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Abstract. A method to simulate concrete structures (quasi-brittle material) with localized nonlinearities is presented. Based on Guyan's condensation, it consists in replacing the elastic zones of the structure by their equivalent rigidities (super-elements). The nonlinear computation is then performed only on the zones of interest (ie, damaged). As new damaged zones may appear, the proposed method monitors the evolution of the system and re-integrates previously condensed areas if necessary. This method, applied on different tests cases, allows a substantial computation economy.

Introduction

During the last decades, models based on damage mechanics have become more complex in order to give a finer representation of the physics. Applying those models on the simulation of large-scale structures to understand their behavior at the fine scale requires more and more computational resources, and the scalability of simulation methods is now a key issue. Several classes of methods have been developed to access to fine simulations on those large-scale problems.

For instance, global-local methods combine coarse and fine simulations to separate the different scales of the problem, and therefore limit the global problem complexity [1]. However, they require the definition of a Representative Elementary Volume which can be difficult to determine in case of strain localization. Besides, they only focus on reproducing the global behavior of the structure.

Another approach is the use of parallel computing techniques with domain decomposition methods [2,3]. Although those methods allow solving problems in a limited amount of time by optimizing the use of high-performance computing machines, they do not reduce the global computational load.

Model reduction methods and zooming methods allow limiting the global computational load and some of them focus on reproducing the material behavior at the local scale [4–6]. However, those techniques either require previous knowledge of the areas to be damaged in the simulation, or are unable to reproduce exactly the local-scale behavior.

In this contribution, a method is presented to simulate damage in quasi-brittle structures. It uses a two-level Guyan condensation technique in a way adapted to the initiation and propagation of damage in the material. The method is presented in the first section; applications to a heterogeneous quasi-brittle beam and a notched concrete beam using different damage models are then presented.

Adaptive structural zooming method

General principle. The considered problem is a large structure of which material follows a damage constitutive law. It is assumed that damage will appear during the simulation, and eventually propagate to adjacent areas. This method has been developed considering that the typical scale of the damaged areas is small with respect to the structural scale. However, no assumption is made on the location of the damaged areas. Fig. 1 presents the general scheme of the method applied to the simulation of an academic two-dimensional problem.

