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Abstract

We conduct an experiment in which subjects participate in a first-price auction against an automaton that bids randomly in a given range. The subjects first place a bid in the auction. They are then given an incentivized elicitation of their beliefs of the opponent's bid. Despite having been told that the bid of the opponent was drawn from a uniform distribution, we find that most of the subjects report beliefs that have a peak in the interior of the range. The result is robust across seven different experimental treatments. While not expected at the outset, these single-peaked beliefs have precedence in the experimental psychology judgments literature. Our results suggest that an elicitation of probability beliefs can result in responses that are more concentrated than the objectively known or induced truth. We provide indicative evidence that such individual belief reports can be rationalized by well-defined subjective beliefs that differ from the objective truth. Our findings offer an explanation for the conservatism and overprecision biases in Bayesian updating. Finally, our findings suggest that probabilistic forecasts of uncertain events might have less variance than the actual events.

Keywords: belief elicitation, quadratic scoring rule, overprecision, conservatism

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1 Introduction

Often when experimenters describe a stochastic distribution to subjects, it is assumed that they accept the distribution as given.[1] We conduct an experiment where the subjects participate in a first-price auction against a random-draw opponent with a strategy known by the subject: their opponent's bid is uniformly distributed on a given range. The subjects first place a bid in the auction. They are then given an incentivized elicitation of their beliefs of the opponent's bid. Despite having been told that their opponent's bid is drawn from a uniform distribution, we find that subjects tend to report beliefs that have a single peak in the interior of the range. While not expected at the outset, these single peaked beliefs have precedence in the judgments literature.

When subjects estimate physical quantities (length, weight, loudness, etc.), the judgments[2] often exhibit a bias toward the mean of the distribution of the stimuli (Hollingworth, 1910; Poulton, 1979). For instance, if subjects are tasked to make judgments of lengths of lines, lines longer than the mean tend to be underestimated and lines shorter than the mean tend to be overestimated. In other words, there is a tendency to judge physical quantities to be closer to the mean than they actually are. This effect is sometimes referred to as the *central tendency bias*.[3] Considered in aggregate, the reported distribution of judgments of line lengths tends to be less variable than the true distribution of line lengths.

The central tendency bias has been found in various other judgment settings, for instance weight (Jones and Hunter, 1982), distance (Radvansky, Carlson-Radvansky, and Irwin, 1995), loudness (Algom and Marks, 1990), and temporal duration (Jazayeri and Shadlen, 2010). Huttenlocher, Hedges, and Vevea (2000) also find the central tendency bias in judgments of the fatness of computer-generated images of fish, the greyness of squares, and the lengths of lines.[4] We say that the elicitation of beliefs in our experiment exhibits the central

[1]One exception is Camerer and Weigelt (1988), who find that subjects can bring "homemade" priors to the experiment that are different from those specified in the experiment. We also note that Prospect Theory (Kahneman and Tversky, 1979) suggests that subjects can behave as if probabilities were different from those given by the experimenter. This relationship is characterized by the probability weighting function.

[2]While judgments of uncountable physical quantities are common in the psychology literature, see Duffy, Gussman, and Smith (2019) for an experiment where subjects estimate line lengths in a choice setting.

[3]This is also sometimes referred to as the regression effect (Stevens and Greenbaum, 1966) or the contraction bias (Jou et al., 2004). The representativeness heuristic (Kahneman and Frederick, 2002; Kahneman and Tversky, 1973) makes similar predictions. Finally, the extremeness aversion literature (Simonson and Tversky, 1992; Chernev, 2004; Neumann, Böckenholt, and Sinha, 2016) finds that subjects tend to avoid extreme options in choice settings.

[4]Huttenlocher el al. (2000) also offer an explanation for this experimental regularity. Also see Huttenlocher, Hedges, and Duncan (1991). The authors propose a *category adjustment model*, which posits that subjects compensate for their imperfect memory and imperfect perception by employing information about the distribution of stimuli. We note that, according to the category adjustment model, subjects learn the distribution through experience. Huttenlocher et al. refer to this as a *Bayesian* model since the information about the distribution is employed to maximize the precision of the judgments. However, Duffy and Smith (2019a) analyze data from a replication of Huttenlocher et al. (2000) and find that non-Bayesian explanations have more support in the data than the category adjustment model. Duffy and Smith (2018) analyze the data from Duffy, Huttenlocher, Hedges, and Crawford (2010), and come to a similar conclusion about the predictions of the category adjustment model. See also Crawford (2019) and Duffy and Smith (2019b).

tendency bias because there appears to be such a bias toward the mean of the distribution.

The central tendency bias is not commonly studied in economics settings. However, some authors report a bias toward the center of an ordered action space in games. For instance, Arad and Rubinstein (2012) and Arad and Penczynski (2018) find a tendency to overweight the central battlefields in Colonel Blotto games, despite there being no strategic advantage of doing so.

In the domain of beliefs, Kareev, Arnon, and Horwitz-Zeliger (2002) find that subjects tend to perceive a stochastic distribution learned through sampling to be less variable than it truly is.[5] Theoretically, they explain this finding by arguing that subjects estimate the true variance by sample variance, which, if uncorrected for the degrees of freedom, is a downward-biased estimate of variance. The main contribution of this paper is the finding that underestimation of true variance is present also when the distribution is learned by description rather than by sampling. The theoretical justification described above does not apply in this case. Our finding therefore suggests that the central tendency bias in reporting of judgments in general and beliefs in particular might be driven by an intrinsic reporting bias toward the center of the support range.

We also go a step further and investigate where such bias might come from in our setting. We first rule out that it is an artefact of experimental design operating through incentive incompatibility of the belief elicitation procedure or through payoff hedging. We then provide indicative evidence that such individual belief reports can be rationalized by well-defined subjective beliefs that differ from the objective truth. This view is also supported by a finding that, between subjects, beliefs are correlated with auction bids.

Apart from contributing to the literature documenting biases in reported judgments and beliefs, our finding, if corroborated by future research, has a profound implication for designing experiments. It implies that subjects do not necessarily accept the distribution as given by the experimenter, even when the distribution is as simple as the uniform distribution. Moreover, our findings offer another explanation for the conservatism and overprecision biases in Bayesian updating. To the extent that our results generalize to other distributions, our findings also suggest that probabilistic forecasts (rather than point forecasts) of events will have less variance than the true variance of these events. We hope to stimulate more empirical work on these issues.

[5] Also see Marchiori, Di Guida, and Erev (2015).

2 Experimental design

2.1 Overview

We conducted an experiment where subjects engaged in a first-price auction. Subjects were told the distribution of their random-draw opponent's strategy: a uniform distribution on a range of possible integer bids. The subjects then placed a bid in the auction. Subsequently, subjects were given an incentivized elicitation of their beliefs of the random-draw opponent's strategy and their beliefs of winning the auction. The random-draw opponent's bid was determined by a physical draw of a token at the end of the experiment. A total of 379 subjects participated in the experiment.

2.2 Belief elicitations

After submitting a bid in the auction, subjects reported their beliefs of the distribution of their random-draw opponent's strategy. The strategy space of the random-draw opponent was divided into 5 bins of equal size. Subjects allocated probability weights into the bins. An automatic checker verified that these amounts correctly summed to 100. Screenshots of the elicitation procedures are provided in Appendix D.[6]

It is well-known that eliciting beliefs from subjects, particularly the full distribution of beliefs, can be a difficult matter.[7] We elicited the distribution of beliefs of the opponent's strategy by using the quadratic scoring rule (QSR), apparently first suggested by Brier (1950). As the QSR can be difficult for subjects to understand, instead of presenting the payoff formula, we provided subjects with general and intuitive advice on how to perform well on this task.[8] We also prepared a sheet with the payoff formula to be shown to subjects who requested seeing it after the experiment.[9] The maximum that subjects could earn on this task was 20 ECUs, where 1 ECU=€0.20.

Subjects were also asked to report their beliefs of winning (and not winning) the auction given their bid. Given the known distribution of the strategy of their opponent, there is a well-defined objective probability

[6]Figure D1 shows a treatment in which the random-draw opponent's strategy was the uniform distribution on $\{1, \ldots, 60\}$. Figures D2 and D3 show situations in which random-draw opponent's strategy was the uniform distribution on $\{1, \ldots, 100\}$ and the values did not sum to 100 or the cells were empty.

[7]See Winkler and Murphy (1970), Savage (1971), Matheson and Winkler (1976), Allen (1987), Manski (2004), Karni (2009), Offerman, Sonnemans, Van de Kuilen, and Wakker (2009), Blanco, Engelmann, Koch, and Normann (2010), Armantier and Nicolas (2013), Hossain and Okui (2013), Schlag and van der Weele (2013), Andersen, Fountain, Harrison, and Rutström (2014), Schotter and Trevino (2014), Schlag, Tremewan, and van der Weele (2015), Schlag and van der Weele (2015), and Harrison, Martínez-Correa, Swarthout, and Ulm (2017).

[8]See Question 1 in Appendix C for the complete instructions. We note that Qiu and Weitzel (2016) also do not show their subjects the formal expression for the scoring rule.

[9]This information sheet is available on OSF at https://osf.io/fy3mt/. Only 2 out of 379 subjects requested this information.

of winning the auction given the bid. Similar to the previous elicitation, there was an automatic checker that verified that the weights correctly summed to 100.[10] These beliefs were also incentivized with the QSR. The maximum that subjects could earn on this task was 20 ECUs. Screenshots are provided in Appendix D.[11]

2.3 Auction treatments

Subjects placed a bid against a random-draw opponent in one of seven different auction treatments in a between-subject design. In every treatment, the payoff was an induced value known by the subject minus the bid in the event of winning the auction, and 0 otherwise. Ties were resolved in favor of subjects.

The *auction 100/100* treatment is our baseline.[12] Subjects' value of the object was 100 ECUs. The random-draw opponent had a strategy of placing a bid drawn from the uniform distribution on $\{1, ..., 100\}$. Subjects selected a bid from $\{1, ..., 100\}$. This treatment elicited bids using a visual representation of the strategy space and of the lottery induced by each choice.

The *auction 100/100 without visualization* treatment was identical to the baseline except that auction bids were elicited simply by typing a response from $\{1, ..., 100\}$ rather than with the aid of the visual representation.

In the *BRET* treatment, we implemented the Bomb Risk Elicitation Task (BRET, Crosetto and Filippin, 2013). In the *BRET*, subjects faced a 10×10 matrix with 100 numbered boxes. Of these, 99 were empty, while one contained a time bomb programmed to "explode" at the end of the task, i.e., *after* choices have been made. The bomb had an equal probability to be in any of the 100 boxes. Subjects decided how many of the 100 boxes to collect in the increasing order of their numbers. If the collection did not contain the bomb, the payoff was equal to the number of collected boxes, and 0 otherwise. The *BRET* is isomorphic to the *auction 100/100* treatment, with the number of *uncollected* boxes corresponding to the bid, and the same visual representation as the baseline.

In the *auction 80/100* treatment, subjects' value of the object was 80 ECUs. The strategy of the computerized opponent and subjects' bidding space was identical to the baseline. The bid was elicited using the same visual representation as in the baseline, and included the possibility of overbidding. The instructions contained a warning against bidding more than 80.

The *auction 60/100* treatment is identical to the *auction 80/100* treatment with the exception that subjects'

[10]See Question 2 in Appendix C for the complete instructions.

[11]See Figures D4, D5 and D6 for screenshots where values sum up to 100, do not sum to 100 or the input boxes are empty.

[12]In the name of the treatments, the number before "/" refers to the subject's value of the object and the number after "/" to the upper bound of the bidding range of the automaton.

value of the object was 60 ECUs and the instructions contained a warning against bidding more than 60.

The *auction 60/60* treatment set the subjects' value of the object to 60 ECUs. The computerized opponent bid uniformly on $\{1, ..., 60\}$. Subjects selected a bid from $\{1, ..., 60\}$. Bids were elicited with a visualization analogous to the baseline, but with the size of the matrix reduced to 10×6 boxes that were numbered $\{1, ..., 60\}$.

The *auction 60/60 expand* treatment is identical to the *auction 60/60* treatment with the exception that subjects selected a bid from $\{1, ..., 100\}$ and the visualization accommodated this by presenting a 10×10 matrix. However, the difference from the baseline is that the auction winning probabilities were derived from the random-draw opponent bidding range being $\{1, ..., 60\}$ rather than $\{1, ..., 100\}$. Instructions contained a warning against bidding more than 60.

2.4 Experimental details and earnings

The timing of the experiment is as follows. First, subjects were required to pass an unincentivized test of their understanding of first-price auctions and uniform probability distributions.[13] The subjects then placed their auction bid under 1 of the 7 experimental treatments. The subjects then responded to an elicitation of their beliefs of the distribution of the random-bid opponent's strategy and their beliefs of the probability of winning the auction given their bid.

The auction values, bids and prices were expressed in ECUs, where 1 ECU=€0.20. Each belief elicitation question could earn as many as 20 ECUs. One of these two elicitations was randomly selected for payment. Subjects were paid their earnings in the auction, the randomly drawn belief elicitation and a €2.50 show-up fee.

The sessions were conducted in German in the laboratory of the Max Plank Institute for Economics in Jena, Germany.[14] They lasted approximately 30 minutes and subjects earned €8.50 on average. We note that this amount was above the the hourly wage available to our student subjects at the time of the experiment.

[13] See Appendix C for the complete list of questions the subjects had to accurately complete.

[14] Appendix C contains the English translation of the instructions given to the subjects. The original German wording is available from the corresponding author upon request.

Table 1: Mean weights within bins

Treatment	Bin 1	Bin 2	Bin 3	Bin 4	Bin 5
Auction 100/100	11.92	21.92	30.38	23.72	12.07
Auction 100/100 w/o	11.68	21.39	28.98	23.05	14.90
BRET	18.16	25.87	23.82	18.45	13.69
Auction 80/100	16.53	26.09	28.72	18.06	10.59
Auction 60/100	15.85	25.77	28.08	18.95	11.35
Auction 60/60	12.25	19.03	29.68	23.59	15.44
Auction 60/60 expand	14.94	22.45	24.15	21.61	16.85
Pooled	14.33	23.02	27.65	21.27	13.73

Notes: We list the means of the weights reported within each of the 5 bins for each treatment and pooled across all treatments.

3 Results

3.1 Summary statistics

We define a *response* to be the collection of probability weights allocated to the 5 bins. The response in the lowest bin is labeled *Bin 1*, next, *Bin 2*, and so on. The 5 bins in *auction 60/60* and *auction 60/60 expand* treatments refer to ranges $1-12, 13-24, 25-36, 37-48, 49-60$. The 5 bins in the remaining treatments refer to ranges $1-20, 21-40, 41-60, 61-80, 81-100$. Table 1 summarizes the means of the weights within each bin. Figure 1 gives an overview of the full dataset, with each point representing the weight allocated by a subject to the given bin and the boxplots representing the distribution. The central tendency is robust and immediately evident at the aggregate level.

