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OFDM for Optical Wireless Systems under Severe Clipping Conditions

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***Abstract*—In this paper, we analytically model the impact of clipping and quantization for optical wireless communication systems which employ direct-current biased optical orthogonal frequency division multiplexing (DCO-OFDM). Analytical expression for signal-to-distortion-quantization ratio has been evaluated, as a function of clipping ratio, considering uniform quantization for both digital-to-analog converter (DAC) and analog-to-digital converter (ADC). A lower bound on required bit resolution for DAC/ADC, as a function of clipping ratio has also been determined for a target bit-error-rate (BER). Gain in modulation power at the output of the DAC under severe clipping conditions, has been analytically evaluated. Furthermore, an iterative method for clipping mitigation is presented, which can be employed to relax the bit resolution required by the DAC, and further to mitigate the clipping distortions.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) has been widely used in modern communication systems owing to a number of advantages, such as, its inherent resilience to inter-symbol interference, high spectral efficiency etc. Recently optical-OFDM (O-OFDM) has been explored as a candidate for optical wireless communication (OWC) systems because of benefits like immunity to fluorescent-light noise near DC region [1], [2]. Moreover, with the advent of incoherent high power optoelectronic light emitting diodes (LEDs), and highly sensitive photo-detectors (PDs), OWC has gained an interest in the recent past [3]. In OWC systems, O-OFDM can be employed using intensity modulation-direct detection (IM-DD) techniques. In IM-DD, the data carrying intensity waveform (time-domain O-OFDM signal) is modulated onto the brightness of the LED, and the light intensity is photo-detected at the receiver, thus, producing a photocurrent which is directly proportional to the received light intensity. Further, IM-DD is particularly attractive for OWC systems because of low cost and ease of implementation [2], [3].

In IM-DD, the intensity waveform which is used to modulate the brightness of the LED is constrained to be real and non-negative. Therefore, O-OFDM techniques have been tailored for IM-DD implementation, the two main approaches being: direct-current biased O-OFDM (DCO-OFDM) [1] and asymmetrically-clipped O-OFDM (ACO-OFDM) [4]. In DCO-OFDM, a DC bias is added to the bipolar time-domain signal to attain a unipolar time-domain signal. However, addition of the DC bias results in an increased power consumption. Whereas, in ACO-OFDM, only the odd sub-carriers are modulated (sacrificing the even ones) to obtain an asymmetric time-domain signal, for which the negative amplitudes can be clipped without any loss of useful information. In terms of performance, DCO-OFDM is spectrally more efficient compared to ACO-OFDM, while ACO-OFDM is more power efficient compared to DCO-OFDM for low order constellations.

Conventional O-OFDM techniques (DCO- and ACO-OFDM) are generally attributed to have a high peak-to-average power ratio (PAPR) [2]. High PAPR is of particular significance in the context of IM-DD, since the LEDs have a limited linear dynamic range. Moreover, the limited bit resolution of digital-to-analog converters (DACs) and analog-to-digital converters (ADCs) also limit the performance of the OWC systems because quantization noise is exacerbated when a signal with high PAPR is impinged on them [5]. Therefore, the non-linear distortions of DAC and ADC will be pivotal in terms of achieving high data rates.

Due to limited linear dynamic range of LEDs, the time-domain DCO-OFDM signal is clipped double-sidedly in digital domain before DAC. Clipping of time-domain DCO-OFDM signal will induce clipping distortions in the system, thus, drastically impacting the system performance.

