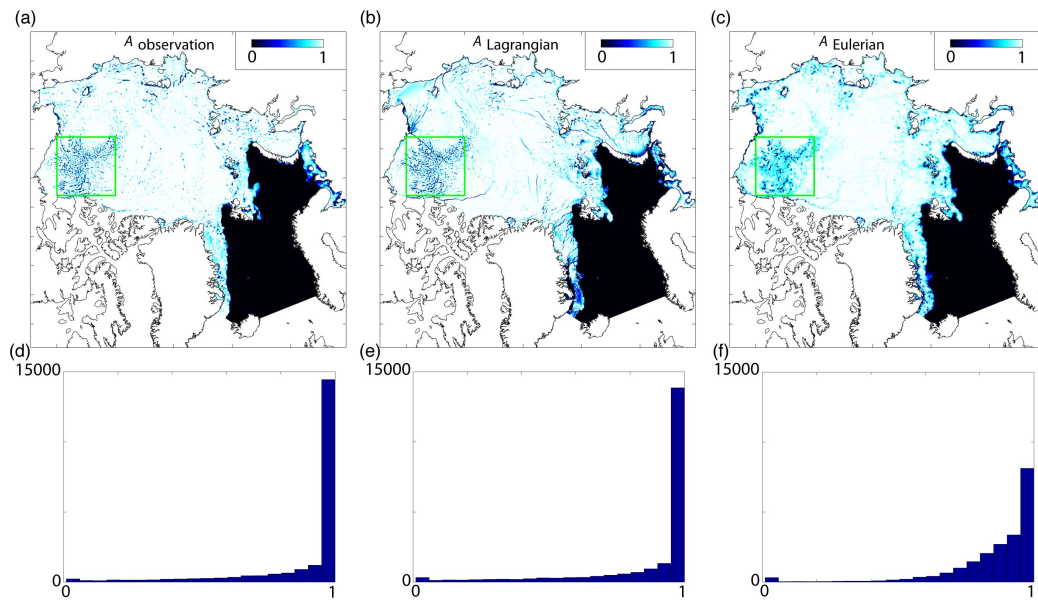


Figure 3: Sea ice concentration fields for the 15th March 2008 from observations (a) and obtained from simulations initialized on the 5th March with A_{obs} and h_{obs} using a Lagrangian advection scheme (b) or an Eulerian upwind advection scheme (c). The corresponding distributions of ice concentration (d, e, f) are computed on an arbitrary region in the Beaufort Sea indicated by a green rectangle. The numerical diffusion produced by the use of the Eulerian upwind scheme significantly impacts the statistics of ice concentration.