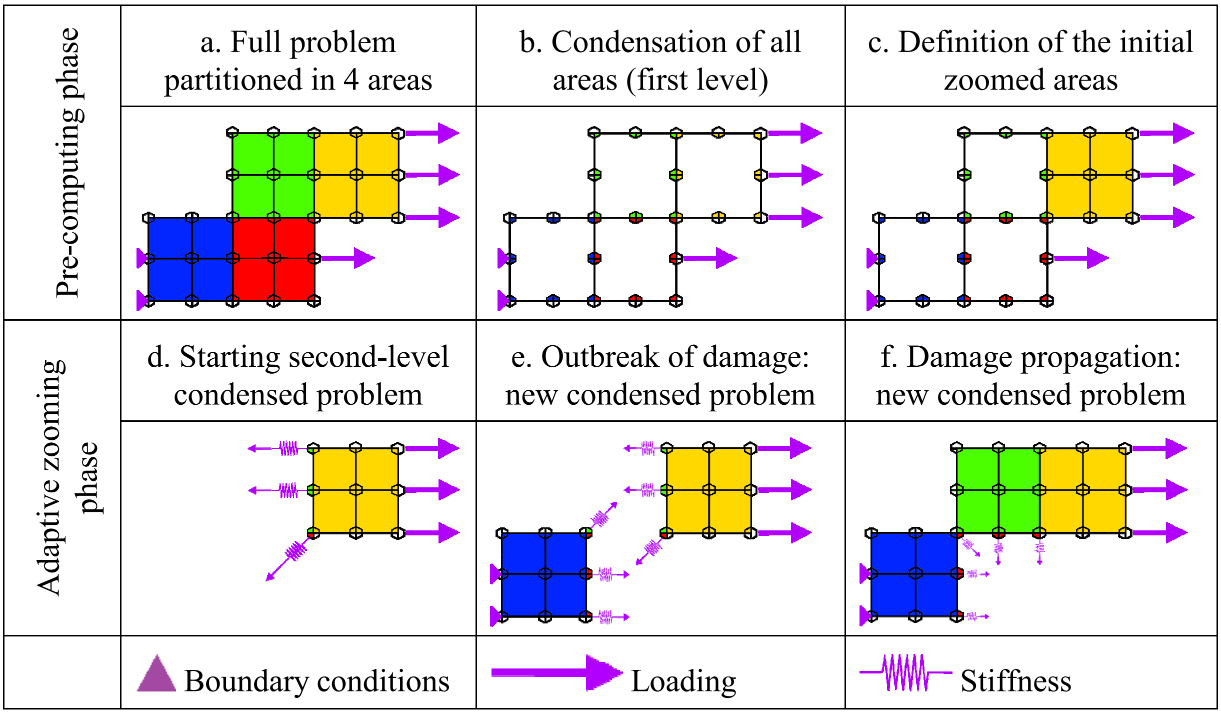

Figure 1: Structural zooming method applied to a 2D problem with 4 areas.

This method provides a framework for an adaptive, local-scale oriented use of a two-level Guyan's condensation method when solving damage problems [7]. It follows several steps:

- Pre-computing phase:
  - The structural problem is first partitioned in *N* subdomains (4 in Fig. 1.a): Each of these areas will either be zoomed or condensed later depending on the predicted behavior. The decomposition has no impact on the results but large impact on computational performance.
  - Each area is condensed on its boundaries using the static condensation method (Fig. 1.b): the equivalent stiffness and equivalent loading are calculated to be used during the simulation. It's the first-level condensation.
  - A linear pre-calculus of the structure determines, by the use of a damage outbreak criterion, which areas are zoomed at the start of the simulation, and which areas are condensed (yellow area zoomed, Fig. 1.c):
    - Areas with predicted nonlinear behavior are zoomed.
    - Areas with predicted linear behavior are condensed.
  - All condensed areas are combined. Then, the combination is condensed on the interface between zoomed and condensed areas: the second-level equivalent stiffness and equivalent loading are calculated using Guyan's method at a second level.
  - The model of each zoomed area is included in the simulation (Fig. 1.d).
  - The second-level equivalent stiffness and loading are combined with the stiffness of the zoomed areas: the second-level condensed problem is created. (The equivalent

stiffness can be considered as additional boundary conditions on the zoomed areas, Fig. 1.d).

- Adaptive zooming phase: The nonlinear simulation is run.
  - At each step, the zoomed problem is solved. Its dimension is smaller than the full problem by an amount which depends on the ratio between condensed and zoomed areas, i.e. between elastic and non-linear parts.
  - New potential nonlinearities are checked during the simulation through 2 criteria:
    - An outbreak criterion, to detect appearance of damage in new areas, due to possible stress redistribution upon cracking. This criterion, which requires computing the mechanical model on the condensed areas, is regularly checked during the simulation. With damage models, it is typically based on the elastic limit of the equivalent strain variable.
    - A propagation criterion, to detect damage propagation from the damaged zoomed areas to adjacent areas. This criterion is checked at the end of every step. With damage models, it is typically based on a non-zero value of the damage variable.
  - New areas are zoomed when nonlinearities are likely to appear: the nonlinear second-level condensed problem is reconstructed:
    - If the outbreak criterion is matched, and damage “should have” appeared in condensed areas, the simulation goes back to the last time step when the outbreak criterion has been checked, and the concerned areas are zoomed from the following step (appearance of damage in the blue area, Fig. 1.e). This approach, called “just-in-time” zooming, avoids zooming on areas that are arbitrarily close to the end of their elastic phase, but never actually initiate damage.
    - If the propagation criterion has been matched, and damage is likely to propagate to condensed areas, the concerned areas are zoomed from the following time step (damage propagation from the yellow to the green area, Fig. 1.f).