Herby, in this paper, we analytically model the non-linear distortions that occur in OWC systems, considering DCO-OFDM. Extending the work by Dimitrov *et al.* [3], in which the authors evaluated the impact of clipping distortion to obtain an effective signal-to-noise ratio (SNR) assuming infinite resolution DAC/ADC, we evaluate the signal-to-distortion-quantization noise ratio (SDQNR), which along with clipping distortion and thermal noise also incorporates the quantization noise impact. Double-sidedly clipping is incorporated by symmetrically clipping the time-domain DCO-OFDM signal to a pre-defined value. Further, uniform quantization is considered to assimilate the impact of DAC/ADC. Moreover, clipping's impact on required bit resolution for the DAC/ADC has also been examined. It has been observed that if the DCO-OFDM signal is severely clipped, then, following added benefits can be achieved: (i) gain in modulation power at the output of DAC, (ii) significant reduction in PAPR, (iii) reduction in the DC bias required to overcome the non-negative constraint, (iv) reduction in quantization noise, and (v) relaxation of bit resolution requirement for DAC. However, if small clipping ratio is considered, then clipping distortions would aggravate. So, in order to overcome the clipping distortions, we propose an iterative method for clipping distortion mitigation at the receiver, which is in line with [6]. Simulation results provided demonstrate that if a smaller clipping ratio is used in conjunction with clipping distortion mitigation, the required effective number of bits for the DAC can be reduced, thus reducing the overall system cost.

The remainder of the paper is organized as follows: section II presents the systems model, section III provides statistical analysis of non-linearities like clipping and quantization. Section IV highlights the effects of clipping, and also presents an iterative method for clipping distortion mitigation. Simulation results are depicted in section V, finally based on the obtained results, conclusions are drawn in section VI.

In the sequel, we use lower case, boldface letters for time-domain vectors e.g., $\boldsymbol{x}$, and upper case boldface letters (e.g., $\mathbf{F}$) for matrices. Discrete Fourier transform (DFT) of time-domain vectors is represented by upper case, boldface calligraphic letters e.g., $\boldsymbol{\mathcal{X}}$.

II. System Model

Consider DCO-OFDM transmission with N sub-carriers, in which the frequency-domain data-symbol vector, $\boldsymbol{\mathcal{X}}$, is modulated according to M-quadrature amplitude modulation (QAM) constellation. $\boldsymbol{\mathcal{X}}$ is enforced with Hermitian symmetry to obtain a real-valued time-domain DCO-OFDM signal, $\boldsymbol{x}$, using an inverse DFT on $\boldsymbol{\mathcal{X}}$, i.e., $\boldsymbol{x}$ = IDFT$\{\boldsymbol{\mathcal{X}}\}$ = $\mathbf{F}^{\mathrm{H}}\boldsymbol{\mathcal{X}}$. $\mathbf{F}$ represents the DFT matrix whose (k,l)th element is given by $\mathbf{F}(k,l) = N^{-1/2}e^{-j2\pi kl/N}; (k,l) \in 0,1,\cdots,N-1$, and $(.)^{\mathrm{H}}$ represents the Hermitian operator. By exploiting central limit theorem, for $N \to \infty$, $\boldsymbol{x}$ can be modeled as Gaussian distributed, i.e., $\mathcal{N}(0,\sigma_x^2)$. For brevity, we use $p(x)$ to represent the probability density function (pdf) of $\boldsymbol{x}$, given as

$$p(x) \approx \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left(-\frac{x^2}{2\sigma_x^2}\right) \tag{1}$$

where σ_x^2 is the mean power given as $\sigma_x^2 = \mathrm{E}\left(|\boldsymbol{x}|^2\right)$, $\mathrm{E}(\cdot)$ being the statistical expectation.

Cyclic prefix (CP) can be appended in the DCO-OFDM signal to combat the inter-symbol and inter-carrier interferences, and to transform the dispersive optical channel to a flat fading optical channel [3], [7]. However, in what follows, CP has been omitted, since it has negligible impact on the required electrical SNR, and the spectral efficiency [3], [8].