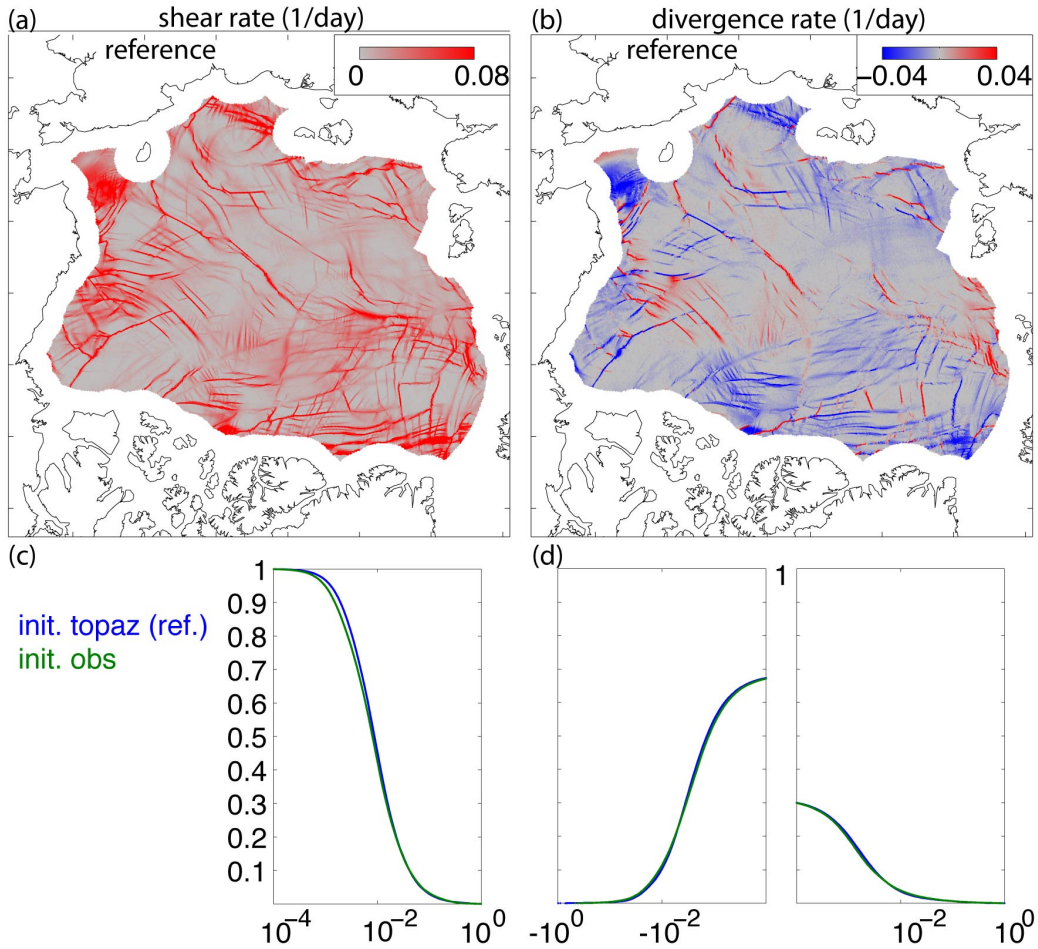


Figure 4: Sea ice (a) shear rate and (b) divergence rate (in 1/day) computed over the last 3 days of the reference simulation (from 12 to 15 March 2008) for all the elements in the Central Arctic that are located at least 150 km from the nearest coasts. The corresponding cumulative distributions (c, d), in other words the probabilities of exceedance, are shown for the reference simulation (i.e., with initial conditions A_{topaz} and h_{topaz} , blue line) and for the simulation with the initial conditions A_{obs} and h_{obs} (green line). In both cases, the cohesion $c=4$ kPa and the compactness parameter $\alpha = -20$ (see Table 6 for the list of all the parameters).