Quasi-brittle beam with heterogeneous elastic modulus field

Presentation. The test case is a quasi-brittle beam undergoing three-point bending. It uses the modified von Mises damage model for quasi-brittle materials [8]. The beam (100 x 10 cm) is modelled in two dimensions under the hypothesis of plane stresses (and considered 5 cm wide). A regular quadrilateral mesh of size 5 mm is used with bilinear finite elements. The simulations are run with Cast3M [9].

The Young’s modulus field is non-uniform in the beam, and is generated using a random Gaussian auto-correlated field, with a mean of 30 GPa and a standard deviation of 2 GPa [10]. This randomly distributed elastic modulus field is used to represent the heterogeneity of the material and the existence of local strong and weak points. It implies, in particular, that the damage initiation location is unknown. The mean compressive strength and tensile strength are arbitrarily set respectively to 30.3 MPa and 2.75 MPa (with a ratio of 11 between them). To avoid mesh dependency, a nonlocal method is applied: The stress-based regularization method, with an internal length l_{C0} of 7.5 mm has been chosen to limit the spreading of nonlocal damage [11]. To avoid stress concentration, the loading and boundary conditions are applied on small elastic areas. The loading is an increasing vertical displacement up to $2.43.10^{-4}$ m (which corresponds to a previously calculated load peak).

Zooming parameters. The structural zooming method is applied to this test case: the 2D mesh is divided in 32 areas (plus one additional area for the elastic loading zone). The propagation criterion for this model is formulated as such: a new adjacent area is zoomed if damage ω reaches a nonzero value in any element inside a 1.5 l_{C0} thick band from its boundary (a numerical tolerance of 0.01 is applied on damage value). This value avoids creating discontinuities in the nonlocal domain around damaged areas. The outbreak criterion writes: A condensed area is zoomed if the nonlocal von Mises equivalent strain reaches the elastic limit k_0 in any of its elements: using the nonlocal value is computationally more expensive, but keeps coherence with the zoomed areas. The outbreak criterion is checked every eight time steps.

Global behavior. Fig. 2 compares the load-deflection evolutions obtained with the zoomed simulation to the reference simulation, using the same dataset without zooming method.

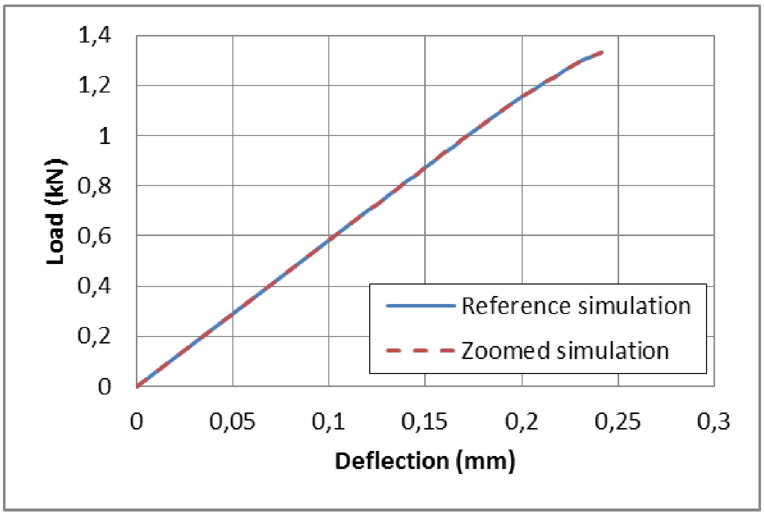

Figure 2: Comparison of load-deflection evolutions of the reference and zoomed simulations.

Both curves follow the same evolution with strictly identical values at each loading step.

Local behavior. Damage profiles at 90% of the loading and at the end of the simulations are presented in Fig. 3, with enhanced views of the central part of the beam. Both damage fields are identical in both simulations: the structural zooming method reproduces correctly the reference behavior at the local scale.