Due to limited input limits of DAC, and limited linear dynamic range of the LED, the digital time-domain DCO-OFDM signal, $\boldsymbol{x}$, is subjected to amplitude clipping at given upper (ξ_{upper}) and lower (ξ_{lower}) levels prior to DAC stage. For given clipping levels $(\xi_{\mathrm{upper}}, \xi_{\mathrm{lower}})$, the clipped time-domain DCO-OFDM signal, $\boldsymbol{x}_{\mathrm{clip}}$, has the form

$$x_{\mathrm{clip}}(n) = \\ \mathrm{clip}\left\{x(n),\gamma\right\} = \begin{cases} \xi_{\mathrm{upper}}, & x(n) > \xi_{\mathrm{upper}} \\ x(n), & \xi_{\mathrm{lower}} \le x(n) \le \xi_{\mathrm{upper}} \\ \xi_{\mathrm{lower}}, & x(n) < \xi_{\mathrm{lower}} \end{cases} \tag{2}$$

where $x_{\mathrm{clip}}(n)$ is the nth element of the clipped time-domain DCO-OFDM signal, $\boldsymbol{x}_{\mathrm{clip}}$. Note that we have considered symmetric clipping, i.e., $\xi_{\mathrm{lower}} = -\xi_{\mathrm{upper}}$. The severity of clipping can be determined by a factor referred to as *clipping ratio*, γ, and is associated with the clipping amplitudes $(\xi_{\mathrm{upper}}, \xi_{\mathrm{lower}})$ as $\gamma \triangleq \xi_{(\cdot)}/\sigma_x$.

$\boldsymbol{x}_{\mathrm{clip}}$ is impinged on a uniform DAC with Q_b bits to obtain DCO-OFDM intensity waveform, $\boldsymbol{x}_t$. The quantization noise variance of the DAC can be given as σ^2_{DAC}. The DCO-OFDM intensity waveform, $\boldsymbol{x}_t$, is inherently bipolar, therefore, a DC bias, β_{DC}, is required to obtain a unipolar intensity waveform, $\boldsymbol{x}_B$, i.e., $\boldsymbol{x}_B = \boldsymbol{x}_t + \beta_{\mathrm{DC}}$. $\boldsymbol{x}_B$ can thus be used to modulate the intensity of the optical carrier. To avoid further lower level clipping from the LED, the condition for the bias, i.e., $\beta_{\mathrm{DC}} = |\min \boldsymbol{x}_t| = \gamma\sigma_x$ is considered.

In what follows, we assume perfect synchronization [2], [9], [10], further, we also consider that the intensity waveform, $\boldsymbol{x}_B$, corresponds to the dynamic range of the LED and the input limits of the DAC [2]. Non-linearity of the LED operating within its dynamic range can be mitigated using pre-distortion techniques [11], henceforth, a linear response of the LED is considered.

The data carrying intensity waveform is photo-detected at the receiver using a photo-detector (PD), which is then converted to an electrical signal. It has been considered that no additional clipping has been introduced by the PD. The received signal is electronically amplified using a transimpedance amplifier (TIA) and then impinged on Q_b bit uniform ADC. The quantization noise variance for the ADC is represented by σ^2_{ADC}. Moreover, the thermal noise of the TIA is modeled with double-sided power spectral density of $N_0/2$. The received electrical signal, $\boldsymbol{y}$, can be given as

$$\boldsymbol{y} = g_{\mathrm{h}}\boldsymbol{x}_{\mathrm{clip}} + \boldsymbol{w} \tag{3}$$

where the noise, $\boldsymbol{w}$, has a variance of $\sigma^2_{\mathrm{noise}}$ given as $\sigma^2_{\mathrm{noise}} = N_0\mathrm{B}$, with B being the double sided bandwidth of the frequency-domain DCO-OFDM signal, and g_{h} being the optical path gain coefficient of the channel. The SNR at the receiver can be defined as SNR = $g_{\mathrm{h}}^2\sigma_x^2/\sigma^2_{\mathrm{noise}}$. (3) incorporates the simplified description of the optical wireless channel, since it has been considered as flat over entire DCO-OFDM frame [3]. N point DFT operation on (3) is used to translate the received electrical signal, $\boldsymbol{y}$, to the received frequency-domain DCO-OFDM symbols, expressed as [3]