Analyzing the data at the individual level, we first explore the extent of the general deviation from the payoff-optimal uniform response. We define a response to be *non-uniform* if a distribution of weights other than $(20, 20, 20, 20, 20)$ was given. Results are reported in Table 2. Despite there being incentives for correctly responding with uniform weights, more than 72% of subjects give a response other than the uniform distribution. To capture how far the response is from the uniform distribution, we measure the largest vertical distance (d) of the cdf of the reported belief from the cdf of the uniform distribution at the four boundaries dividing the 5 bins. Cumulative beliefs of more than two thirds of subjects deviate from the uniform distribution by at least 10 percentage points, those of more than two fifths by at least 20 percentage points and those of more than a fifth by at least 30 percentage points.[15] This result is robust to different specifications, see Table

[15]Interestingly, not having the visualization (in the *Auction 100/100 w/o*) does not make the deviation from the uniform worse. If

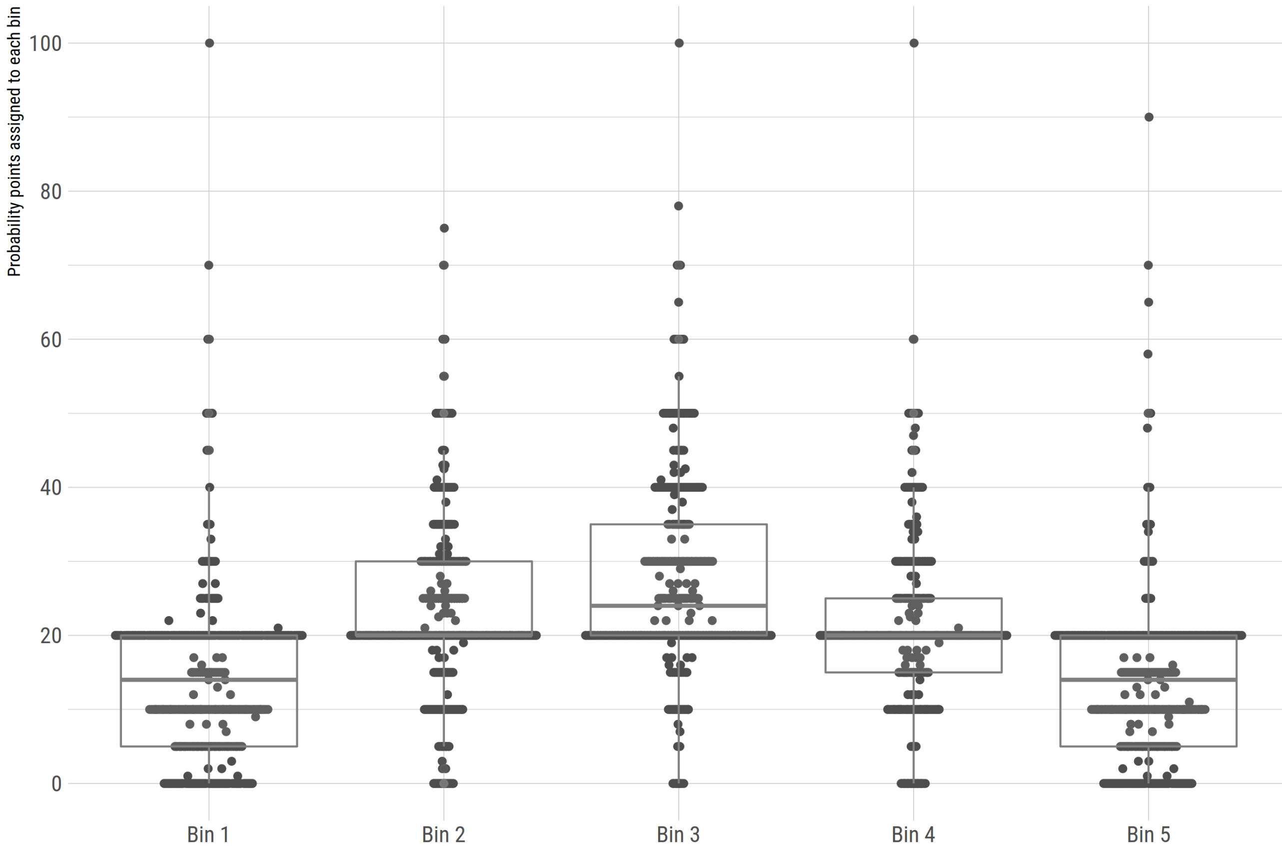


Figure 1: The full dataset at a glance. Each point represents the weight allocated by a subject to the given bin. The boxplots show the 25 $^{\text{th}}$, 50$^{\text{th}}$ and 75$^{\text{th}}$ percentile of the distribution.

B1 in Appendix B.

3.2 Central single peaked distributions

We next explore to what extent deviations from the uniform distribution are due to the central tendency bias. We define a response to have a *strict central single peak (Strict-CSP)* if the weight in Bin 1 is strictly less than that in Bin 2, the weight in Bin 2 is strictly less than that in Bin 3, the weight in Bin 4 is strictly less than that in Bin 3, and the weight in Bin 5 is strictly less than that in Bin 4. If we define w_i as the weight allocated into Bin i, then we can write the definition of Strict-CSP as $w_1 < w_2 < w_3 > w_4 > w_5$. To loosen this definition such that it

anything, the deviation rate and the threshold distance rates are lower than the average of the other 6 treatments, but the difference is not statistically significant using the Fisher's exact test (the p-values corresponding to the four statistics presented in Table 2 are 0.427, 0.880, 0.474 and 0.177, respectively).

Table 2: Non-uniform responses and distances from the uniform distribution

Treatment	Non-Uniform	$d > 0.1$	$d > 0.2$	$d > 0.3$	Subjects
Auction 100/100	46 (76.7%)	41 (68.3%)	20 (33.3%)	11 (18.3%)	60
Auction 100/100 w/o	40 (67.8%)	39 (66.1%)	22 (37.3%)	9 (15.3%)	59
BRET	44 (71.0%)	39 (62.9%)	20 (32.3%)	13 (21.0%)	62
Auction 80/100	25 (78.1%)	25 (78.1%)	18 (56.3%)	11 (34.4%)	32
Auction 60/100	40 (72.7%)	39 (70.9%)	28 (50.9%)	14 (25.5%)	55
Auction 60/60	41 (69.5%)	38 (64.4%)	29 (49.2%)	16 (27.1%)	59
Auction 60/60 expand	39 (75.0%)	35 (67.3%)	23 (44.2%)	10 (19.2%)	52
Pooled	275 (72.6%)	256 (67.5%)	160 (42.2%)	84 (22.2%)	379

Notes: We list the number (and percentage) of subjects who report non-uniform responses. We also list the number (and percentage) of subjects whose reported beliefs deviate from the uniform distribution by at least a threshold sup-norm distance $d \in \{0.1, 0.2, 0.3\}$.

allows a multi-bin peak that includes Bin 3, we define a response to have a *weak central single peak (Weak-CSP)* if the inequalities are allowed to be weak, but the weight in Bin 1 is strictly less than the weight in Bin 3 and the weight in Bin 5 is strictly less than the weight in Bin 3.[16] To loosen the definition of Strict-CSP in a different direction, namely by allowing a strict peak also in other non-boundary bins, we define a response to have a *strict semi-central single peak (Strict-Semi-CSP)* if it satisfies the conditions for Strict-CSP or there is a strict single peak in either Bin 2 or Bin 4.[17] Finally, in the least restrictive definition, that subsumes both Weak-CSP and Strict-Semi-CSP, we define a response to have a *weak semi-central single peak (Weak-Semi-CSP)* if it satisfies the conditions for Weak-CSP or there is a weak single peak in either Bin 2 or Bin 4.[18] Table 3 describes the distribution of responses according to these definitions. Within each treatment and across all treatments, there appears to be a single peak in the interior of the distribution. Over 50% of responses satisfy our definition of a weak semi-central single peak. Figure 2 gives a visual feeling of the data by plotting the allocation of subjects to types of peaked response by treatment, assigning subjects to the most restrictive type.[19]

To further explore incidence of a central single peak in the responses, we take pairwise tests of the differences of the weights between adjacent bins $(w_2 - w_1; w_3 - w_2; w_4 - w_3; w_5 - w_4)$. We perform this analysis with paired t-tests and signed rank tests. Since we make multiple pairwise tests, we perform the Bonferroni

[16] In other words, a response is Weak-CSP if $w_1 \leq w_2 \leq w_3 \geq w_4 \geq w_5$, $w_1 < w_3$, and $w_3 > w_5$. Note that if a response is an Strict-CSP, it is also a Weak-CSP.

[17] A response is Strict-Semi-CSP if it is Strict-CSP or it is a response where $w_1 < w_2 > w_3 > w_4 > w_5$ or $w_1 < w_2 < w_3 < w_4 > w_5$.

[18] A response is Weak-Semi-CSP if it is Weak-CSP or it is a response where $w_1 < w_2 \geq w_3 \geq w_4 \geq w_5$ and $w_2 > w_5$, or it is a response where $w_1 \leq w_2 \leq w_3 \leq w_4 > w_5$ and $w_1 < w_4$. Note that if a response is a Strict-Semi-CSP, it is also a Weak-Semi-CSP.

[19] For instance, a Strict-CSP subject is, by definition, *also* a Weak-CSP subject; in this case, the subject is assigned to the Strict-CSP type.

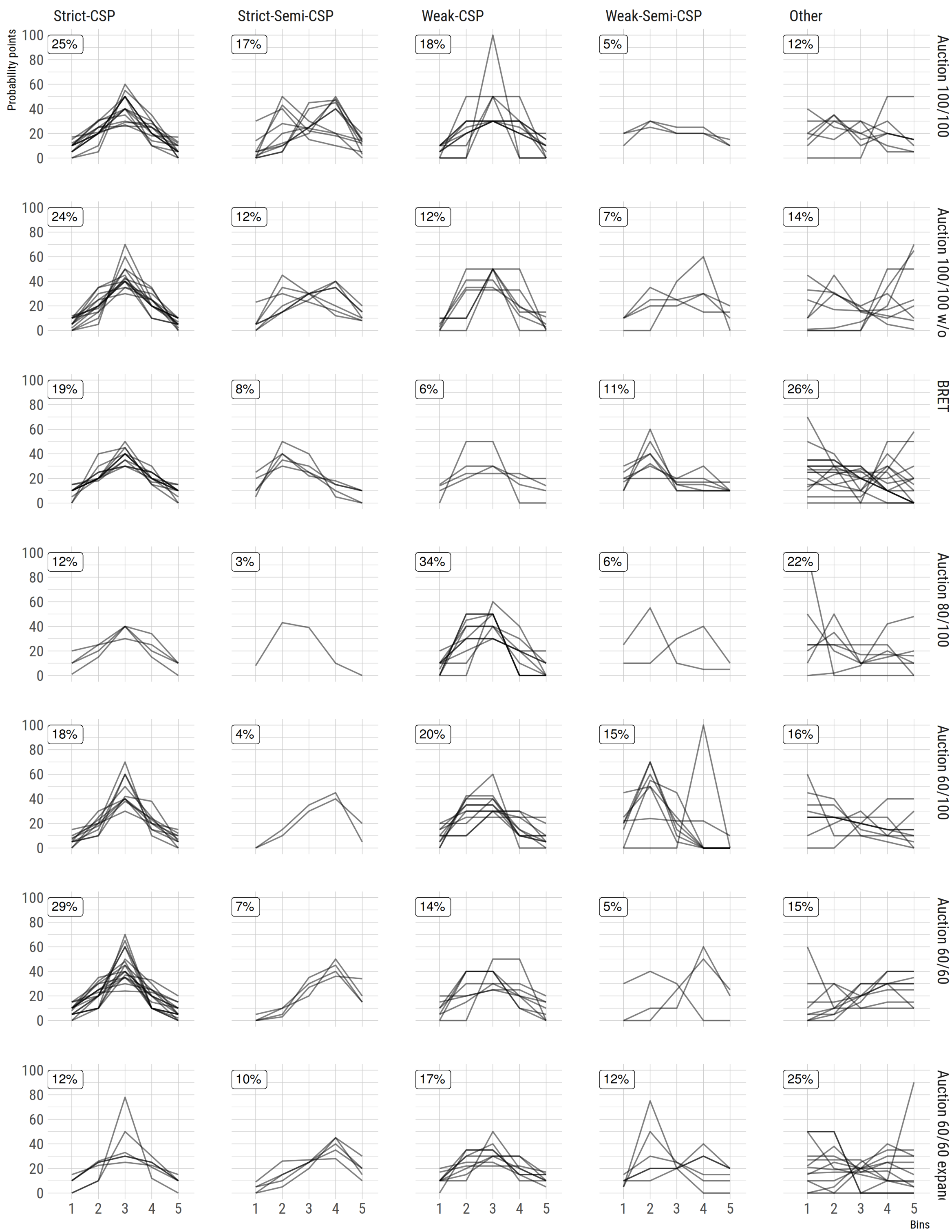


Figure 2: Non-uniform responses at a glance. Each piece-wise line represents a subject. The percentages are frequencies of the types within a treatment.

Table 3: Frequency of peaked responses

Treatment	Strict-CSP	Weak-CSP	Strict-Semi-CSP	Weak-Semi-CSP	Subjects
Auction 100/100	15 (25.0%)	26 (43.3%)	25 (41.7%)	39 (65.0%)	60
Auction 100/100 w/o	14 (23.7%)	21 (35.6%)	21 (35.6%)	32 (54.2%)	59
BRET	12 (19.4%)	16 (25.8%)	17 (27.4%)	28 (45.2%)	62
Auction 80/100	4 (12.5%)	15 (46.9%)	5 (15.6%)	18 (56.3%)	32
Auction 60/100	10 (18.2%)	21 (38.2%)	12 (21.8%)	31 (56.4%)	55
Auction 60/60	17 (28.8%)	25 (42.4%)	21 (35.6%)	32 (54.2%)	59
Auction 60/60 expand	6 (11.5%)	15 (28.8%)	11 (21.2%)	26 (50.0%)	52
Pooled	78 (20.6%)	139 (36.7%)	112 (29.6%)	206 (54.4%)	379

Notes: We list the number (and percentage) of responses that we categorize as having a strict central single peak, a weak central single peak, a strict semi-central single peak, and a weak semi-central single peak.

Table 4: Paired t-tests and signed rank tests of differences in weights between adjacent bins

	Bin 2 - Bin 1	Bin 3 - Bin 2	Bin 4 - Bin 3	Bin 5 - Bin 4
t-statistic	10.83	4.81	-6.68	-11.00
Corrected p-value	< 0.001	< 0.001	< 0.001	< 0.001
Signed rank statistic	10664.5	4747.5	-7276	-10037
Signed rank z-statistic	12.26	5.44	-7.30	-12.52
Corrected p-value	< 0.001	< 0.001	< 0.001	< 0.001

Notes: We list the t-statistic for a paired t-test and the signed rank test statistic for adjacent bins. We report the Bonferroni corrected p-values of these tests. Since we have 4 pairwise tests, we multiply each p-value by 4. Each test is based on 379 observations.

correction. As we make four pairwise comparisons, we multiply every p-value by 4. We summarize this analysis in Table 4. We note that each statistic involving Bins 1 or 2 is positive and each statistic involving Bins 4 or 5 is negative. This suggests that the weights tend to be increasing below Bin 3 and decreasing above Bin 3. In other words, this is evidence of a central single peak. Moreover, even with the Bonferroni correction of multiplying every pairwise p-value by 4, each test is significant at 0.001. This result is robust to different specifications, see Table B2 in Appendix B.

4 What Drives the Non-Uniform Responses?

In this section, we try to shed light on what drives the non-uniform responses, and particularly CSP responses. In subsections 4.1 and 4.2, we examine the possibility that such responses are an artifact of the experimental design. In 4.1, we ask whether such responses might be driven by the impact of risk non-neutrality within the Quadratic Scoring Rule (QSR) belief elicitation. In 4.2, we analogously examine a potential role of payoff hedging. In subsections 4.3 and 4.4, we examine the extent to which such responses might reflect truly held beliefs. In 4.3, we ask whether such responses are consistent with bids. In 4.4, we examine whether the non-uniform responses are consistent with reported probabilities of winning the auction.