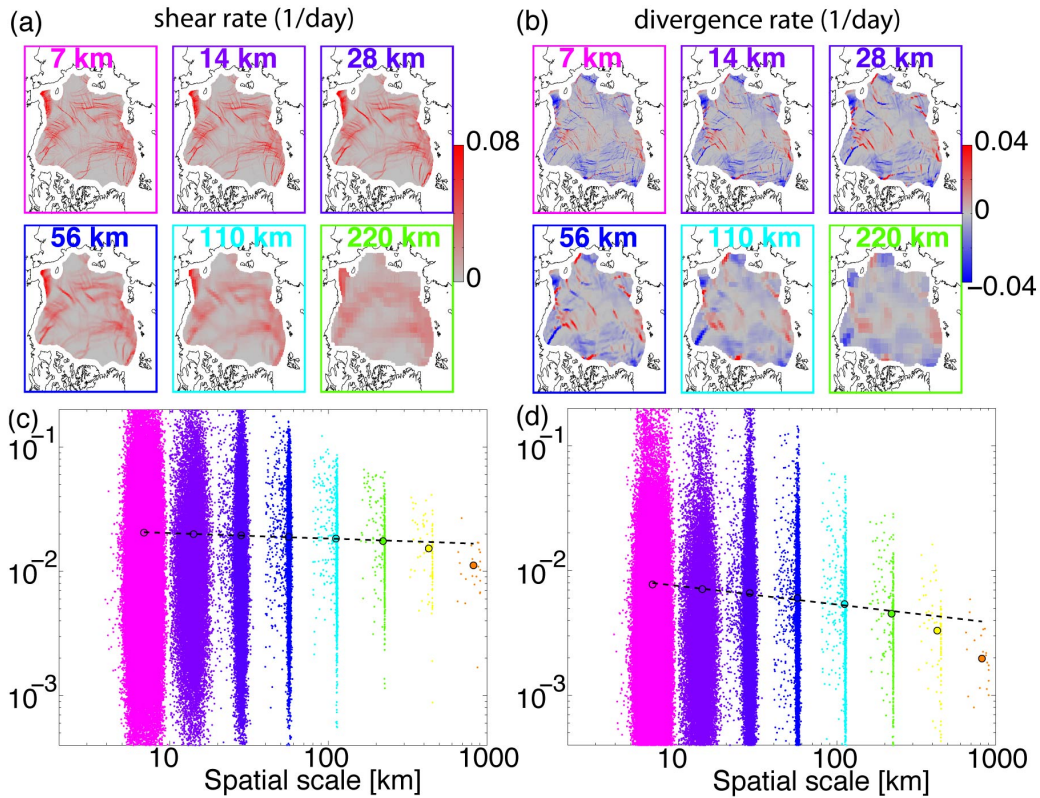


Figure 5: Sea ice (a) shear rate and (b) divergence rate (in 1/day) computed over the last 3 days of the reference simulation (from 12 to 15 March 2008) for spatial scales ranging from 7 to 220 km (each color corresponds to a different scale). The coarse-graining procedure defines boxes of different sizes and compute for each box the mean deformation over all the elements that have their center in the box. The values of the shear rate and divergence rate are then reported as a function of the spatial scale, here defined as the square root of the area covered by the selected elements (c, d, respectively). The mean values are represented by circles and the dashed lines are power law fits of the first six mean values (here, from 7 to 220 km).

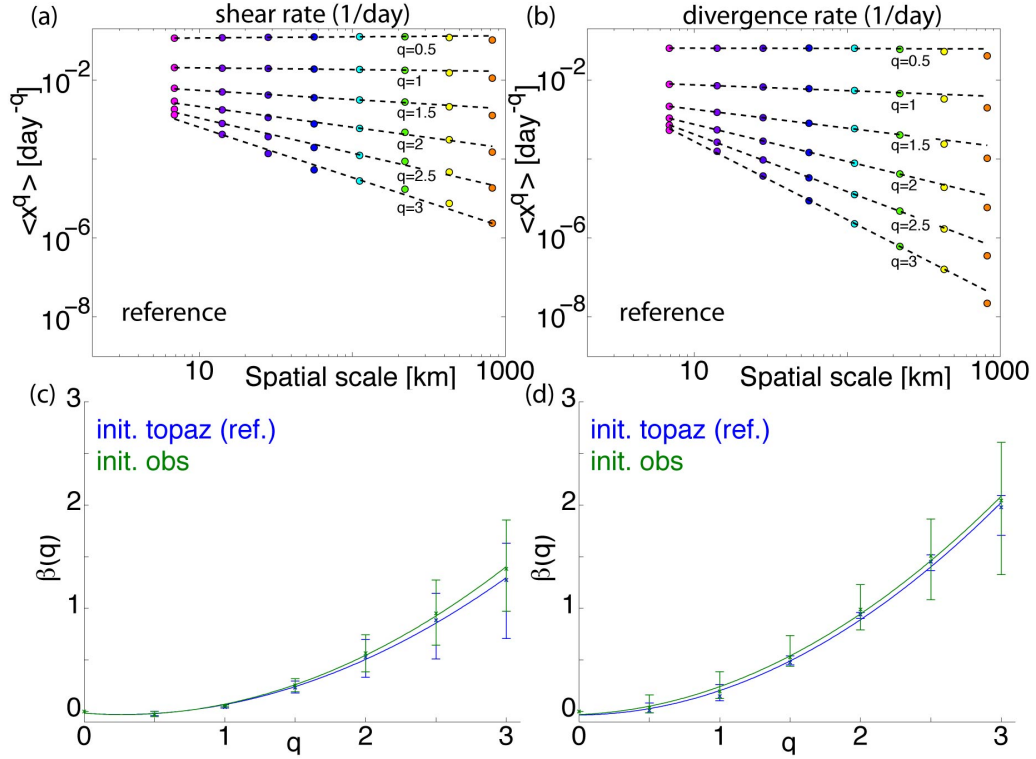


Figure 6: Multifractal analysis: Moments of the deformation rates $\langle \dot{\epsilon}^q \rangle$ as a function of the scale for $q = 0.5$ to 3 , for shear (a) and divergence (b) from the reference simulation. Dashed lines are power law fits of the sixth first values (here, from 7 to 220 km). The slope β of these dashed lines are plotted as a function of the moment order q for the shear (c) and divergence (d) along with the best (in the least-square sense) quadratic fits $\beta(q) = aq^2 + bq$ (solid lines). The curvature a indicates the degree of multifractality. The bars on the graph are not error bars but indicate for each moment order q the minimum and maximum slope β obtained with only two of the six first values.