$$\boldsymbol{\mathcal{Y}} = g_{\mathrm{h}}\boldsymbol{\mathcal{X}}_{\mathrm{clip}} + \boldsymbol{\mathcal{W}} \tag{4}$$

where $\boldsymbol{\mathcal{X}}_{\mathrm{clip}}$ and $\boldsymbol{\mathcal{W}}$ are the frequency-domain counterparts of $\boldsymbol{x}_{\mathrm{clip}}$ and $\boldsymbol{w}$, respectively. An M-QAM demapping operation is performed on $\boldsymbol{\mathcal{Y}}$ to obtain an estimate of transmitted data after equalization.

III. Statistical Analysis of Non-Linearities

In this section, we statistically analyse the impact of non-linearities in OWC DCO-OFDM system. The non-linearities considered are (i) the clipping distortion, and (ii) quantization impact of DAC and ADC.

A. Clipping Distortion

Clipping can be performed on time-domain DCO-OFDM signal to counteract the high PAPR, relax the bit resolution requirement for DAC/ADC, and further, to operate within the linear regime of the LED. According to Ochiai and Imai [12], the clipped time-domain OFDM signals can be modeled as a sum of two uncorrelated parts, therefore we have

$$\boldsymbol{x}_{\text{clip}} = \alpha \boldsymbol{x} + \boldsymbol{d} \tag{5}$$

where α is a linear attenuation factor, and $\boldsymbol{d}$ is the clipping distortion statistically uncorrelated to $\boldsymbol{x}$. The factor, α, can be evaluated by exploiting the Bussgang Theorem, as follows

$$\alpha = \frac{\text{cov}\left[\boldsymbol{x}, \boldsymbol{x}_{\text{clip}}\right]}{\sigma^2} = \text{erf}\left(\frac{\gamma}{\sqrt{2}}\right) \tag{6}$$

where $\text{cov}[\cdot]$ is the covariance, and $\text{erf}(\psi)$ is the error function given as $\text{erf}(\psi) = 2/\sqrt{\pi}\int_0^{\psi}\exp(-t^2)\text{d}t$. Observe that α reflects the shrinkage of the signal power caused by clipping. However, the impact of power shrinkage is dominant only if the clipping ratio is small. Furthermore, for a given clipping ratio, γ, the clipping probability for DCO-OFDM can be given as $\mathcal{P}_{\text{clip}} = 1 - 2\ \text{erf}(\gamma/\sqrt{2})$.

Contrary to the approximation that $\boldsymbol{x}$ is Gaussian distributed i.e., $\boldsymbol{x} \sim \mathcal{N}(0, \sigma_x^2)$, the clipped time-domain DCO-OFDM signal, $\boldsymbol{x}_{\text{clip}}$, cannot be modeled as Gaussian distributed, rather it follows a truncated Gaussian distribution, $p(x_{\text{clip}})$, given by $p(x_{\text{clip}}) =$

$$\begin{cases} \frac{1}{2}\text{erfc}\left(\frac{\xi_{\text{upper}}}{\sqrt{2\sigma_x^2}}\right)\delta(x_{\text{clip}} - \xi_{\text{upper}}), & x_{\text{clip}}(n) = \xi_{\text{upper}} \\ \frac{1}{\sqrt{2\pi\sigma_x^2}}\exp\left(-\frac{x^2}{2\sigma_x^2}\right), & \xi_{\text{lower}} < x_{\text{clip}}(n) < \xi_{\text{upper}} \\ \left[1 - \frac{1}{2}\text{erfc}\left(\frac{\xi_{\text{lower}}}{\sqrt{2\sigma_x^2}}\right)\right]\delta(x_{\text{clip}} - \xi_{\text{lower}}), & x_{\text{clip}}(n) = \xi_{\text{lower}} \end{cases} \tag{7}$$

where $\text{erfc}(\psi)$ is complementary error function given as $\text{erfc}(\psi) = 1 - \text{erf}(\psi)$.