4.1 Impact of Risk Non-neutrality on Belief Elicitation via the QSR

In this subsection, we investigate the possibility that a subject holding the uniform true belief provides a non-uniform response because of incentives inherent in the QSR and its description.

We begin by noting that the instructions do not provide a payoff formula for the QSR. Instead, subjects are told: "*You will be paid based on how closely your estimates match your opponent's bid. The exact formula (the so-called quadratic scoring rule) is complicated and the experimenters will be happy to explain it after the end of the experiment to those who are interested. However, in order to maximize your expected earnings from this procedure, you should report these likelihoods truthfully according to what you believe.*" Subjects are then also advised to make their reported probabilities strictly increasing in their truly believed probabilities and to not under- or over-concentrate their reports relative to what they believe. Given this description, it is hard to see why a subject holding the uniform belief would provide a non-uniform response.

However, we consider how subjects with both knowledge of the QSR payoff formula and understanding of its incentive properties would report given their true belief of the bid of the opponent. It is widely known that the QSR is incentive compatible for risk-neutral expected utility (EU) maximizers, whereas it is not incentive compatible for risk-averse or risk-loving EU maximizers. What is crucial for our purpose is to see whether the non-uniform responses could be driven by such deviations from risk neutrality despite the underlying true belief being uniform.

Formally, suppose that the state space is partitioned into n mutually exclusive and exhaustive events (5 in our setting). Let $p_1, ..., p_n$ be the truly believed probabilities and let $r_1, ..., r_n$ be the reported probabilities of the individual events. Any report must satisfy $r_1 + ... + r_n = 1$. The realized payoff is given by a constant minus a penalty linear in the square of the Euclidian distance between the realized and the reported probability vector. The former one is given by $(0, ..0, 1, 0, .., 0)$, with "1" on the position of the realized state. That is, the payoff if state i is realized is given by

$$\pi_i \equiv \alpha - \beta \left((1 - r_i)^2 + \sum_{j \neq i} (0 - r_j)^2 \right)$$
$$= (\alpha - \beta) + 2\beta r_i - \beta \sum_{j=1}^{n} r_j^2$$

with $\alpha, \beta > 0$. Note that the minimum possible value of π_i is $\alpha - 2\beta$ and it is attained when $r_j = 1$ for some $j \neq i$, whereas the maximum possible value of π_i is α and it is attained when $r_i = 1$. In our setting $\alpha = 20$ and $\beta = 10$ imply a possible payoff range from 0 to 20 ECUs. Consider an EU maximizer with a strictly increasing and twice continuously differentiable utility function $u(\cdot)$. We then have the following result (proof of which can be found in Appendix A):[20]

Proposition 1 *Suppose that $u'(\cdot)$ is bounded away from 0 and ∞ on $[\alpha - 2\beta, \alpha]$. Then any optimal report $(r_1^*, .., r_n^*)$ satisfies:*

1. *for any event i, $r_i^* = 0$ if and only if $p_i = 0$, $0 < r_i^* < 1$ if and only if $0 < p_i < 1$ and $r_i^* = 1$ if and only if $p_i = 1$;*

2. *for any two distinct events i and j, if $p_i > p_j$, then $r_i^* > r_j^*$; by a contrapositive, if $r_i^* = r_j^*$, then $p_i = p_j$;*

3. *for any two distinct events i and j, if $u''(x)/u'(x) \leq \beta^{-1}$ for any $x \in [\alpha - 2\beta, \alpha]$ and $p_i = p_j$, then $r_i^* = r_j^*$;*

4. *for any two distinct events i and j with $p_i > p_j > 0$, if $u(\cdot)$ is linear, then $r_i^* = p_i$ and $r_j^* = p_j$; if $u(\cdot)$ is strictly concave, then $r_i^*/r_j^* < p_i/p_j$; and if $u(\cdot)$ is strictly convex, then $r_i^*/r_j^* > p_i/p_j$.*

The most important implication of this proposition is that if a subject believes that all 5 bins are equally likely, then, barring a sufficiently high risk-loving, it is optimal to provide the uniform response. That is, the non-uniform responses that we observe cannot be a consequence of deviations from risk neutrality if the underlying true belief is uniform.[21]

[20]This proposition is similar to the theoretical results presented in Harrison et al. (2017). However, there are three novelties here: (1) the first statement in part 2; while the same result follows from Lemma 1 in Harrison et al. (2017) under risk aversion, we prove it generally for any risk preference (the contrapositive statement in part 2 is identical to Lemma 4 in Harrison et al. (2017)); (2) the upper bound on risk-loving in part 3; while Harrison et al. (2017) prove the same result under risk aversion (Lemma 3), we partially extend it to risk loving as well; (3) part 4; in comparison to Proposition 4 in Harrison et al. (2017), this is a different way of expressing the result that risk aversion leads to a "flattening" of reported beliefs toward equal reports, whereas risk loving has the opposite effect; implications of the two approaches for data inference are arguably identical, though.

[21]To gauge what is meant by "sufficiently high risk-loving", note that by part 3 of Proposition 1, the result of equal reporting of equal true probabilities goes through for any utility function with constant absolute risk loving of at most $\beta^{-1} = 0.1$ if payoffs are

Proposition 1 also implies that if a subject has a well-defined non-uniform subjective belief, they optimally provide a non-uniform report whose shape mirrors the belief in terms of ordinal comparisons of probabilities. Risk aversion has the effect of "flattening" the response relative to the belief, whereas risk loving has a "magnifying" effect instead. Either way, however, the shape of the response mirrors the shape of the underlying belief. Therefore, if being an EU maximizer, a report exhibiting a central tendency implies that the subjective belief exhibits a central tendency, too. The empirical regularity that most of the subjects are risk averse implies that, if risk preferences play a role, what we measure is likely a lower bound of the true central tendency. [22]

4.2 Hedging

Next, we investigate the possibility that subjects holding the uniform true belief provide a non-uniform response because they use their payoff from the belief elicitation as a hedge for their payoff from the auction. Obviously, only a risk-averse subject desires to hedge.[23]

The first comment is analogous to the previous subsection. Given that the instructions do not provide a payoff formula for the QSR and given what subjects were advised, the possibility of hedging is arguably not salient to subjects. However, analogously the previous subsection, we consider how subjects with the knowledge of the QSR payoff formula, understanding of its incentive properties and understanding of incentives for hedging would report given their true belief of the bid of the opponent. In particular, we want to see whether the non-uniform responses could be due to risk-aversion-driven hedging despite the underlying true belief being uniform.

Formally, we follow the structure introduced in the previous subsection, but this time adapted to the parameterization used in the experiment. That is, $n = 5$, $\alpha = 20$ and $\beta = 10$. Moreover, the events now correspond to the individual bins, with event i corresponding to bin i, $i = 1, .., 5$. Let k be the bin that contains the bid. Also, let s be the auction surplus, that is, the difference between value and bid. We assume that this surplus is strictly positive.[24] For $i < k$, the belief elicitation bid is augmented by the auction surplus, resulting

denominated in ECUs, or of at most 0.5 if payoffs are denominated in Euros. Working with the constant absolute risk aversion utility function $u(x) = e^{0.1x}$ when counting in ECUs ($u(x) = e^{0.5x}$ when counting in Euros), and considering a 50/50 lottery between the two most extreme possible payoffs from the payoff elicitation, namely 0 and 20 (€0 and €4), the certainty equivalent of this lottery for a decision maker with such utility function is approximately 14.34 ECUs (€2.87). Arguably, few subjects are as risk-loving as this.

[22] It is also useful to note that Andersen, Fountain, Harrison, and Rutström (2014) show that the magnitude of the bias in reporting beliefs under the QSR due to risk preferences is of a second order importance.

[23] Recall that each subject is paid by the QSR for either eliciting beliefs of the opponent's bid or for eliciting beliefs of winning the auction, but not both. Therefore the payoff from the latter elicitation does not enter into consideration when thinking about hedging using the payoff from the former elicitation.

[24] This is the case for all 379 subjects in our sample.

in the overall payoff of

$$\pi_i \equiv s + 10 + 20r_i - 10\sum_{j=1}^{5} r_j^2.$$

An analogous payoff expression applies also to bin k if the bid of the opponent does not exceed the bid of the subject. Denote this payoff π_{kH} and let q be the objective probability of such outcome conditional on the bid of the opponent being in bin k. Note that $q > 0$ by the design of the tie-breaking rule. For $i > k$, the belief elicitation bid is *not* augmented by the auction surplus, resulting in the overall payoff of

$$\pi_i \equiv 10 + 20r_i - 10\sum_{j=1}^{5} r_j^2.$$

An analogous payoff expression applies also to bin k if the bid of the opponent exceeds the bid of the subject. Denote this payoff π_{kL}. The objective probability of such outcome conditional on the bid of the opponent being in bin k is $1-q$. We then have the following result (proof of which can be found in Appendix A):

Proposition 2 *Suppose that $u(\cdot)$ is strictly concave and $u'(\cdot)$ is bounded away from 0 and ∞ on $[0, 20+s]$. Also suppose that the true belief of the subject is uniform. Then any optimal report $(r_1^*, .., r_5^*)$ satisfies:*

1. *if $k > 1$, then for any $i < k$ it holds that $r_i^* = r_L^*$; moreover, if $q < 1$, then $r_L^* < r_k^*$ and $r_L^* < 0.2$; if $q = 1$ and $k < 5$, then $r_L^* = r_k^* < 0.2$; if $q = 1$ and $k = 5$, then $r_L^* = r_k^* = 0.2$;*
2. *if $k < 5$, then for any $i > k$ it holds that $r_i^* = r_H$ with $r_H > r_k^*$ and $r_H > 0.2$.*

In our data, subjects are not guaranteed to win the auction given their bid, ruling out the case $q = 1$ and $k = 5$. As a result, this proposition implies that if a risk-averter has the uniform belief, then the reported probability in all bins below k (if any) should be the same and strictly less than 20%, the reported probability in all bins above k (if any) should be the same and strictly more than 20% and the reported probability for bin k should be somewhere in between or, if bidding at the upper boundary of bin k, it should be equal to the reported probability for the bins below k (if any). Among 379 subjects in our data, 7 follow the predictions of Proposition 2 to the letter. Once we allow also for cases with $r_H = r_k$, this number increases to 9. Once we also allow for various probability reports below the bin k or above the bin k to vary up to 1 percentage point[25] or 3 percentage points or 5 percentage points, this number increases to 10 or 11 or 13 (3.4% of the sample), respectively. Since this fraction is negligible compared to the fraction of subjects with non-uniform responses,

[25]This could occur, despite the subject's best effort to equalize the reported probabilities, due to the integer reporting grid.

we conclude that the non-uniform responses that we observe cannot be a consequence of payoff hedging if the underlying true belief is uniform.

4.3 "External" Inconsistency of Responses

In this and the next subsection, we try to shed light on the question of whether the non-uniform responses, and particularly the CSP responses, reflect truly-held subjective beliefs or not. If not, such responses could be noisy reports of underlying well-defined subjective beliefs, or they could reflect a lack of well-defined subjective beliefs or confusion, responses we refer to as "pure noise."

We begin by asking whether there is at least some evidence of the non-uniform responses being consistent with an "external" behavior, particularly bidding. We define the response of a subject to be *externally consistent* if this subject's bid maximizes EU under *some* (strictly increasing) utility function and under *some* belief of the opponent's bid that, when aggregated within each of the 5 bins, results in the given response. This is, admittedly, a fairly non-restrictive definition. However, it is arguably the only possible one in the absence of knowledge of subject domain-specific risk attitude and in the absence of assumptions on how beliefs summarized by a response are distributed within the individual bins. Based on this definition, the only subjects that might have externally inconsistent responses are: (a) those whose bid exceeds the value when the probability of winning at this bid is strictly positive; (b) those whose bid results in a zero probability of winning when there is a higher bid at which the auction could be profitably won with a positive probability; (c) those whose bid does not exceed the value, but could be reduced without affecting a positive probability of winning. No subject overbid their value, making (a) irrelevant. Only 3 subjects have a guaranteed zero probability of winning given their response and only 2 of these fit (b). Another 4 subjects fit (c). That is, only 6 out of 379 subjects (about 1.6% of the sample) display externally inconsistent responses. The remaining 373 subjects have externally consistent responses.

Given that the concept of external consistency imposes relatively little discipline on the responses, we also check for the relationship between responses and bids across subjects. This exercise is motivated by considering how one's EU-maximizing bid would change if the belief of the opponent's bid were to shift to the right (toward higher bids of the opponent) in the sense of first-order stochastic dominance (a downward shift in cdf). Holding the utility function constant, such shift of beliefs might be expected to lead to a higher bid.[26]

[26]This is not the case for all possible belief shifts to the right in the sense of first-order stochastic dominance. However, a simple sufficient, but not a necessary, condition for the bid to (weakly) increase is that the after-shift belief cdf $G(\cdot)$ satisfies, relative to the pre-shift belief cdf $F(\cdot)$, that $G(b_1)/F(b_1) \leq G(b_2)/F(b_2)$ for any $b_1 < b_2$ such that $F(b_1) > 0$. To see this, let $u(\cdot)$ be the utility

To examine the extent to which this theoretical comparative static is borne in between-subject comparisons, we rank-correlate across subjects their bid with the probability implied by the response that the bid of the opponent does not exceed the upper boundary of Bin i, repeating the exercise for $i \in \{1, .., 4\}$. To make this exercise meaningful as a reflection of the comparative static discussed above, we must assume that subjects reporting lower probabilities up to Bin i are not systematically more risk averse than subjects reporting higher probabilities. We perform the rank-correlation computations for four groups of subjects. In the first group, we use all 373 externally consistent subjects. In the second group, we use only the externally consistent subjects with non-uniform responses. Next, we split this group into subjects with Weak-Semi-CSP responses (the third group) and others (the fourth group). In *Auction 60/60* and *Auction 60/60 expand*, we rescale both bids and belief bin boundaries by a factor of $100/60$ so that they are comparable to the other treatments. The resulting rank-correlations are presented in Table 5.

We observe that the rank correlations are negative and, with the exception of one case, statistically (highly) significant (even after a Bonferroni correction for multiple testing). That is, reporting lower probabilities up to Bin i (hence assigning higher probabilities above Bin i) is indeed significantly correlated with higher bids. This is a very broad and robust observation. It holds within all four considered groups, which comprise both wide and successively narrower sub-samples of the data. This observation provides further evidence that non-uniform responses in general, and Weak-semi-CSP responses in particular, cannot be dismissed as pure noise. However, it is unclear to what extent the responses reflect true *ex ante* beliefs that subjects hold before they decide on their bid and which have a causal impact on their bids. Given that subjects had to decide on their bids before being probed for their beliefs, their responses might also be driven by an *ex-post* justification of the submitted bids.

4.4 "Internal" Inconsistency of Responses

Despite the finding that non-uniform responses cannot in general be dismissed as pure noise, there might be between-subject differences in whether these responses reflect truly-held subjective beliefs, or noisy reports of underlying well-defined subjective beliefs, or pure noise. In this subsection, we provide evidence that sheds light on these distinctions.

function normalized such that $u(0) = 0$. Then the expected payoff from bidding b when having the belief $F(\cdot)$ is $u(v-b)F(b)$. We then have that if for $b_1 < b_2$ with $F(b_1) > 0$ it is the case that $u(v-b_1)F(b_1) \leq u(v-b_2)F(b_2)$, it must also be the case that $u(v-b_1)G(b_1) \leq u(v-b_2)G(b_2)$. As a result, whenever b_2 generates at least as high an expected utility as b_1 under $F(\cdot)$, it does so also under $G(\cdot)$, and a similar implication analogously holds for strict inequalities as well. Hence the peak of the expected utility function cannot move to the left under G relative to F, and will typically move to the right.