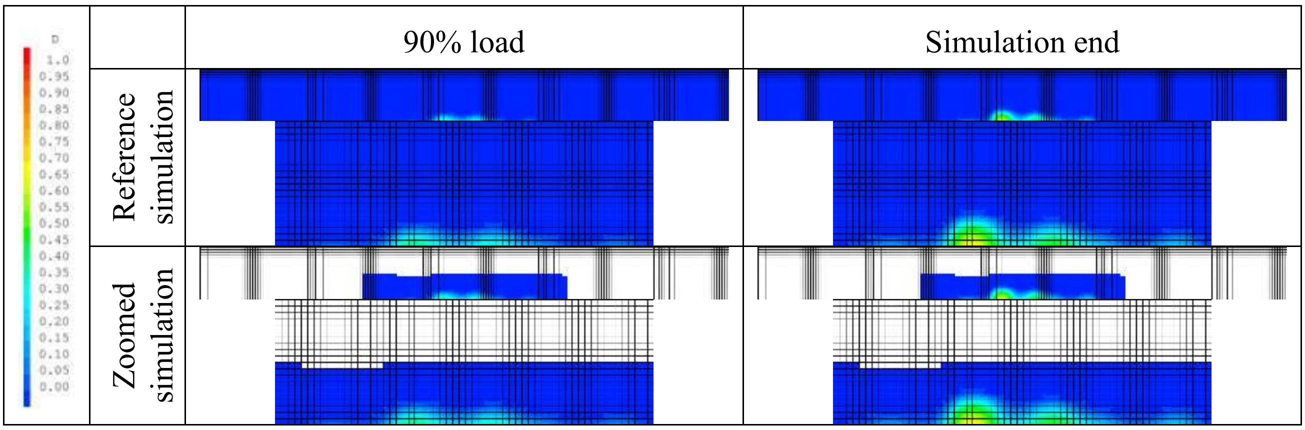

Figure 3 : Damage profiles at 90% load and at the end of the simulation.

Method efficiency. Several zoomed simulations have been run with different problem decompositions, to observe the evolution of the speed-up. Table 1 presents the processor times and relative errors on applied load for the simulations. It is to be noted that, except in very particular cases, the error introduced by the approximation of equivalent stiffness matrices and the dependency to problem decomposition is negligible. The simulations using the adaptive structural zooming method provide identical results to the reference, for a reduced computational time. Although speed-up cannot be considered as a performance criterion on a quantitative point of view as it is largely dependent on the problem characteristics, it still demonstrates that the structural zooming method provides a substantial computational economy while reproducing the results of a fully refined simulation.

Table 1: Processor times for the simulation of the quasi-brittle beam.

Simulation	Reference	Zoomed / 16 areas	Zoomed / 32 areas	Zoomed / 64 areas
Max. error (load)		$8.24\ 10^{-5}$	$8.57\ 10^{-5}$	$8.57\ 10^{-5}$
Processor time [s]	741	292	207	248
Speed-up		2.54	3.58	2.99

Notched concrete beam

Presentation. The test case is a notched plain concrete beam undergoing three-point bending. It uses a different mechanical model, the Mazars isotropic damage model for concrete [12]. All simulation hypotheses and model parameters are obtained from previous experimentally-verified work by Dufour et al. [13]. The beam (40 x 10 cm) is modelled in two dimensions under the hypothesis of plane stresses (the experimental beam is 5 cm wide). A notch (2 x 20 mm) is made at mid-span on the bottom of the beam. 2.5 mm long quadrilaterals with bilinear finite elements are used for the mesh (6464 elements). The simulations are run with Cast3M [8].

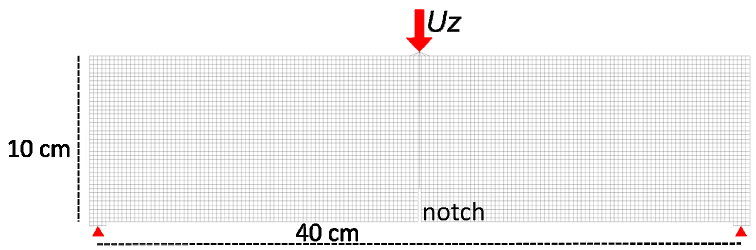

Figure 4: Mesh of the 2D notched beam (40x10 cm), with boundary and loading conditions.