Owing to (7), it can be observed that clipping results in a modification of the mean power of time-domain DCO-OFDM signal. As a consequence, the mean power of the clipped time-domain DCO-OFDM signal is now the sum of effective power of the clipped time-domain DCO-OFDM signal, and the power induced by the clipping distortion, i.e.,

$$\bar{\sigma}_x^2 = \sigma_{x,\text{eff}}^2 + \sigma_{\text{clip}}^2 \tag{8}$$

where $\bar{\sigma}_x^2$ is the power of clipped time-domain DCO-OFDM signal, $\sigma_{x,\text{eff}}^2$ is the effective power of the clipped time-domain DCO-OFDM signal, and σ_{clip}^2 is the power of clipping distortion. Moreover, the effective power of the clipped time-domain DCO-OFDM signal, $\sigma_{x,\text{eff}}^2$, can be evaluated as

$$\sigma_{x,\text{eff}}^2 = \sigma_x^2\left[\int_{\xi_{\text{lower}}}^{\xi_{\text{upper}}}\frac{1}{\sqrt{2\pi\sigma_x^2}}\exp\left(-\frac{x^2}{2\sigma_x^2}\right)\text{d}x\right]^2 = \sigma_x^2\text{erf}^2\left(\frac{\gamma}{\sqrt{2}}\right). \tag{9}$$

Further, $\boldsymbol{x}_{\text{clip}}$ can be re-written as $\boldsymbol{x}_{\text{clip}} = \boldsymbol{x} - \Delta\boldsymbol{x}$, where $\Delta\boldsymbol{x}$ is the clipped signal portion. Note that the clipped portion, $\Delta\boldsymbol{x}$, results in the reduction of the mean power of time-domain DCO-OFDM signal. The power of the clipped portion can be calculated as

$$\begin{aligned}\sigma_{\Delta x}^2 = &\int_{-\infty}^{\xi_{\text{lower}}}(x^2 - \xi_{\text{lower}}^2)p(x)\text{d}x \\ &+ \int_{\xi_{\text{upper}}}^{+\infty}(x^2 - \xi_{\text{upper}}^2)p(x)\text{d}x.\end{aligned} \tag{10}$$

For symmetric clipping, (10) can be simplified to

$$\begin{aligned}\sigma_{\Delta x}^2 &= 2\int_{\xi_{\text{upper}}}^{+\infty}(x^2 - \xi_{\text{upper}}^2)p(x)\text{d}x \\ &= \sigma_x^2\left[\sqrt{\frac{2}{\pi}}\gamma\exp\left(-\frac{\gamma^2}{2}\right) + (1-\gamma^2)\text{erfc}\left(\frac{\gamma}{\sqrt{2}}\right)\right].\end{aligned} \tag{11}$$

By applying the law of conservation of energy and combining (6), (8) and (11), the power of clipping distortion can be obtained as

$$\begin{aligned}\sigma_{\text{clip}}^2 &= \sigma_x^2 - \sigma_{x,\text{eff}}^2 - \sigma_{\Delta x}^2 \\ &= \sigma_x^2\left[1 - \text{erf}^2\left(\frac{\gamma}{\sqrt{2}}\right) - \sqrt{\frac{2}{\pi}}\gamma\exp\left(-\frac{\gamma^2}{2}\right)\right. \\ &\qquad \left. -(1-\gamma^2)\text{erfc}\left(\frac{\gamma}{\sqrt{2}}\right)\right].\end{aligned} \tag{12}$$

Observe that the attenuation factor, α, the variance of clipping distortion, σ_{clip}^2 are independent of the QAM constellation order M, and the size of IDFT/DFT, N, rather are a function of the clipping ratio, γ.