Table 5: Spearman's rank-correlations of (scaled) bid with belief of the bid b_c of the opponent

Subject Group	Statistic	$\Pr(b_c \leq 20)$	$\Pr(b_c \leq 40)$	$\Pr(b_c \leq 60)$	$\Pr(b_c \leq 80)$	Subjects
Externally consistent	Spearman's ρ	-0.241	-0.344	-0.286	-0.139	373
	Corrected p-value	< 0.001	< 0.001	< 0.001	0.028	
Non-uniform	Spearman's ρ	-0.303	-0.419	-0.372	-0.212	269
	Corrected p-value	< 0.001	< 0.001	< 0.001	0.002	
Weak-Semi-CSP	Spearman's ρ	-0.215	-0.333	-0.317	-0.154	205
	Corrected p-value	0.008	< 0.001	< 0.001	0.110	
Other non-uniform	Spearman's ρ	-0.552	-0.634	-0.551	-0.476	64
	Corrected p-value	< 0.001	< 0.001	< 0.001	< 0.001	

Notes: We list the Spearman's ρ rank-correlations of (scaled) bid with belief that the bid b_c of the opponent does not exceed 20, 40, 60 or 80, respectively, for groups of subjects listed in the table. All listed groups exclude the 6 externally inconsistent subjects. In *auction 60/60* and *auction 60/60 expand*, we rescale both bids and belief bin boundaries by a factor of 100/60. We report the Bonferroni corrected p-values of these tests. Since we have 4 pairwise correlations for each group, we multiply each p-value by 4.

To streamline exposition, in this subsection we refer to one's belief report about the bid of the opponent as *response 1* (this is what we have referred to as "response" above), and we refer to one's winning probability report as *response 2*. We define the pair of responses of a subject to be *internally consistent* if there exists a belief about the opponent's bid such that: *a)* it can be aggregated within the 5 bins to reconstruct response 1; *b)* given these beliefs, the implied probability of winning the auction at the submitted bid is equal to response 2. If a subject's responses are internally consistent, there is an argument for considering such responses to reflect truly held beliefs.

If a subject's responses are not internally consistent, we next ask whether they could be considered to be noisy reports of well-defined subjective beliefs. In such case, assuming that noise in the two responses is independent, we expect the two responses to become internally consistent once noise is removed from response 1 (*correction 1*) or response 2 (*correction 2*). If the pair of responses of an internally inconsistent subject becomes internally consistent after at least one of the two corrections, we consider that subject to have well-defined subjective beliefs. In the opposite case, we consider that subject's beliefs not to be well defined and hence the responses to be pure noise.[27]

Applicability of this methodology is limited by the fact that the only way to identify the parts of the two responses accounted for by noise is to posit the true subjective beliefs. If such beliefs were allowed to be subject-specific, one could obtain a 100% internal consistency rate after correction 1 or after correction 2 by choosing a subjective belief that perfectly matches response 2 or response 1, respectively. Hence to render the exercise meaningful, we restrict attention to subject non-specific beliefs. Among infinitely many possibilities, we restrict attention only to the case of the subjective beliefs being uniform. This choice is motivated by the shape of the objective distribution. This means that, under correction 1, response 1 is replaced by the uniform response. Analogously, under correction 2, response 2 is replaced by the objective probability of winning the auction at the submitted bid given the uniform distribution of the opponent's bid. As a consequence, we might end up classifying the responses of some internally inconsistent subjects with non-uniform subjective beliefs as pure noise as opposed to noisy reports of well-defined subjective beliefs just because we did not consider the correct subjective beliefs. Also, we might end up with such erroneous pure-noise classification even if the true subjective beliefs are uniform but the noise in the two responses is so large that none of the two corrections establishes internal consistency. Hence the incidence of pure noise responses in our data should

[27] Performing both noise corrections simultaneously mechanically establishes internal consistency. That is why we do not consider such exercise.

be taken as an upper bound on the true incidence of not-well-defined beliefs.

Table 6 reports results of the consistency and correction classification. We observe that, overall, about 65% of responses are internally consistent. For the uniform-response subjects, it is almost 77%, whereas for the non-uniform-response subjects, it is about 60%. For the Weak-Semi-CSP subjects in particular, 66% of responses are internally consistent. This implies that about three quarters of uniform responses, two thirds of single-peaked responses and 60% of non-uniform responses can be thought of as representing true beliefs. Performing one or the other correction establishes internal consistency for about another quarter of responses in each of these three groups (for the uniform response group, this happens by construction). That is, (at least) about a quarter of the non-uniform responses in general, and the single-peaked responses in particular, can be thought of as being noisy reports of a well-defined underlying (uniform or non-uniform) subjective belief. Finally, (at most) about 14% of the non-uniform responses in general and 10% of the single-peaked responses in particular can be thought of as pure noise.

This exercise implies that a majority of non-uniform responses might indeed reflect true subjective beliefs. Only for a minority (at most one seventh) of non-uniform responses there is little evidence of any other explanation but pure noise. And as discussed above, even this fraction should be taken as an upper bound on the true incidence of pure noise responses in the data.

5 Implications for biases in Bayesian updating

In this section, we discuss implications of the central tendency bias on reported biases in Bayesian updating when there are more than 2 ordered states of the world. It is well-known that subjects often have difficulty properly executing the Bayes' rule.[28] One well-documented bias is *conservatism*, i.e., the tendency of subjects to insufficiently adjust their posteriors to new information (Phillips and Edwards, 1966).[29] Our results suggest an explanation for conservatism in settings where the new information is relatively extreme. If subjects perceive the prior distribution to be less variable than that which the experimenter attempts to induce, we should observe, in response to extreme new information, subjective posteriors closer to priors than the experimenter

[28] A sample of this large literature would include Kahneman and Tversky (1973), Bar-Hillel (1980), Grether (1980, 1992), Weber (1994), Gigerenzer and Hoffrage (1995), El-Gamal and Grether (1995), Zizzo, Stolarz-Fantino, Wen, and Fantino (2000), Sedlmeier and Gigerenzer (2001), Kahneman and Frederick (2002), Charness, Karni, and Levin (2007, 2010), Oechssler, Roider, and Schmitz (2009), Achtziger and Alós-Ferrer (2013), Hawkins et al. (2015), and Cassey et al. (2016).

[29] Also see Beach (1968), Marks and Clarkson (1972), De Swart, J. H. (1972), Griffin and Tversky (1992), Erev, Wallsten, and Budescu (1994), Oechssler, Roider, and Schmitz (2009), and Corner, Harris, and Hahn (2010).

Table 6: Internal consistency of beliefs

Subject Group	Consistent	Inconsistent					Subjects
		All	Consistent after			Inconsistent after	
			Correction 1	Correction 2	Correction 1 or 2	either correction	
All	246 (64.9%)	133 (35.1%)	53 (14.0%)	50 (13.2%)	96 (25.3%)	37 (9.8%)	379
Uniform	80 (76.9%)	24 (23.1%)	0 (0.0%)	24 (23.1%)	24 (23.1%)	0 (0.0%)	104
Non-uniform	166 (60.4%)	109 (39.6%)	53 (19.3%)	26 (9.5%)	72 (26.2%)	37 (13.5%)	275
— Weak-Semi-CSP	136 (66.0%)	70 (34.0%)	34 (16.5%)	23 (11.2%)	50 (24.3%)	20 (9.7%)	206
— Other non-uniform	30 (43.5%)	39 (56.5%)	19 (27.5%)	3 (4.4%)	22 (31.9%)	17 (24.6%)	69

Notes: We list the number (and percentage) of belief reports that we categorize as being internally consistent and internally inconsistent. We further break down the latter group into belief reports that become consistent after *Correction 1*, after *Correction 2*, after *Correction 1* or *Correction 2*, and those that do not become consistent after either of the two corrections.

expects.[30] To see how our results can imply behavior consistent with conservatism, consider the following example. Suppose that there are 5 ordered states of the world, $\omega_i = i$ for every $i \in \{1, ..., 5\}$. Also suppose that there is a uniform prior on the states, $Pr(\omega_i) = 0.2$ for every $i \in \{1, ..., 5\}$. The subject will receive one signal, $s_i \in \{1, ..., 5\}$. The signal matches the state with probability 0.8, i.e., $Pr(s_i|\omega_j) = 0.8$ if $i = j$. The signal does not match the state with probability 0.2, where there is a uniform on the remaining states, i.e., $Pr(s_i|\omega_j) = 0.05$ if $i \neq j$. Therefore, upon observing the extreme signal s_5, a Bayesian subject will report a posterior of

$$Pr(\omega_5|s_5) = \frac{Pr(s_5|\omega_5)Pr(\omega_5)}{\sum_{i=1}^{5} Pr(s_5|\omega_i)Pr(\omega_i)} = 0.8.$$

Further, the expected value of the state will be

$$EV = \sum_{i=1}^{5} \omega_i * Pr(\omega_i|s_5) = 4.5.$$

However, suppose the subject has beliefs that are not as the experimenter specified. Rather than uniform, consider the case where the subject has prior beliefs that exhibit the central tendency bias, $Pr(\omega_1) = 0.15$, $Pr(\omega_2) = 0.2$, $Pr(\omega_3) = 0.3$, $Pr(\omega_4) = 0.2$, and $Pr(\omega_5) = 0.15$. Upon observing the extreme signal s_5, the subject will report a posterior of $Pr(\omega_5|s_5) = 0.738$ and an expected value of $EV = 4.38$. Since the reported expected value is insufficiently sensitive to the extreme signal, the experimenter will conclude that the subject exhibits behavior consistent with conservatism.

Another bias, *overprecision* (or Bayesian overconfidence), describes a situation where the variance of posterior beliefs is less than that justified by the acquired information. For instance, there is evidence that subjects facing a uniform distribution of events tend to hold updated beliefs that, even though unbiased on average, are characterized by a lower variance than what Bayesian updating implies (Harrison and Swarthout, 2019). Rather than a feature of the updating process, however, overprecision could be rooted in (or at least affected by) the *ex ante* beliefs, as long as they do not reflect the assumed underlying distribution but they are instead characterized by a central tendency bias as shown by our results.

Consider the same example above, but this time receiving the signal s_3. The Bayesian posterior belief starting from a uniform prior should be:

[30]See He and Xiao (2017) for a related model. Also, Benjamin, Rabin, and Raymond (2016) and Kovach (2015) propose other models of conservatism.

$$Pr(\omega_1|s_3) = 0.05; Pr(\omega_2|s_3) = 0.05; Pr(\omega_3|s_3) = 0.8; Pr(\omega_4|s_3) = 0.05; Pr(\omega_5|s_3) = 0.05.$$

When the agent holds prior beliefs that exhibit the central tendency bias as in the example above, the posterior beliefs are instead:

$$Pr(\omega_1|s_3) \simeq 0.027; Pr(\omega_2|s_3) \simeq 0.036; Pr(\omega_3|s_3) \simeq 0.873; Pr(\omega_4|s_3) \simeq 0.036; Pr(\omega_5|s_3) \simeq 0.027.$$

While unbiased on average ($EV = 3$), posterior beliefs are characterized by overprecision. In fact, the distribution is more concentrated than the Bayesian posterior beliefs given a correct prior (the variance is equal to $\sigma^2 \simeq 0.29$ instead of $\sigma^2 = 0.50$.)

Thus the central tendency bias in prior beliefs, as found in our experiment, may offer an explanation for conservatism and overprecision in settings where there are more than 2 ordered states of the world. In other words, another explanation for conservatism and overprecision, rather than subjects not believing or not fully exploiting the information received, is that subjects have prior beliefs that are more concentrated in the middle of the ordered state space than the objective distribution.

6 Discussion

In our experiment, we describe the distribution of bids of a random-bid auction opponent as being uniform. We subsequently elicit beliefs of this distribution. To our surprise, we find evidence that the reported beliefs are largely non-uniform and that many tend to have a central single peak.

The judgments literature has found such a central tendency bias in other settings. Our results suggest that the central tendency bias is more general than the collection of one-at-a-time judgments. Rather, the central tendency bias can also be observed when subjects report their beliefs of a probability distribution that is known to be uniform. Even more strikingly, this happens despite the subjects demonstrating the ability to compute probabilities of events drawn from the uniform distribution. Moreover, many of the central tendency bias experiments are not incentivized, whereas we find evidence of this bias in an incentivized belief elicitation procedure.

We show that the central tendency bias in not an artefact of our experimental design (via incentives inherent in the QSR or multiple-stage payoff hedging). Rather, there is some indicative evidence that the reported beliefs exhibiting a central tendency bias capture, or at least approximate, truly held beliefs for most of the subjects. This evidence draws upon consistency of reported beliefs with another belief report and with the auction bid. If confirmed by future research, such finding would imply that it is not easy to control beliefs in laboratory.

The central tendency bias offers an explanation for well-known biases in Bayesian updating, such as conservatism and overprecision. In studies of biases in Bayesian updating, it is usually assumed that the subjects correctly internalize the induced prior distribution. Our results suggest that this is not necessarily the case. Conservatism and overprecision can also be rationalized by subjects holding prior beliefs that are more concentrated in the middle of the ordered state space than the objective distribution. The central tendency bias also suggests that probabilistic forecasts (rather than point forecasts) of events might have less variance than the actual events. While there is mixed evidence of this in macroeconomic forecasts (Smyth and Ash, 1981; Stekler, 1975),[31] we are interested to learn whether this implication of our results can be found in studies of predictions of uncertain events.

Although we observe the central tendency bias in seven different experimental settings, we are interested to learn the extent to which our results are robust to different stochastic distributions, different elicitation specifications (for instance, bins that do not have identical sizes), and other experimental details. We hope that future experimental work can shed light on the extent to which our results are robust.

References

Achtziger, Anja, and Alós-Ferrer, Carlos (2013): "Fast or rational? A response-times study of Bayesian updating," *Management Science*, 60(4), 923–938.

Akaike, Hirotugu (1974): "A new look at the statistical model identification," *IEEE Transactions on Automatic Control*, 19(6), 716–723.

Algom, Daniel and Marks, Lawrence E. (1990): "Range and regression, loudness scales, and loudness processing: Toward a context-bound psychophysics," *Journal of Experimental Psychology: Human Perception and Performance*, 16(4), 706–727.

[31] Engelberg, Manski, and Williams (2009) also examine the probabilistic forecasts of experts but do not compare the variances of the predictions with the actual variances.

Allen, Franklin (1987): "Discovering personal probabilities when utility functions are unknown," *Management Science*, 33(4), 542–544.

Andersen, Steffen, Fountain, John, Harrison, Glenn W., and Rutström, E. Elisabet (2014): "Estimating subjective probabilities," *Journal of Risk and Uncertainty*, 48(3), 207–229.

Arad, Ayala and Penczynski, Stefan (2018): "Multi–Dimensional Reasoning in Competitive Resource Allocation Games: Evidence from Intra–Team Communication," working paper Tel Aviv University.

Arad, Ayala and Rubinstein, Ariel (2012): "Multi-dimensional iterative reasoning in action: The case of the Colonel Blotto game," *Journal of Economic Behavior and Organization*, 84(2), 571–585.