The compressive strength and tensile strength are respectively equal to 41.4 MPa and 3.03 MPa: those values are obtained from the model with experimentally-suited parameters. To avoid mesh dependency, as in the first test case, the stress-based regularization method is applied with an internal length l_{C0} of 7.5 mm. The loading and boundary conditions are applied on small elastic areas. The loading is an increasing vertical displacement up to $2\text{x}10^{-4}$ m.

Zooming parameters. The structural zooming method is applied to this test case: the 2D mesh is divided in 64 areas (plus one additional area for the elastic loading zone). With the Mazars damage model, the propagation criterion is formulated: a new adjacent area is zoomed if damage D reaches a nonzero value in any element inside a 1.5 l_{C0} thick band from its boundary (a numerical tolerance of 0.01 is applied on damage value). The outbreak criterion writes: A condensed area is zoomed if the nonlocal Mazars equivalent strain reaches the elastic limit ε_0 in any of its elements. The outbreak criterion is checked every four time steps, as damage is likely to appear in multiple areas.

Global behavior. Fig. 5 compares the load-CMOD evolutions obtained with the just-in-time zooming method (zoomed simulation) with a reference simulation, run with the same dataset but not using the zooming method. The CMOD is measured as the relative horizontal displacement at the notch mouth.

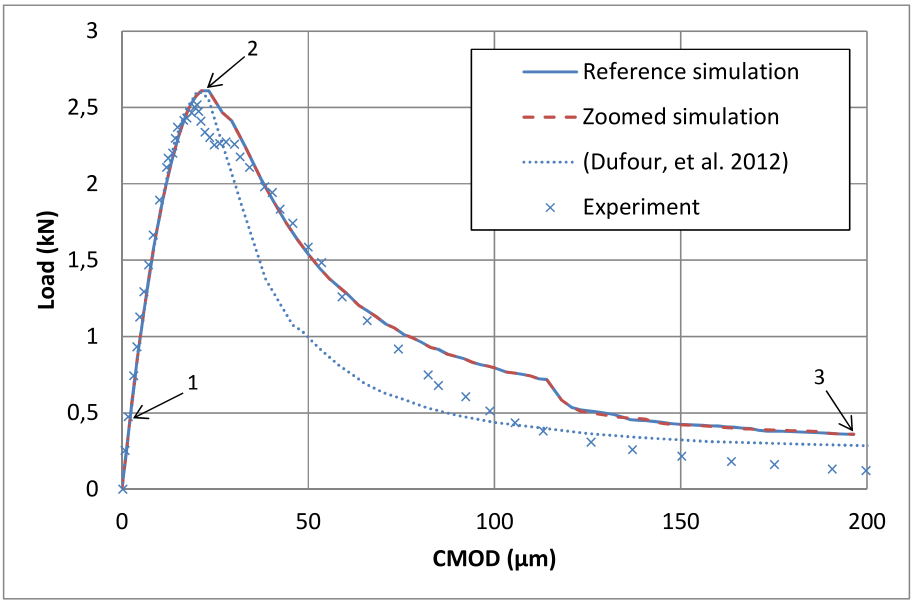

Figure 5: Comparison of load-CMOD evolutions of the reference and zoomed simulation.

Both simulations (reference and zoomed) follow the same evolution:

- At point 1, damage initiates at the center of the beam.
- At point 2, the load peak is reached.
- At point 3, the last loading increment is reached (end of the simulations).

Also, between 115 µm and 118 µm of CMOD, a slight drop in the applied load is observed. This is due to damage reaching the top of the beam, which induces softening behavior and a fast decrease of the applied bending load.

The results of this study appear to be slightly closer to the experiment than those obtain in the work of Dufour et al. [13]. This is due to the use of the stress-based regularization method, which models more accurately the behavior around the notch tip and the fracture process zone than the Gaussian regularization method [11].