B. Quantization

Here, we consider uniform DAC and ADC with Q_b bits. The clipped time-domain DCO-OFDM signal, $\boldsymbol{x}_{\text{clip}}$, is impinged upon DAC with $\mathcal{L}$ uniform levels to convert the digital signal, $\boldsymbol{x}_{\text{clip}}$, into analog time-domain intensity waveform, $\boldsymbol{x}_t$. The effective number of bits, Q_b, required for $\mathcal{L}$ level quantization can be obtained as $Q_b = \log_2(\mathcal{L})$. Moreover, for uniform DAC, the quantization noise is modeled as additive, uniformly distributed white noise, with a variance of

$$\sigma_{\text{DAC}}^2 = (1 - \mathcal{P}_{\text{clip}})\frac{\xi_{\text{upper}}^2}{3(2^{2Q_b})} = (1 - \mathcal{P}_{\text{clip}})\frac{\gamma^2\sigma_x^2}{3(2^{2Q_b})}. \tag{13}$$

where $\mathcal{D}$ represents the input limits of the quantizer (DAC). Here we consider that the upper and lower clipped levels of the time-domain DCO-OFDM signal are constrained by the input limits of the DAC, $\mathcal{D}$, therefore, we have $\mathcal{D} \triangleq \xi_{\text{upper}} - \xi_{\text{lower}} = 2\xi_{\text{upper}}$. Moreover, (13) foresees that for larger amplitude thereshold of the signal, the quantization noise variance would be high, and vice versa. Interestingly, decreasing the clipping ratio, γ can reduce the quantization noise, at an expense of increase in clipping distortion. Therefore, a compromise between clipping distortion and quantization noise can be achieved to operate the system in conditions which results in a maximum SNR.

At the receiver, the photo-detected signal has a reduced signal variance, σ_{rx}^2 because of the optical path gain coefficient, g_{h}, therefore, $\sigma_{rx}^2 = g_{\text{h}}^2\sigma_x^2$. The received electrical signal, $\boldsymbol{y}$ is then fed to the ADC with Q_b bits to obtain a digital electrical signal. The quantization noise variance for the ADC is given as

$$\sigma_{\mathrm{ADC}}^2 = \frac{\gamma^2 \sigma_{rx}^2}{3(2^{2Q_b})}. \tag{14}$$

Moreover, (14) assumes that PD does not introduce any further clipping.

Using (12), (13), (14), and considering the impact of thermal noise, an analytical expression for SNR, which we refer to as signal-to-distortion-quantization noise ratio (SDQNR), is evaluated after equalization process, which can be expressed as

$$\mathrm{SDQNR} = \frac{\alpha^2 \sigma_x^2 \zeta_{\mathrm{B}}}{\sigma_{\mathrm{clip}}^2 + \sigma_{\mathrm{DAC}}^2 + \frac{\sigma_{\mathrm{noise}}^2}{g_{\mathrm{h}}^2 \zeta_{\mathrm{DC}}} + \frac{\sigma_{\mathrm{ADC}}^2}{g_{\mathrm{h}}^2}} \tag{15}$$

where ζ_{B} is the utilization factor of double sided bandwidth, B, which is equal to $\zeta_{\mathrm{B}} = N/(N-2)$. Moreover, ζ_{DC} represents the attenuation of the useful electrical signal power due to the DC bias and is equal to $\zeta_{\mathrm{DC}} = 1/(1+\gamma^2)$. Note that, SDQNR represents the ratio between the attenuated signal power and the total noise power in the system.