Armantier, Olivier and Treich, Nicolas (2013): "Eliciting beliefs: Proper scoring rules, incentives, stakes and hedging," *European Economic Review*, 62, 17–40.

Bar-Hillel, Maya (1980): "The base-rate fallacy in probability judgments," *Acta Psychologica*, 44(3), 211–233.

Beach, Lee (1968): "Probability magnitudes and conservative revision of subjective probabilities," *Journal of Experimental Psychology*, 77(1), 57–63.

Benjamin, Daniel J., Rabin, Matthew, and Raymond, Collin (2016): "A model of nonbelief in the law of large numbers," *Journal of the European Economic Association*, 14(2), 515–544.

Blanco, Mariana, Engelmann, Dirk, Koch, Alexander K., and Normann, Hans-Theo (2010): "Belief elicitation in experiments: Is there a hedging problem?" *Experimental Economics*, 13(4), 412–438.

Brier, Glenn W. (1950): "Verification of forecasts expressed in terms of probability," *Monthly Weather Review*, 78(1), 1–3.

Camerer, Colin and Weigelt, Keith (1988): "Experimental tests of a sequential equilibrium reputation model," *Econometrica*, 56(1), 1–36.

Cassey, Peter, Hawkins, Guy E., Donkin, Chris and Brown, Scott D. (2016): "Using alien coins to test whether simple inference is Bayesian," *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 42(3), 497–503.

Charness, Gary, Karni, Edi, and Levin, Dan (2007): "Individual and group decision making under risk: An experimental study of Bayesian updating and violations of first-order stochastic dominance," *Journal of Risk and Uncertainty*, 35(2), 129–148.

Charness, Gary, Karni, Edi, and Levin, Dan (2010): "On the conjunction fallacy in probability judgment: New experimental evidence regarding Linda," *Games and Economic Behavior*, 68(2), 551–556.

Chernev, Alexander (2004): "Extremeness aversion and attribute-balance effects in choice," *Journal of Consumer Research*, 31(2), 249–263.

Corner, Adam, Harris, Adam, and Hahn, Ulrike (2010): "Conservatism in belief revision and participant skepticism," *Proceedings of the Annual Meeting of the Cognitive Science Society*, 32, 1625–1630.

Crawford, L. Elizabeth (2019): "Reply to Duffy and Smith's (2018) reexamination," *Psychonomic Bulletin and Review*, 26(2), 693–698.

Crosetto, Paolo, and Filippin, Antonio (2013): "The 'bomb' risk elicitation task," *Journal of Risk and Uncertainty*, 47(1), 31–65.

De Swart, J. H. (1972): "Effects of diagnosticity and prior odds on conservatism in a bookbag-and-pokerchip situation," *Acta Psychologica*, 36(1), 16–31.

Duffy, Sean, Gussman, Steven, and Smith, John (2019): "Judgments of length in the economics laboratory: Are there brains in choice?" Working paper, Rutgers University-Camden.

Duffy, Sean, Huttenlocher, Janellen, Hedges, Larry V., and Crawford, L. Elizabeth (2010): "Category effects on stimulus estimation: Shifting and skewed frequency distributions," *Psychonomic Bulletin and Review*, 17(2), 224–230.

Duffy, Sean and Smith, John (2018): "Category effects on stimulus estimation: Shifting and skewed frequency distributions-A reexamination," *Psychonomic Bulletin and Review*, 25(5), 1740–1750.

Duffy, Sean and Smith, John (2019a): "On the Category Adjustment Model: Another look at Huttenlocher, Hedges, and Vevea (2000)," Working paper, Rutgers University-Camden.

Duffy, Sean and Smith, John (2019b): ”Omitted-variable bias and other matters in the defense of the category adjustment model: A reply to Crawford (2019),” Working paper, Rutgers University-Camden.

El-Gamal, Mahmoud A. and Grether, David M. (1995): ”Are people Bayesian? Uncovering behavioral strategies,” *Journal of the American Statistical Association*, 90(432), 1137–1145.

Engelberg, Joseph, Manski, Charles F., and Williams, Jared (2009): ”Comparing the point predictions and subjective probability distributions of professional forecasters,” *Journal of Business and Economic Statistics*, 27(1), 30–41.

Erev, Ido, Wallsten, Thomas S. and Budescu, David V. (1994): ”Simultaneous over-and underconfidence: The role of error in judgment processes,” *Psychological Review*, 101(3), 519–527.

Gigerenzer, Gerd and Hoffrage, Ulrich (1995): ”How to improve Bayesian reasoning without instruction: Frequency formats,” *Psychological Review*, 102(4), 684–704.

Grether, David M. (1980): ”Bayes rule as a descriptive model: The representativeness heuristic,” *Quarterly Journal of Economics*, 95(3), 537–557.

Grether, David M. (1992): ”Testing Bayes rule and the representativeness heuristic: Some experimental evidence,” *Journal of Economic Behavior and Organization*, 17(1), 31–57.

Griffin, Dale and Tversky, Amos (1992): ”The weighing of evidence and the determinants of confidence,” *Cognitive Psychology*, 24(3), 411–435.

Harrison, Glenn W., Martínez-Correa, Jimmy, Swarthout, J. Todd and Ulm, Eric R. (2017): ”Scoring rules for subjective probability distributions,” *Journal of Economic Behavior and Organization*, 134, 430–448.

Harrison, Glenn W. and Swarthout, J. Todd (2019): ”Belief Distributions, Overconfidence and Bayes Rule” *mimeo, Georgia State University*.

Hawkins, Guy E., Hayes, Brett K., Donkin, Chris, Pasqualino, Martina and Newell, Ben R. (2015): ”A Bayesian latent-mixture model analysis shows that informative samples reduce base-rate neglect,” *Decision*, 2(4), 306–318.

He, Xue Dong and Xiao, Di (2017): ”Processing consistency in non-Bayesian inference,” *Journal of Mathematical Economics*, 70, 90–104.

Hollingworth, Harry Levi (1910): ”The central tendency of judgment,” *Journal of Philosophy, Psychology and Scientific Methods*, 7(17), 461–469.

Hossain, Tanjim and Okui, Ryo (2013): ”The binarized scoring rule,” *Review of Economic Studies*, 80(3), 984–1001.

Huttenlocher, Janellen, Hedges, Larry V., and Duncan, Susan (1991): ”Categories and particulars: Prototype effects in estimating spatial location,” *Psychological Review*, 98(3), 352–376.

Huttenlocher, Janellen, Hedges, Larry V., and Vevea, Jack L. (2000): ”Why do categories affect stimulus judgment?” *Journal of Experimental Psychology: General*, 129, 220–241.

Jazayeri, Mehrdad and Shadlen, Michael N. (2010): ”Temporal context calibrates interval timing,” *Nature Neuroscience*, 13(8), 1020–1026.

Jones, Lynette A. and Hunter, Ian W. (1982): ”Force sensation in isometric contractions: a relative force effect,” *Brain Research*, 244(1), 186–189.

Jou, Jerwen, Leka, Gary E., Rogers, Dawn M., and Matus, Yolanda E. (2004): ”Contraction bias in memorial quantifying judgment: Does it come from a stable compressed memory representation or a dynamic adaptation process?” *American Journal of Psychology*, 117 (4), 543–564.

Kahneman, Daniel and Frederick, Shane (2002): ”Representativeness revisited: Attribute substitution in intuitive judgment,” In Gilovich, Thomas, Griffin, Dale, and Kahneman, Daniel (Eds.), *Heuristics and biases: The Psychology of Intuitive Judgment*, Cambridge University Press, 49–81.

Kahneman, Daniel and Tversky, Amos (1973): ”On the psychology of prediction,” *Psychological Review*, 80(4), 237–251.

Kahneman, Daniel and Tversky, Amos (1979): ”Prospect Theory: An Analysis of Decision under Risk,” *Econometrica*, 47 (2), 263–292.

Karni, Edi (2009): ”A mechanism for eliciting probabilities,” *Econometrica*, 77(2), 603–606.

Kareev, Yaakov, Arnon, Sharon, and Horwitz-Zeliger, Reut (2002): ”On the misperception of variability,” *Journal of Experimental Psychology: General*, 131(2), 287–297.

Kovach, Matthew (2015): ”Sticky Beliefs: A Characterization of Conservative Updating,” Working paper Virginia Tech.

Manski, Charles F. (2004): ”Measuring expectations,” *Econometrica*, 72(5), 1329–1376.

Marchiori, Davide, Di Guida, Sibilla, and Erev, Ido (2015): ”Noisy retrieval models of over-and undersensitivity to rare events,” *Decision*, 2(2), 82–106.

Marks, D. F. and Clarkson, J. K. (1972): ”An explanation of conservatism in the bookbag-and-pokerchips situation,” *Acta Psychologica*, 36(2), 145–160.

Matheson, James E. and Winkler, Robert L. (1976): ”Scoring rules for continuous probability distributions,” *Management Science*, 22(10), 1087–1096.

Neumann, Nico, Böckenholt, Ulf, and Sinha, Ashish (2016): ”A meta-analysis of extremeness aversion,” *Journal of Consumer Psychology*, 26(2), 193–212.

Oechssler, Jörg, Roider, Andreas and Schmitz, Patrick W. (2009): ”Cognitive abilities and behavioral biases,” *Journal of Economic Behavior and Organization*, 72(1), 147–152.

Offerman, Theo, Sonnemans, Joep, Van de Kuilen, Gijs, and Wakker, Peter P. (2009): ”A truth serum for non-bayesians: Correcting proper scoring rules for risk attitudes,” *Review of Economic Studies*, 76(4), 1461–1489.

Phillips, Lawrence D. and Edwards, Ward (1966): ”Conservatism in a simple probability inference task,” *Journal of Experimental Psychology*, 72(3), 346–354.

Poulton, E. Christopher (1979): ”Models for biases in judging sensory magnitude,” *Psychological Bulletin*, 86(4), 777–803.

Qiu, Jianying and Weitzel, Utz (2016): ”Experimental evidence on valuation with multiple priors,” *Journal of Risk and Uncertainty*, 53(1), 55–74.

Radvansky, Gabriel A., Carlson-Radvansky, Laura A., and Irwin, David E. (1995): "Uncertainty in estimating distances from memory," *Memory and Cognition*, 23(5), 596–606.

Savage, Leonard J. (1971): "Elicitation of personal probabilities and expectations," *Journal of the American Statistical Association*, 66(336), 783–801.

Schlag, Karl H., Tremewan, James, and Van der Weele, Joël J. (2015): "A penny for your thoughts: a survey of methods for eliciting beliefs," *Experimental Economics*, 18(3), 457–490.

Schlag, Karl H. and van der Weele, Joël (2013): "Eliciting probabilities, means, medians, variances and covariances without assuming risk neutrality," *Theoretical Economics Letters*, 3, 38–42.

Schlag, Karl H. and van der Weele, Joël J. (2015): "A method to elicit beliefs as most likely intervals," *Judgment and Decision Making*, 10(5), 456–468.

Schotter, Andrew and Trevino, Isabel (2014): "Belief elicitation in the laboratory," *Annual Review of Economics*, 6(1), 103–128.

Sedlmeier, Peter and Gigerenzer, Gerd (2001): "Teaching Bayesian reasoning in less than two hours," *Journal of Experimental Psychology: General*, 130(3), 380–400.

Simonson, Itamar and Tversky, Amos (1992): "Choice in context: Tradeoff contrast and extremeness aversion," *Journal of Marketing Research*, 29(3), 281–295.

Smyth, David J. and Ash, J. C. K. (1981): "The underestimation of forecasts and the variability of predictions and outcomes," *Bulletin of Economic Research*, 33(1), 37–44.

Stekler, Herman O. (1975): "Why do forecasters underestimate?" *Economic Inquiry*, 13(3), 445–449.

Stevens, S. S. and Greenbaum, Hilda B. (1966): "Regression effect in psychophysical judgment," *Perception and Psychophysics*, 1(5), 439–446.

Weber, Elke U. (1994): "From subjective probabilities to decision weights: The effect of asymmetric loss functions on the evaluation of uncertain outcomes and events," *Psychological Bulletin*, 115(2), 228–242.

Winkler, Robert L. and Murphy, Allan H. (1970): ”Nonlinear utility and the probability score.” *Journal of Applied Meteorology*, 9(1), 143–148.

Zizzo, Daniel John, Stolarz-Fantino, Stephanie, Wen, Julie, and Fantino, Edmund (2000): ”A violation of the monotonicity axiom: Experimental evidence on the conjunction fallacy,” *Journal of Economic Behavior and Organization*, 41(3), 263–276.

A Proofs

Proof of Proposition 1. The decision-maker solves

$$\max_{r_1,..,r_n \in [0,1]} \sum_{i=1}^{n} p_i u \left(\alpha - \beta + 2\beta r_i - \beta \sum_{j=1}^{n} r_j^2 \right) \text{ s.t. } r_1 + ... + r_n = 1.$$

At the beginning, we are going to ignore the constraint $r_1 + ... + r_n = 1$ and focus on the resulting "unconstrained" problem subject only to the usual probability bounds of 0 and 1. Note that π_i is strictly increasing and concave in r_i on $[0,1]$ with $\partial\pi_i/\partial r_i|_{r_i=1} = 0$, whereas π_j, $j \neq i$, is strictly decreasing and concave in r_i on $[0,1]$ with $\partial\pi_j/\partial r_i|_{r_i=0} = 0$. This implies that, starting from $r_i = 0$, a small increase in r_i has a positive first-order effect on π_i without any counterbalancing negative first order effect on π_j, $j \neq i$. Since $u'(\cdot)$ is bounded away from 0 and ∞, this implies that $r_i^* > 0$ if $p_i > 0$. Likewise, starting from $\hat{p}_i = 1$, a small decrease in r_i has a positive first-order effect on π_j, $j \neq i$, without any counterbalancing first order effect on π_i, implying that $r_i^* < 1$ if $p_i < 1$. Moreover, if $p_i = 0$, it is trivial to see that $r_i^* = 0$ even without considering the lower probability bound of 0, and if $p_i = 1$, it is trivial to see that $r_i^* = 1$ even without considering the upper probability bound of 1. This series of observations jointly implies the equivalent of part 1 of the Proposition for the unconstrained problem. It also implies that the probability bounds of 0 and 1 are never binding in this problem and can therefore be ignored.

For the equivalent of part 2 of the Proposition for the unconstrained problem, suppose that $p_i > p_j$ and, by contradiction, $r_i^* \leq r_j^*$. First, suppose that $r_i^* = r_j^*$. By part 1 of the Proposition, it then must be the case that $0 < r_i^* = r_j^* < 1$. Now, starting from this point, consider a small increase in r_i and an exactly offsetting small decrease in r_j. This change has a positive first-order effect on π_i, an offsetting negative first-order effect on π_j of the same absolute size and no first-order effect on π_k for $k \neq i, j$. Since $p_i > p_j$ and $u'(\cdot)$ is bounded away from 0 and ∞, such perturbation increases EU, contradicting optimality of r^*. Second, suppose that $r_i^* < r_j^*$, implying that $\pi_i^* < \pi_j^*$. Now consider resetting r_i and r_j such that $r_i = r_j^*$ and $r_j = r_i^*$. This change does not affect π_k for $k \neq i, j$. As a result, the EU changes by

$$[p_i u(\pi_j^*) + p_j u(\pi_i^*)] - [p_i u(\pi_i^*) + p_j u(\pi_j^*)] = (p_i - p_j)[u(\pi_j^*) - u(\pi_i^*)] > 0.$$

But this means that such a change increases EU, contradicting optimality of r^*. Hence if $p_i > p_j$, it must be the case that $r_i^* > r_j^*$.