Local behavior. The evolution of the damage profiles in the reference and zoomed simulation is presented in Fig. 6. Both damage profile evolutions are identical through loading, and agree with previously published results. The structural zooming method reproduces correctly the reference behavior of the damaged areas at the local scale.

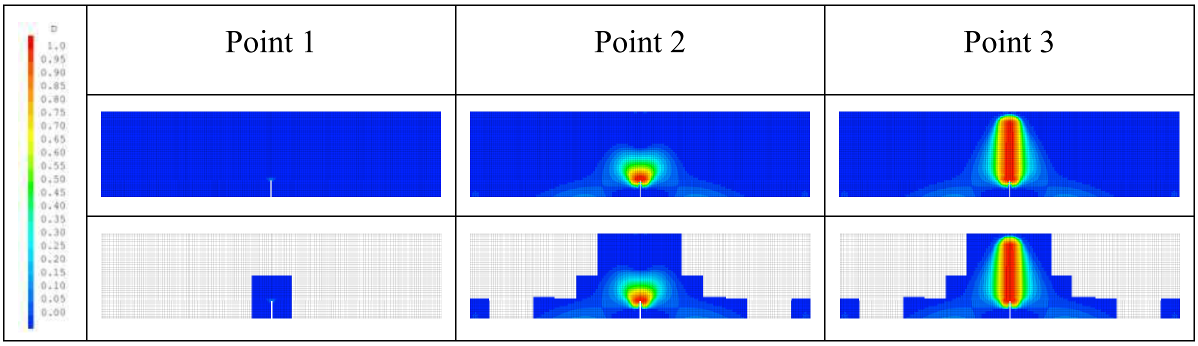

Figure 6: Damage profiles at points 1 (damage initiation), 2 (peak load), and 3 (end) for the reference simulation and the simulation with structural zooming.

Method efficiency. To evaluate the dependency of speed-up to the problem decomposition, additional simulations with other decompositions have been run. Table 2 presents the processor times obtained.

Table 2: Processor times for the simulation of the notched concrete beam.

Simulation	Reference	Zoomed / 16 areas	Zoomed / 32 areas	Zoomed / 64 areas
Processor time [s]	14120	10403	7776	7147
Speed-up		1.36	1.82	1.98

The simulation using the structural zooming method with 64 zoomable areas provides identical results to the reference simulation, for half the computational time. In this particular case, the speed-up values are lower than in the first case because the damaged areas represent a larger part of the structure. Although those speed-ups may seem low compared to those attainable with other classes of methods, several elements must be considered:

- A large majority of the zoomed areas is actually damaged, meaning that the computing effort is effectively focused on the nonlinear part of the structure.
- The solution is identical to the reference simulation, at both global and local scale.
- The structural zooming method has been developed for the simulation of large-scale structures, and more specifically, to allow access to a finer level of modelling on these large-scale structures. In particular, the speed-up will increase as the scale of nonlinear areas will be small compared to the scale of the complete structure.

Conclusions

An adaptive structural zooming method for the simulation of damage in quasi-brittle structures has been presented. It uses a two-level Guyan’s reduction approach to condense all areas with a linear elastic behavior in the simulated structure, so that only the nonlinear areas finely modelled. The structural zooming method monitors the evolution of the system through physically-based criteria and starts zooming on new areas just before damage appears or spreads. This method can be applied to any mechanical model provided suitable zooming criteria: for instance, the speed-up would expectedly be higher with energy-regularized models.

This method has been applied to two quasi-brittle structures using different mechanical models. In both cases it has reproduced both the global and local behaviors of the reference simulation for a reduced computational load. In particular, the computational load that can be attained depends on the size of the actually damaged areas more than the global size of the structure. It is therefore actually usable to give access to refined modelling on a larger scale of structures.

Ongoing research will include optimization of the different parameters (problem decomposition, criteria checks) and a method to condense already damaged areas with no more damage evolution (broken areas).

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