The characteristic behaviour of SDQNR, as a function of clipping ratio, γ, for different bit resolution of DAC and ADC, Q_b is depicted in Fig. 1 for $\mathrm{SNR} = 30$dB considering 16-QAM constellation and $g_{\mathrm{h}}^2 = 1$ for simplicity. SNR of 30dB defines the overall noise floor in the system. It can be observed from Fig. 1 that there is an optimal value of clipping ratio, γ_{opt}, for each Q_b which can maximize the SDQNR. To the right of γ_{opt}, i.e., $\gamma > \gamma_{\mathrm{opt}}$, SDQNR is limited by quantization and thermal noise, whereas, on the left side γ_{opt}, i.e., $\gamma < \gamma_{\mathrm{opt}}$, SDQNR is constrained by clipping distortion. Note that Fig. 1 has been evaluated considering a DC bias of $\beta_{\mathrm{DC}} = \gamma\sigma_x$ for each γ. Furthermore, once SDQNR is computed, the system performance in terms of BER can easily be evaluated by using the closed-form expression for BER of square M-QAM constellation [13] given as

$$\mathrm{BER} = \frac{2}{\log_2 M}\left[1 - \frac{1}{\sqrt{M}}\right] \mathrm{erfc}\left[\sqrt{\frac{3}{2(M-1)}\mathrm{SDQNR}}\right]. \tag{16}$$

Additionally, we also evaluate the correlation between the required bit resolution of DAC/ADC to achieve a target BER and the clipping ratio, γ. Consider a target BER of 10^{-3}, for M-QAM constellation, the required $\mathrm{SDQNR}_{\mathrm{req}}$ to achieve target BER can be determined by (16), which can be further used to evaluate the lower bound (as thermal noise is ignored) on the required bit resolution, $Q_{b,\mathrm{req}}$, required for DAC/ADC, as

$$Q_{b,\mathrm{req}} = \frac{1}{2}\log_2\left\{\mathrm{SDQNR}_{\mathrm{req}} \frac{\left[\zeta_B \gamma^2 (2 - \mathcal{P}_c)\right]}{3(\alpha^2 - \mathrm{SDQNR}_{\mathrm{req}} A)}\right\} \tag{17}$$

where $A = \sigma_{\mathrm{clip}}^2/\sigma_x^2$. $\mathrm{SDQNR}_{\mathrm{req}}$ to achieve a target BER of 10^{-3} for 16-, 64- and 256-QAM constellation is 16.5dB, 22.5dB and 28.4dB, respectively. Further, the effective number of bits, Q_b for M-QAM constellations, as a function of γ, are depicted in Fig. 2. As aforementioned, the results in Fig. 2 provide a lower bound on the required number of bits, since thermal noise has been ignored while evaluating $\mathrm{SDQNR}_{\mathrm{req}}$.

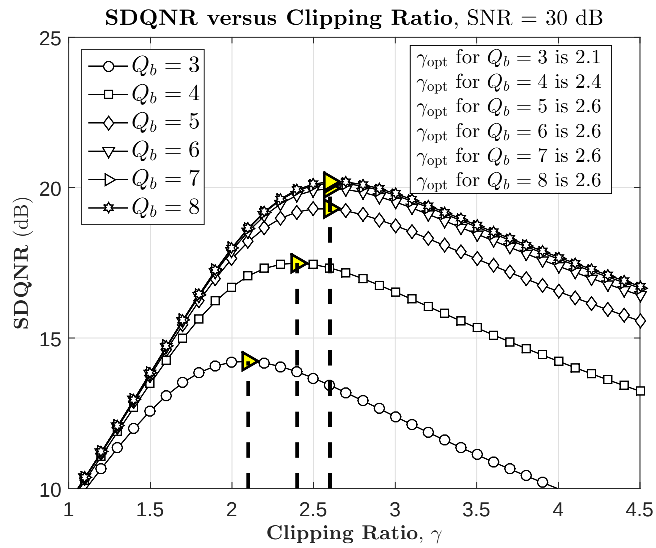

Fig. 1: (in decibels) considering both DAC and ADC versus clipping ratio γ, for different number of quantization bits Q_b and SNR = 30dB and using 16-QAM DCO-OFDM.