Since the probability bound constraints can be ignored as argued above, any solution $(r_1^*, .., r_n^*)$ to the unconstrained problem must satisfy the usual first-order necessary conditions

$$2\beta p_k u'(\pi_k^*) - 2\beta r_k^* \sum_{j=1}^{n} p_j u'(\pi_j^*) = 0, \quad k = 1, .., n, \tag{1}$$

where π_i^* is the resulting payoff if state i is realized. Summing these n conditions gives

$$2\beta \sum_{k=1}^{n} p_k u'(\pi_k^*) - 2\beta \left(\sum_{k=1}^{n} r_k^* \right) \sum_{j=1}^{n} p_j u'(\pi_j^*) = 0,$$

implying that $r_1^* + ... + r_n^* = 1$. As a result, any solution to the unconstrained problem also solves the constrained problem. This observation hence proves the first two parts of the proposition also for the constrained problem.

For the part 3 of the Proposition, if $p_i = p_j = 0$, then $r_i^* = r_j^* = 0$ is implied by part 1 of the Proposition. Now suppose that $p_i = p_j \in (0, 0.5]$. Then part 1 of the Proposition implies that $r_i^*, r_j^* \in (0, 1)$. By (1) it then must be the case that

$$\frac{r_i^*}{r_j^*} = \frac{u'(\pi_i^*)}{u'(\pi_j^*)}. \tag{2}$$

Clearly, if $r_i^* = r_j^*$, then $\pi_i^* = \pi_j^*$, and (2) is satisfied. Next, we are going to show that (2) cannot be satisfied for $r_i^* \neq r_j^*$. Suppose, by contradiction, that there exists r^* with, without loss of generality, $r_i^* > r_j^*$ and $r_i^*, r_j^* \in (0, 1)$ such that (2) is satisfied. Let $\bar{r} \equiv (r_i^* + r_j^*)/2$. Now consider a different report profile r' which differs from r^* by both r_i^* and r_j^* being replaced by $\bar{r}$, while the other reported probabilities are left unchanged. Now, starting from r', gradually increase r_i, by the same amount gradually decrease r_j, and keep all the other reports fixed until r^* is reached. At any point along this trajectory, we have that

$$\begin{aligned} d\left(\ln \frac{r_i}{r_j}\right) &= d \ln r_i - d \ln r_j \\ &= \frac{dr_i}{r_i} + \frac{-dr_j}{r_j} \\ &= \frac{dr_i}{r_i} + \frac{dr_i}{r_j} \\ &= dr_i \left(\frac{1}{r_i} + \frac{1}{r_j} \right) \\ &\geq 4dr_i. \end{aligned}$$

The last inequality follows from the constraint $r_i + r_j \leq 1$. Equality potentially applies only at r', otherwise a strict inequality applies. At any point along this trajectory, we also have that

$$d\left[\ln \frac{u'(\pi_i)}{u'(\pi_j)}\right] = d \ln u'(\pi_i) - d \ln u'(\pi_j)$$
$$= \frac{u''(\pi_i)}{u'(\pi_i)} d\pi_i + \frac{u''(\pi_j)}{u'(\pi_j)}(-d\pi_j). \tag{3}$$

Now note that

$$\begin{aligned} d\pi_i &= 2\beta dr_i - 2\beta(r_i dr_i + r_j dr_j) \\ &= 2\beta dr_i - 2\beta(r_i - r_j)dr_i \\ &= 2\beta(1 - r_i + r_j)dr_i \end{aligned}$$

and

$$\begin{aligned} -d\pi_j &= 2\beta(-dr_j) + 2\beta(r_i dr_i + r_j dr_j) \\ &= 2\beta dr_i + 2\beta(r_i - r_j)dr_i \\ &= 2\beta(1 + r_i - r_j)dr_i, \end{aligned}$$

implying that $d\pi_i > 0$, $-d\pi_j > 0$ and

$$d\pi_i - d\pi_j = 4\beta dr_i.$$

These results and the assumption of part 3 of the Proposition imply that in (3) we have that

$$\begin{aligned} d\left[\ln \frac{u'(\pi_i)}{u'(\pi_j)}\right] &= \frac{u''(\pi_i)}{u'(\pi_i)} d\pi_i + \frac{u''(\pi_j)}{u'(\pi_j)}(-d\pi_j) \\ &\leq \beta^{-1}(d\pi_i - d\pi_j) \\ &= 4dr_i. \end{aligned}$$

Overall, we therefore have at any point along the trajectory from r' to r^* that

$$d\left[\ln \frac{r_i}{r_j} - \ln \frac{u'(\pi_i)}{u'(\pi_j)}\right] \geq 0,$$

with equality potentially applying only at r' and a strict inequality applying otherwise. But this result and (2) then imply that

$$\frac{\bar{r}}{\bar{r}} < \frac{u'(\bar{\pi})}{u'(\bar{\pi})},$$

where

$$\bar{\pi} \equiv \alpha - \beta + 2\beta\bar{r} - \beta\left(\sum_{k\neq i,j} r_k^{*2} + 2\bar{r}^2\right),$$

a clear contradiction. Therefore if $p_i = p_j \in (0, 0.5]$, then, under the assumption of part 3, it must be the case that $r_i^* = r_j^*$.

For the part 4 of the Proposition, if $p_i > p_j > 0$, it follows from parts 1 and 2 of the Proposition that $r_i^* > r_j^* > 0$. If $u(\cdot)$ is linear, then the result that $r_i^* = p_i$ and $r_j^* = p_j$ follows directly from (1) for events i and j. If $u(\cdot)$ is non-linear, then (1) implies that

$$\frac{r_i^*}{r_j^*} = \frac{p_i}{p_j}\frac{u'(\pi_i^*)}{u'(\pi_j^*)}.$$

Recall that π_i (π_j) is strictly increasing in r_i (r_j). As a result, $\pi_i^* > \pi_j^*$. Hence if $u(\cdot)$ is strictly concave, $u'(\pi_i^*)/u'(\pi_j^*) < 1$, implying that $r_i^*/r_j^* < p_i/p_j$, whereas if $u(\cdot)$ is strictly convex, $u'(\pi_i^*)/u'(\pi_j^*) > 1$, implying that $r_i^*/r_j^* > p_i/p_j$. ■

Proof of Proposition 2. The decision-maker solves

$$\begin{aligned}\max_{r_1,\ldots,r_5\in[0,1]} \sum_{i<k} 0.2u\left(s + 10 + 20r_i - 10\sum_{j=1}^{5} r_j^2\right) + 0.2qu\left(s + 10 + 20r_k - 10\sum_{j=1}^{5} r_j^2\right)\\ + 0.2(1-q)u\left(10 + 20r_k - 10\sum_{j=1}^{5} r_j^2\right) + \sum_{i>k} 0.2u\left(10 + 20r_i - 10\sum_{j=1}^{5} r_j^2\right)\\ \text{s.t. } r_1 + \cdots + r_5 = 1.\end{aligned}$$

Following the same steps as in the proof of part 1 of Proposition 1, it follows that the probability bounds are never binding in this problem and can therefore be ignored. Hence any solution $(r_1^*, \ldots, r_5^*)$ to the unconstrained problem must satisfy the usual first-order necessary conditions which, after canceling out a common multiplier 0.2×20, are given by

$$u'(\pi_i^*) - r_i^* \left[\sum_{j \neq k} u'(\pi_j^*) + qu'(\pi_{kH}^*) + (1-q)u'(\pi_{kL}^*)\right] = 0, \quad i \neq k,$$
$$[qu'(\pi_{kH}^*) + (1-q)u'(\pi_{kL}^*)] - r_k^* \left[\sum_{j \neq k} u'(\pi_j^*) + qu'(\pi_{kH}^*) + (1-q)u'(\pi_{kL}^*)\right] = 0,$$

where the asterisk denotes the resulting payoff if the corresponding state is realized. Summing these 5 conditions gives

$$\left[\sum_{j \neq k} u'(\pi_j^*) + qu'(\pi_{kH}^*) + (1-q)u'(\pi_{kL}^*)\right] - \left(\sum_{j=1}^{5} r_j^*\right) \left[\sum_{j \neq k} u'(\pi_j^*) + qu'(\pi_{kH}^*) + (1-q)u'(\pi_{kL}^*)\right] = 0,$$

implying that $r_1^* + ... + r_5^* = 1$. As a result, any solution to the unconstrained problem also solves the constrained problem. The first-order conditions also imply that

$$\frac{r_i^*}{u'(\pi_i^*)} = \frac{r_k^*}{qu'(\pi_{kH}^*) + (1-q)u'(\pi_{kL}^*)}, \quad i \neq k. \tag{4}$$

Since π_j is strictly increasing in r_j and $u(\cdot)$ is strictly concave, it follows that: (1) if $k > 1$ and $q < 1$, then $r_i^* = r_L^*$ for any $i < k$ for some strictly positive (part 1 of Proposition 1) r_L^* and $r_L^* < r_k^*$; (2) if $k > 1$ and $q = 1$, then $r_i^* = r_L^*$ for any $i \leq k$ for some strictly positive r_L^*; (3) if $k < 5$, then $r_i^* = r_H^*$ for any $i > k$ for some strictly positive r_H^* and $r_H^* > r_k^*$. The fact that $r_1^* + ... + r_5^* = 1$ then implies that: (1) if $k > 1$, then $r_L^* < 0.2$, unless $k = 5$ and $q = 1$, in which case $r_L^* = r_k^* = 0.2$; (2) if $k < 5$, then $r_H^* > 0.2$. ■

B Supplemental analysis

We now explore the extent of the general deviation from the optimal uniform responses. We define a response to be *non-uniform* if a distribution of weights other than $(20, 20, 20, 20, 20)$ was given. We also perform χ^2 goodness-of-fit tests for a uniform distribution for each subject. We report the distribution of p-values below. We summarize this analysis in Table B1.

Despite that there are incentives for correctly responding with uniform weights, a majority of subjects offered responses that are significantly different from uniform. More than 70% of subjects gave a response other than a uniform and more than 55% with a response that is significantly different from a uniform at

Table B1: Non-uniform responses and corresponding results of χ^2 tests of uniformity

Treatment	Non-Uniform	$p < 0.05$	$p < 0.01$	$p < 0.001$	Subjects
Focal 60	39 (75.0%)	33 (63.4%)	30 (57.7%)	26 (50.0%)	52
Sym 60	41 (69.5%)	37 (62.7%)	37 (62.7%)	34 (57.6%)	59
Asym 60	40 (72.7%)	36 (65.5%)	34 (61.8%)	33 (60.0%)	55
Asym 80	25 (78.1%)	24 (75.0%)	23 (71.9%)	21 (65.6%)	32
Auct 100 w/o	40 (67.8%)	39 (66.1%)	38 (64.4%)	35 (59.3%)	59
Auct 100 w	46 (76.7%)	42 (70.0%)	39 (65.0%)	34 (56.7%)	60
BRET w	44 (71.0%)	38 (61.3%)	34 (54.8%)	28 (45.2%)	62
Pooled	275 (72.6%)	249 (65.7%)	234 (62.0%)	211 (55.7%)	379

Notes: We list the number (and percent) of subjects who report non-uniform responses. We also list the number of subjects who, according to a χ^2 test of goodness-of-fit have p-values such that their responses are significantly different than a uniform.

0.001.

In the analyses summarized in Tables 3 and 4, we found evidence that beliefs of the random-draw opponent's strategy was single peaked. In order to further investigate the nature of the central single peak, we employ another test of whether responses exhibit the central single peak. Here we perform regressions with weight as the dependent variable and the bin as the independent variable. To accomplish this, we define the Bin code variable as follows: Bin code for Bin 1 is $-\frac{4}{5}$, Bin 2 is $\frac{1}{5}$, Bin 3 is $\frac{6}{5}$, Bin 4 is $\frac{1}{5}$, and Bin 5 is $-\frac{4}{5}$. These codes obviously have a central single peak. Further, these codes sum to zero and will therefore not affect the intercept estimate. We summarize these regressions in Table B2

Table B2: Regressions with weight as dependent variable

	(1)	(2)
Intercept	20.000^{***}	20.000^{***}
	(0.303)	(0.278)
Bin code	–	6.999^{***}
		(0.371)
AIC	15152.8	14827.2

Notes: Notes: We provide the coefficient estimates with the standard errors in parentheses. Both regressions have 1895 observations. AIC refers to the Akaike information criterion (Akaike, 1974). *** denotes $p < 0.001$.

As expected, we estimate the intercept to be 20 in both regressions. In regression (2) we see that the Bin code variable is significant at 0.001. We interpret this as corroborating the evidence found in Table 4 of a central single peak.

C Experimental instructions

C.1 General instructions

Initial screen:

You are about to participate in an experiment in which following the instructions carefully, making good decisions, and with a bit of luck, you can earn a considerable amount of money. Different participants may earn different amounts according to their choices. For your participation in the experiment you will earn an additional show-up fee of 2.5 Euro.

All the monetary values during the experiment are expressed in ECU (Experimental Currency Units). At the end of the experiment, the ECUs you earned will be converted into a cash payoff in Euro using the exchange rate 1 ECU = 20 euro cents and paid in cash privately.

New screen:

The experiment consists of 4 stages in the following order:

1. An Instruction Stage that we are currently going through. At the end of this stage you will be asked some control questions to verify your understanding of the task. After everybody answers correctly, we will proceed with the following stage.

2. A Decision Stage, in which you will make decisions and answer questions relevant towards your payoff.

3. A Demographic Questionnaire, in which you will be asked a few questions about your demographic and academic background.

4. A Feedback Stage, in which your earnings from the experiment will be determined and announced to you privately. You will not be given any feedback on the monetary outcome of your decisions before the Feedback Stage.

New screen:

You are about to participate in an auction against a hypothetical opponent. In this auction, there is a fictitious object for sale that you value at (60/80/100) ECUs. That is, if you win the auction, you obtain the object and receive (60/80/100) ECUs, but from this amount you have to subtract the price you will have to pay for the object.

Your task in this auction is to place a bid. This bid can be any integer number from 1 through (60/100) ECUs. A bid is a binding price offer to pay for the object in case you win the auction. Your opponent also places a bid that will be randomly drawn from the set of integers 1 through (60/100) ECUs. Each of these integers is equally likely to be drawn.

The winner of the auction is the bidder who places the highest bid. In case the two bids coincide, you are the winner. That is, you win if your bid is at least as high as your opponent's bid, otherwise you do not win. The winner pays the price for the object equal to his/her bid. That is, if you win, you receive (60/80/100) ECUs minus your bid. If you do not win, you do not receive anything and you do not pay anything either.

New screen:

Please answer the following control questions. Answers to these questions are not relevant to your earnings. The computer will give you a feedback on whether your responses are correct or not. If you have any problems in answering, please raise your hand and an experimenter will come to assist you. After everyone answers correctly all the questions, we will proceed with the decision stage.

1. Suppose that your opponent bids 25 ECUs.

    a. If you bid 21 ECUs, how much will you earn in ECU?

    b. If you bid 38 ECUs, how much will you earn in ECU?

    c. If you bid 62 ECUs, how much will you earn in ECU?

    d. If you bid 79 ECUs, how much will you earn in ECU?

2. Now suppose that your opponent bids 75 ECUs.

    a. If you bid 21 ECUs, how much will you earn in ECU?

    b. If you bid 38 ECUs, how much will you earn in ECU?

    c. If you bid 62 ECUs, how much will you earn in ECU?

    d. If you bid 79 ECUs, how much will you earn in ECU?