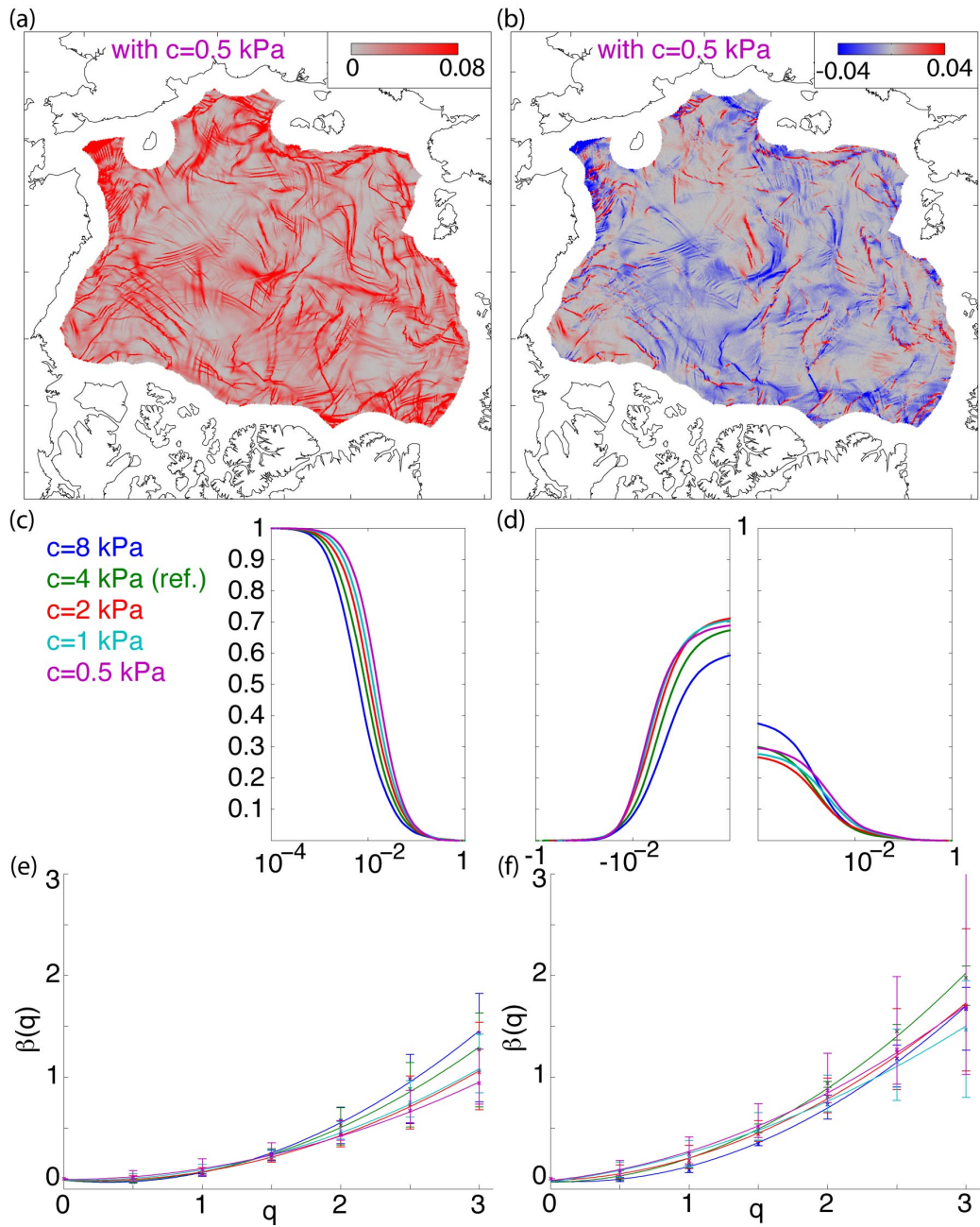


Figure 7: Sea ice (a) shear rate and (b) divergence rate (in 1/day) computed over the last 3 days of the simulation using a cohesion $c=0.5$ kPa. The corresponding cumulative distributions (c, d) and $\beta(q)$ functions (e, f) are shown for a cohesion parameter equal to 8, 4, 2, 1 and 0.5 kPa.

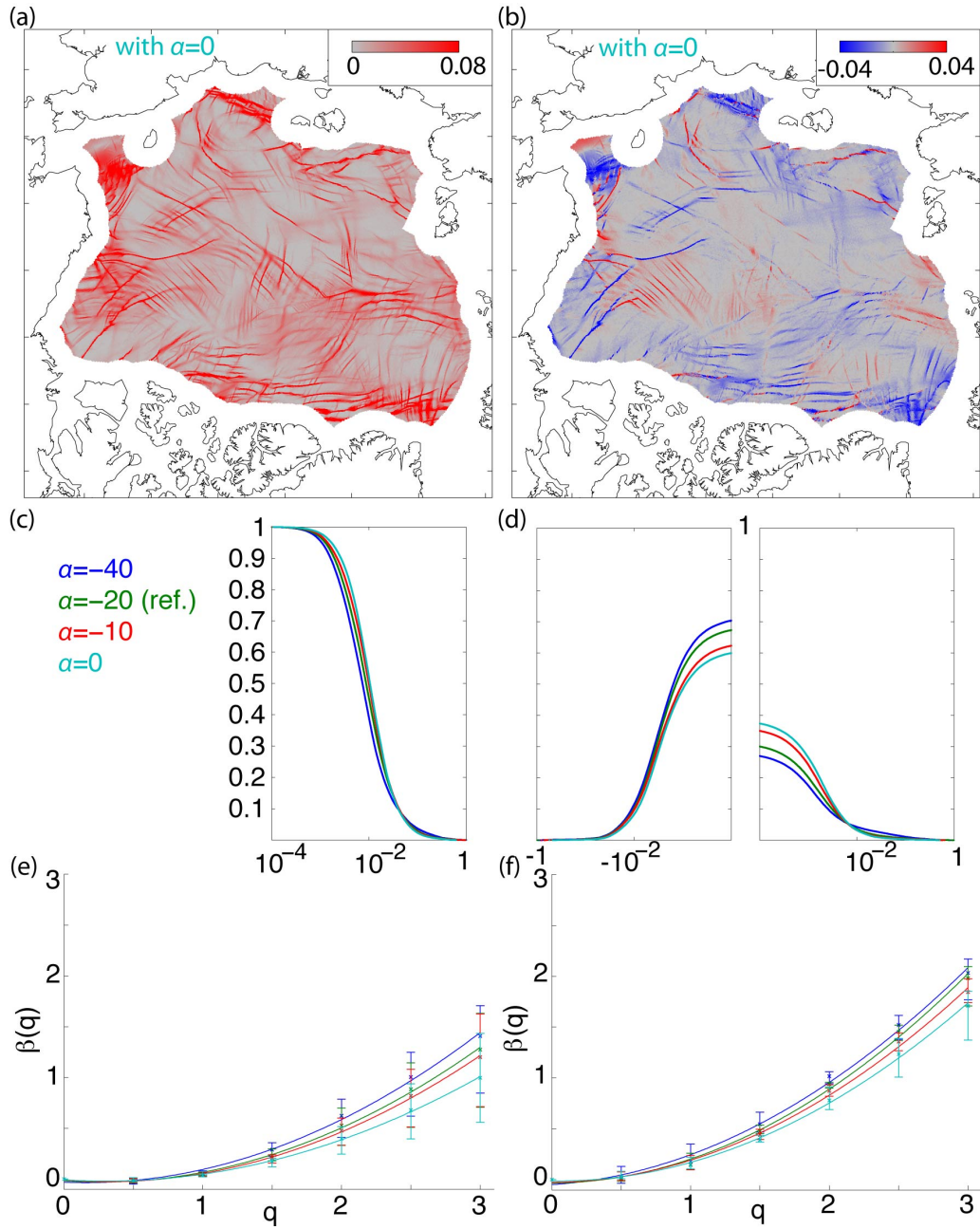


Figure 8: Sea ice (a) shear rate and (b) divergence rate (in 1/day) computed over the last 3 days of the simulation using a compactness parameter $\alpha = 0$. The corresponding cumulative distributions (c, d) and $\beta(q)$ functions (e, f) are shown for a compactness parameter equal to -40, -20, -10 and 0.