3. What is the probability of your opponent's bid being in the range:

a. 39 through 72

b. 22 though 47

c. 1 through 10

d. 16 through 56

e. 62 through 100

4. Your opponent's bid depends on your bid. YES/NO

5. Suppose the bid of your opponent is identical to your bid. Will you earn a positive amount? YES/NO

Please note that the numbers used in these questions are for illustrative purposes only. They are not meant to be a guidance for your choice.

Question 1 stage:

New screen:

Before drawing your opponent's bid, we ask you to answer two short questions that allow you to earn some additional money. You will be paid for one of these two questions. At the end of the experiment, one of the participants will flip a coin to decide which question will be paid. You can earn up to 20 ECUs for the selected question.

Question 1. Please report your belief of your opponent's bid. We will provide five intervals. You are asked to report how likely you think your opponent's bid is to be in each of these intervals. The number in each input field you are asked to fill in is your percentage estimate of the likelihood of your opponent's bid being in that particular interval. The five percentages need to add up to 100. There will be an automatic checker to tell you what the current sum is as you enter the numbers.

You will be paid based on how closely your estimates match your opponent's bid. The exact formula (the so-called quadratic scoring rule) is complicated and the experimenters will be happy to explain it after the end of the experiment to those who are interested. However, in order to maximize your expected earnings from this procedure, you should report these likelihoods truthfully according to what you believe.

Here is some advice on how to fill in the five input fields:

- If you believe that your opponent's bid is more likely to be in a certain range, then assign higher percentages to intervals corresponding to that range and lower percentages to intervals corresponding to other ranges.

- If you believe that your opponent's bid is equally likely to be in several different intervals, then assign the same percentages to those intervals.
- Do not over-concentrate your assigned percentages in one or two intervals if you are not quite sure that your opponent's bid is in these intervals. Otherwise, if it turns out that your opponent's bid is in some other interval, you would earn little money from answering the question.
- On the other hand, do concentrate the entire 100 percent in one or two intervals if you feel confident that your opponent's bid is in this (these) interval(s). This will increase your earnings from answering this question.

New screen:

Here we repeat Question 1 for your convenience. Please provide your answer using the input fields below.

Question 1. Please report your belief of your opponent's bid. We will provide five intervals. You are asked to report how likely you think your opponent's bid is to be in each of these intervals. The number in each input field you are asked to fill in is your percentage estimate of the likelihood of your opponent's bid being in that particular interval. The five percentages need to add up to 100. There will be an automatic checker to tell you what the current sum is as you enter the numbers.

Question 2 stage:

New screen:

Question 2. Please report your perceived probability of winning and of not winning the auction given the choice you made in the main task. The number in each box you are asked to fill in is your percentage estimate of the likelihood of that event. The two percentages need to add up to 100. There will be an automatic checker to tell you what the current sum is as you enter the numbers. You will be paid based on how closely your estimates match the outcome of the auction. The exact formula and the suggestions how to increase your payoffs are the same as in Question 1.

C.2 Treatment-specific instructions

C.2.1 Auction 100/100 with visualization treatment

New screen:

To help you visualize your decision, on your display you will see a square composed of 100 boxes numbered 1 through 100. One of these boxes corresponds to your opponent's bid. You do not know which one it is, however. You only know that it can be any of the 100 boxes with equal probability.

You initiate the bidding process by first entering your intended decision into an input field and clicking on ”Evaluate.” At this point, the originally grey boxes change color. The boxes turning blue are those whose number is less than or equal to your bid. The boxes turning yellow are those whose number exceeds your bid. If the opponent's bid corresponds to one of the blue boxes, you will earn the difference between 100 ECUs and your bid. If the opponent's bid corresponds to one of the yellow boxes, your will earn zero. You are free to evaluate different bids in this way.

When you are confident about your choice, submit it by clicking on the ”Submit” button and then reconfirm it by clicking on ”Confirm”.

At the end of the experiment, after answering some questions and filling out a short questionnaire, we will randomly determine the number of the box corresponding to your opponent's bid. This will be done by one of the participants randomly drawing a token from a bag containing 100 tokens numbered 1 through 100.

New screen:

Please choose your bid. Using the input field and the ”Evaluate” button, you are free to evaluate as many different bids as you wish. The boxes turning blue are those whose number is less than or equal to your bid. The boxes turning yellow are those whose number exceeds your bid. If the opponent's bid corresponds to one of the blue boxes, you will earn the difference between 100 ECUs and your bid. If the opponent's bid corresponds to one of the yellow boxes, your will earn zero. When you are ready to submit your final decision, click on the ”Submit” button and then confirm your choice by clicking on ”Confirm”.

C.2.2 Auction 100/100 without visualization treatment

Identical to Auction 100/100 with visualization treatment but without visualization or mentions of visualization.

C.2.3 Bomb Risk Elicitation Task treatment

New screen:

Your task is to decide on the number of boxes to collect out of 100 such boxes numbered 1 through 100. You collect the boxes starting from box number 1, continuing until the box whose number is equal to the number of boxes you decide to collect. Exactly one of these 100 boxes contains a bomb. You do not know the bomb's location. You only know that it is equally likely to be in any of the 100 boxes.

If the number of the box in which the bomb is located is higher than the number of boxes you collected, you do not collect the bomb and you earn 1 ECU for each collected box. If the number of the box in which the bomb is located is lower than or equal to the number of boxes you collected, you do collect the bomb and you earn zero.

New screen:

To help you visualize your decision, on your display you will see a square composed of 100 numbered boxes. There is a bomb in one of these boxes. You do not know which one it is, however. You only know that it can be in any of the 100 boxes with equal probability.

You initiate the decision process by first entering your intended decision into an input field and clicking on ”Evaluate.” At this point, the originally grey boxes change color. The boxes turning yellow are those that you are deciding to collect. The boxes turning blue are those you are deciding to not collect. If the bomb is in one of the yellow boxes, you will earn zero. If the bomb is in one of the blue boxes, you will earn 1 ECU for every (yellow) box collected. You are free to evaluate different numbers of collected boxes in this way.

At the end of the experiment, after answering some questions and filling out a short questionnaire, we will randomly determine the number of the box containing the bomb. This will be done by one of the participants randomly drawing a token from a bag containing 100 tokens numbered 1 through 100.

Instructions from Questions 1 and 2:

Identical to the Auction 100/100 with visualization treatment but ”your opponent's bid” was replaced by ”the position of the bomb” and ”winning and of not winning the auction” was replaced by ”collecting and not collecting the bomb.”

C.2.4 Auction 80/100 treatment

Identical to Auction 100/100 with visualization treatment but the value to the subject was 80.

Added to first screen in Auction 100/100 with visualization treatment:

Note that if you bid more than 80 you make losses in case you win.

Added to second screen in Auction 100/100 with visualization treatment:

In case you bid more than 80 the boxes in excess turn red signaling that you would make losses if you win.

C.2.5 Auction 60/100 treatment

Identical to Auction 80/100 with visualization treatment but the value to the subject was 60 and the warnings were for bids greater than 60.

C.2.6 Auction 60/60 treatment

New screen:

In this treatment both the value of the subject and of the opponent are set to 60.

Subjects bid choosing a number from 0 to 60, and report beliefs of the opponent's bid which is uniformly distributed in the interval 0-60.

New screen:

You are about to participate in an auction against a hypothetical opponent. In this auction, there is a fictitious object for sale that you value at 60 ECUs. That is, if you win the auction, you obtain the object and receive 60 ECUs, but from this amount you have to subtract the price you will have to pay for the object.

Your task in this auction is to place a bid. This bid can be any integer number from 1 through 60 ECUs. A bid is a binding price offer to pay for the object in case you win the auction. Your opponent also places a bid that will be randomly drawn from the set of integers 1 through 60 ECUs. Each of these integers is equally likely to be drawn.

The winner of the auction is the bidder who places the highest bid. In case the two bids coincide, you are the winner. That is, you win if your bid is at least as high as your opponent's bid, otherwise you do not win. The winner pays the price for the object equal to his/her bid. That is, if you win, you receive 60 ECUs minus your bid. If you do not win, you do not receive anything and you do not pay anything either.

New screen:

To help you visualize your decision, on your display you will see a rectangle composed of 60 boxes numbered 1 through 60. One of these boxes corresponds to your opponent's bid. You do not know which one it is, however. You only know that it can be any of the 60 boxes with equal probability.

You initiate the bidding process by first entering your intended decision into an input field and clicking on "Evaluate." At this point, the originally grey boxes change color. The boxes turning blue are those whose number is less than or equal to your bid. The boxes turning yellow are those whose number exceeds your bid. If the opponent's bid corresponds to one of the blue boxes, you will earn the difference between 60 ECUs and your bid. If the opponent's bid corresponds to one of the yellow boxes, you will earn zero. You are free to evaluate different bids in this way.

At the end of the experiment, after answering some questions and filling out a short questionnaire, we will randomly determine the number of the box corresponding to your opponent's bid. This will be done by one of the participants randomly drawing a token from a bag containing 60 tokens numbered 1 through 60.

C.2.7 Auction 60/60 expand treatment

New screen:

You are about to participate in an auction against a hypothetical opponent. In this auction, there is a fictitious object for sale that you value at 60 ECUs. That is, if you win the auction, you obtain the object and receive 60 ECUs, but from this amount you have to subtract the price you will have to pay for the object.

Your task in this auction is to place a bid. This bid can be any integer number from 1 through 60 ECUs. A bid is a binding price offer to pay for the object in case you win the auction. Your opponent also places a bid that will be randomly drawn from the set of integers 1 through 60 ECUs. Each of these integers is equally likely to be drawn.

The winner of the auction is the bidder who places the highest bid. In case the two bids coincide, you are the winner. That is, you win if your bid is at least as high as your opponent's bid, otherwise you do not win. The winner pays the price for the object equal to his/her bid. That is, if you win, you receive 60 ECUs minus your bid. Note that if you bid more than 60 you make losses in case you win. If you do not win, you do not receive anything and you do not pay anything either.

New screen:

To help you visualize your decision, on your display you will see a square composed of 100 boxes numbered 1 through 100. One of these boxes corresponds to your opponent's bid. You do not know which one it is, however. You only know that it can be any of the 60 boxes with equal probability.

You initiate the bidding process by first entering your intended decision into an input field and clicking on ”Evaluate.” At this point, the originally grey boxes change color. The boxes turning blue are those whose number is less than or equal to your bid. In case you bid more than 60 the boxes in excess turn red signaling that you would make losses if you win. The boxes turning yellow are those whose number exceeds your bid. If the opponent’s bid corresponds to one of the blue or red boxes, you will earn the difference between 60 ECUs and your bid. If the opponent’s bid corresponds to one of the yellow boxes, your will earn zero. You are free to evaluate different bids in this way.

At the end of the experiment, after answering some questions and filling out a short questionnaire, we will randomly determine the number of the box corresponding to your opponent’s bid. This will be done by one of the participants randomly drawing a token from a bag containing 60 tokens numbered 1 through 60.

D Screenshots

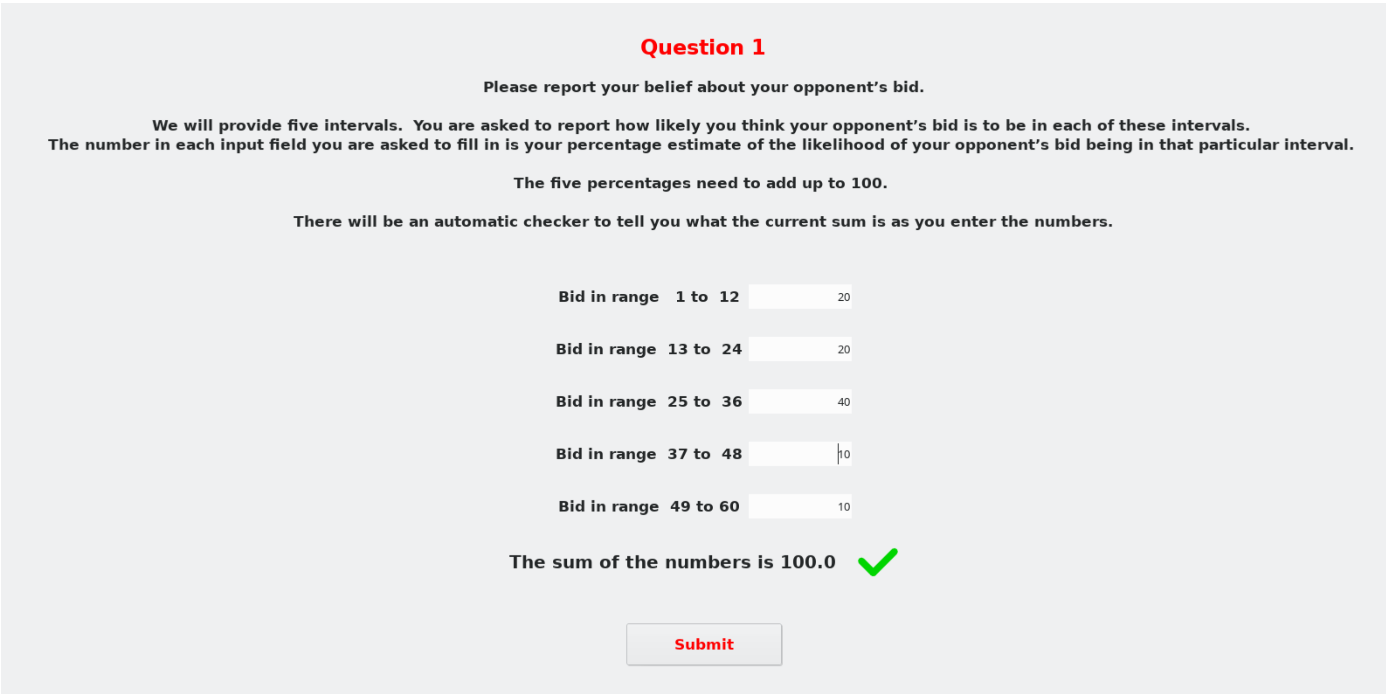
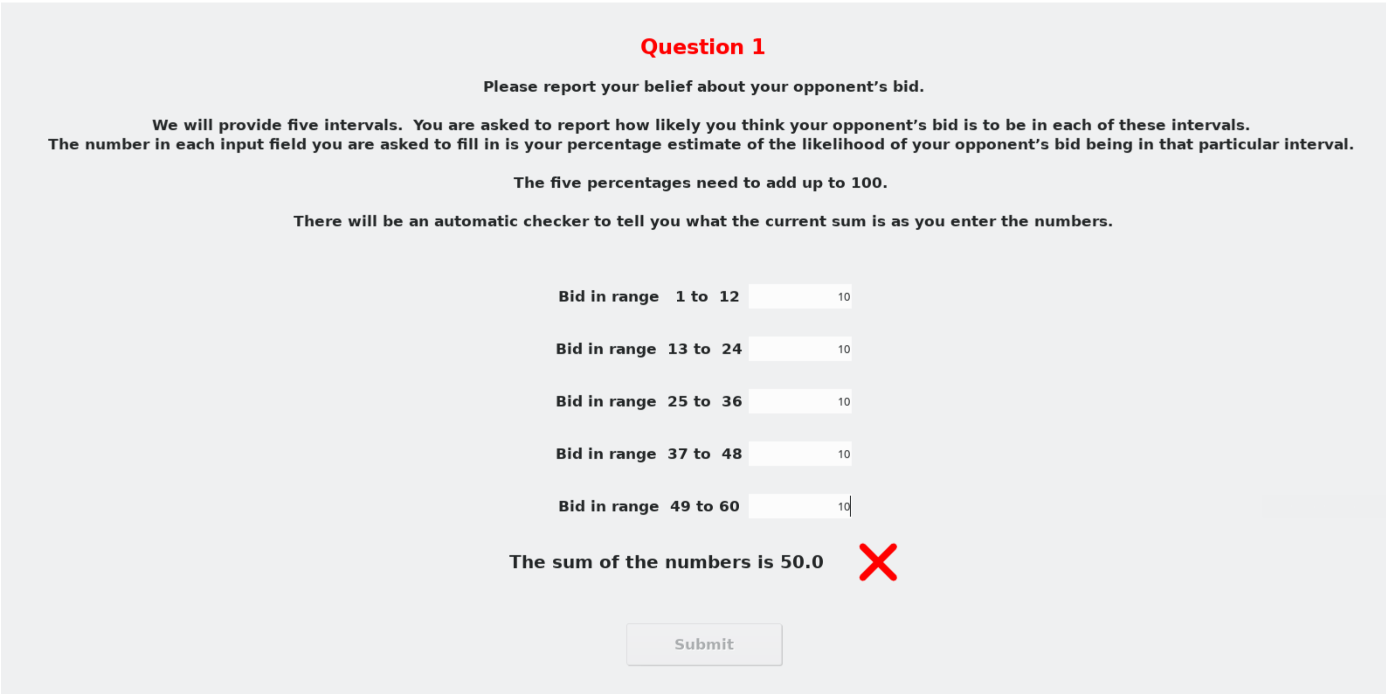


Figure D1: Screenshot of the belief elicitation of the random-draw opponent's strategy after a subject reports weights that correctly sum to 100.


Question 1

Please report your belief about your opponent's bid.

We will provide five intervals. You are asked to report how likely you think your opponent's bid is to be in each of these intervals.
The number in each input field you are asked to fill in is your percentage estimate of the likelihood of your opponent's bid being in that particular interval.

The five percentages need to add up to 100.

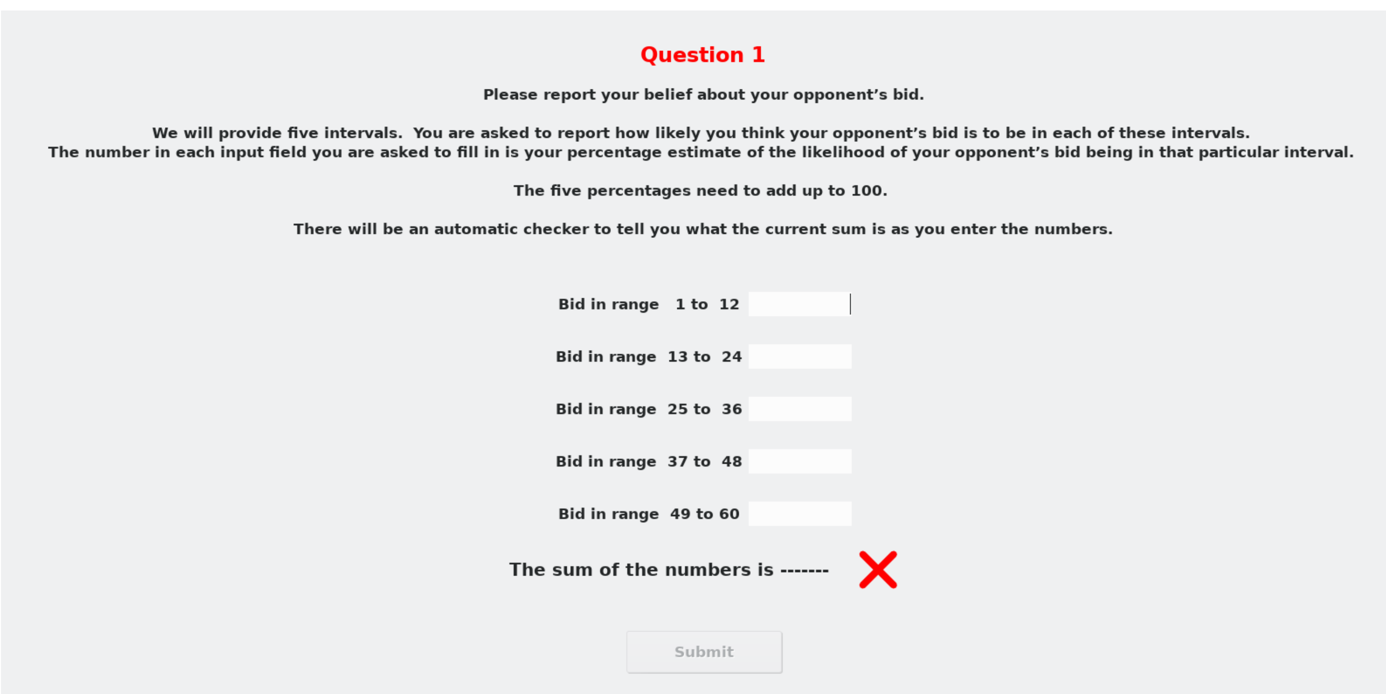
There will be an automatic checker to tell you what the current sum is as you enter the numbers.

Bid in range 1 to 12	10
Bid in range 13 to 24	10
Bid in range 25 to 36	10
Bid in range 37 to 48	10
Bid in range 49 to 60	10

The sum of the numbers is 50.0

Submit

Figure D2: Screenshot of the belief elicitation of the random-draw opponent's strategy where the values do not sum to 100.


Question 1

Please report your belief about your opponent's bid.

We will provide five intervals. You are asked to report how likely you think your opponent's bid is to be in each of these intervals.
The number in each input field you are asked to fill in is your percentage estimate of the likelihood of your opponent's bid being in that particular interval.

The five percentages need to add up to 100.

There will be an automatic checker to tell you what the current sum is as you enter the numbers.

Bid in range 1 to 12

Bid in range 13 to 24

Bid in range 25 to 36

Bid in range 37 to 48

Bid in range 49 to 60

The sum of the numbers is -------

Submit

Figure D3: Screenshot of the belief elicitation of the random-draw opponent's strategy where the input boxes are empty.

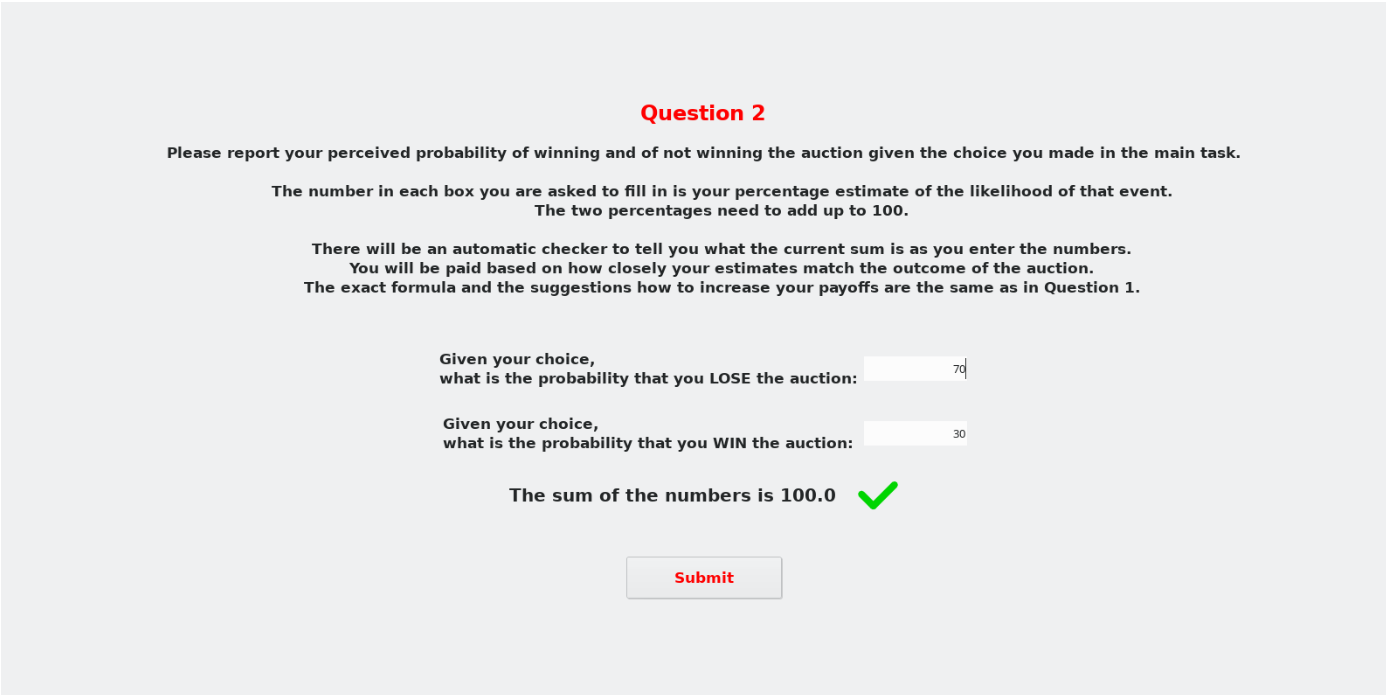


Figure D4: Screenshot of the belief elicitation of the probability of winning the auction where the values sum to 100.

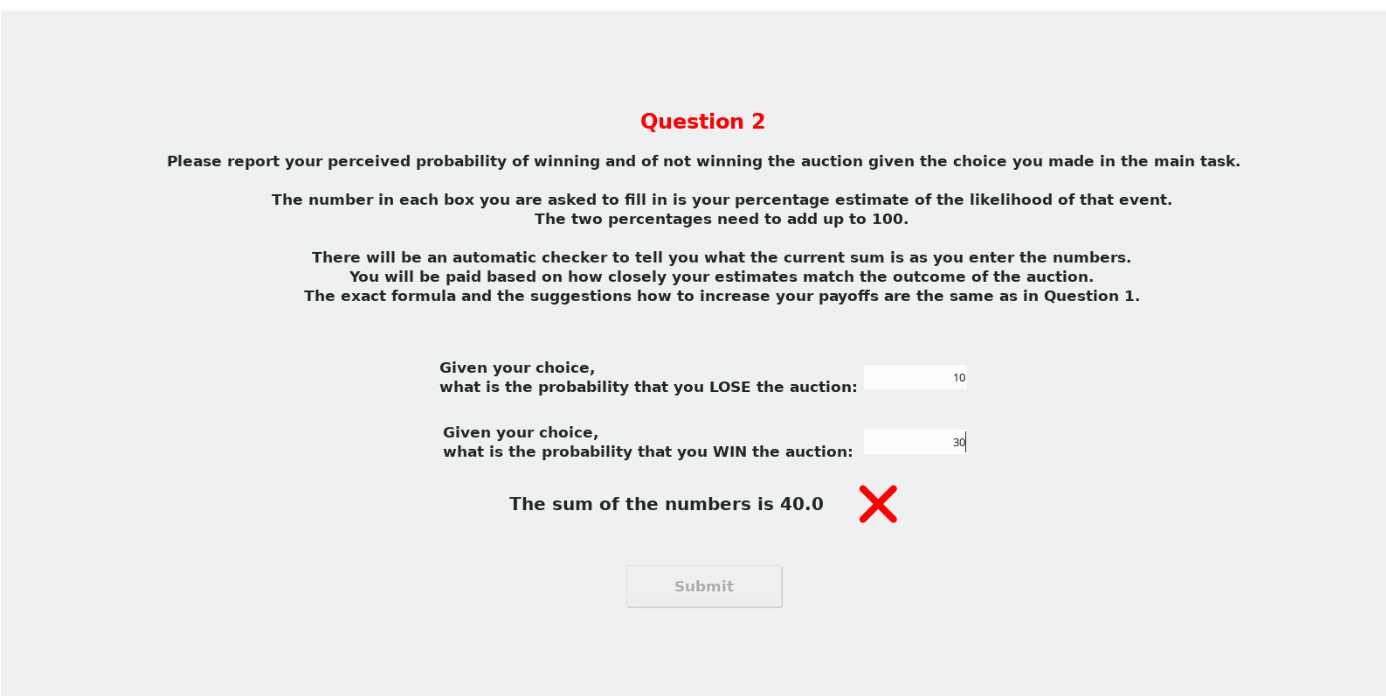


Figure D5: Screenshot of the belief elicitation of the probability of winning the auction where the values do not sum to 100.

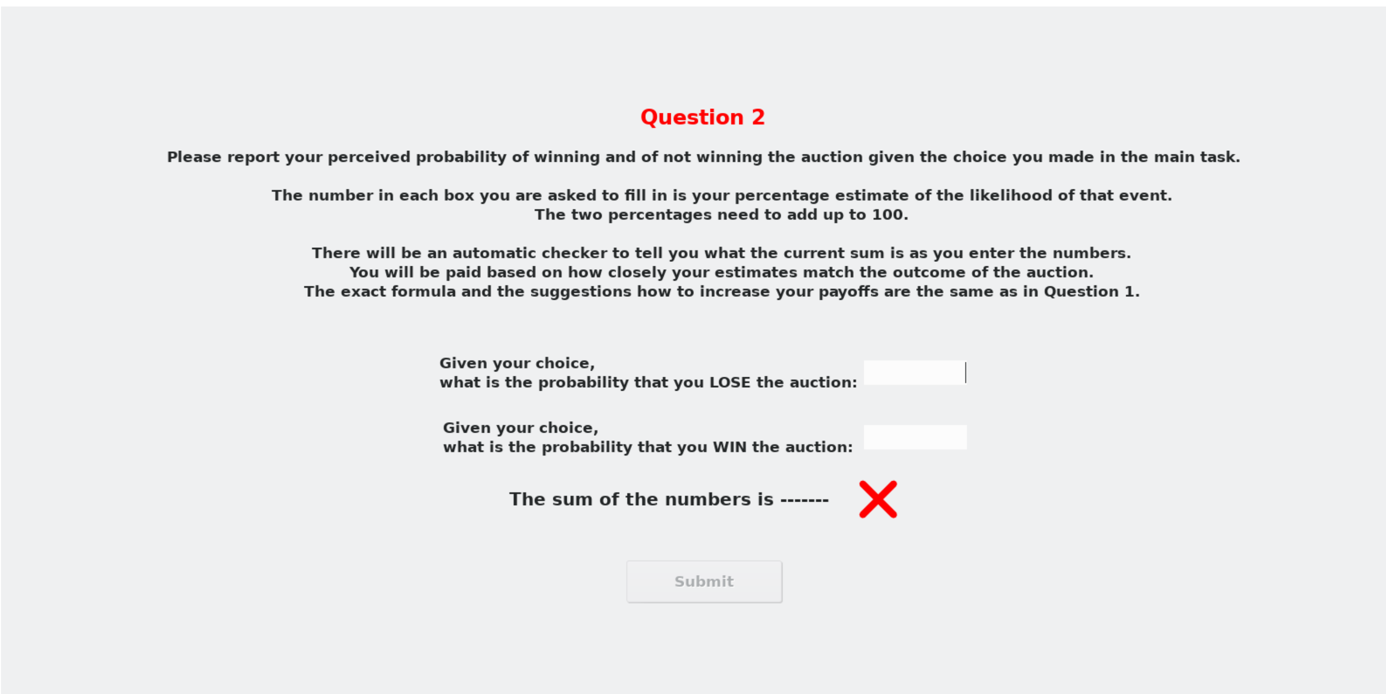


Figure D6: Screenshot of the belief elicitation of the probability of winning the auction where the values are empty.