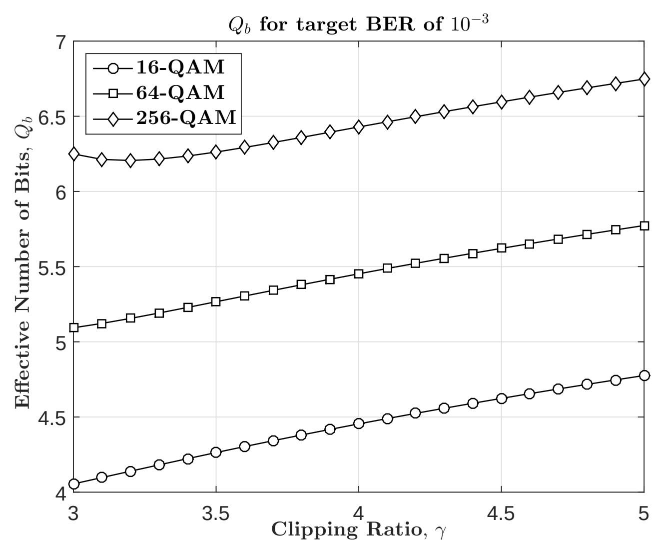

Fig. 2: Required effective number of bits Q_b to achieve a target BER of 10^{-3} using 16-, 64- and 256-QAM constellations.

Furthermore, the BER performances of DCO-OFDM using 16-QAM constellation with different bit resolution DAC, Q_b, and using γ_{opt} evaluated from Fig. 1 are depicted in Fig. 3 against $E_{b(\mathrm{elec})}/N_0$ which is given as $E_{b(\mathrm{elec})}/N_0 = \mathrm{SNR}/(\log_2 M \zeta_{\mathrm{DC}})$. It can be observed that for higher bit resolution, the BER performance improves with an increase in Q_b. Further, for $Q_b = 6$ and γ_{opt}, the performance of DCO-OFDM is comparable with ideal DCO-OFDM (infinite bit resolution DAC and $\beta_{\mathrm{DC}} = 7$dB).

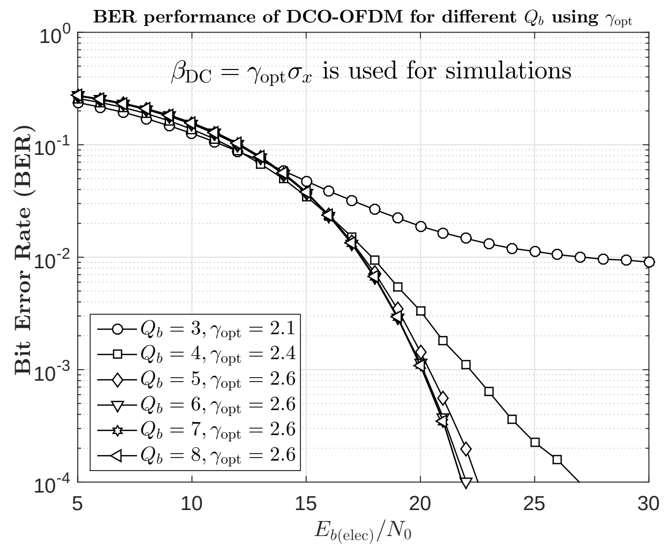

Fig. 3: BER performance of 16-QAM DCO-OFDM considering different number of bits for DAC. The optimised clipping ratio, γ_{opt}, is obtained at SNR = 30dB.

IV. Severe Clipping Effects and Mitigation

In this section, we explore the impact of severe clipping ($\gamma < \gamma_{\text{opt}}$) on the time-domain DCO-OFDM signal. It has been observed that if the time-domain DCO-OFDM signal is clipped such that clipping ratio is $\gamma < \gamma_{\text{opt}}$, then, we can achieve a number of benefits like gain in modulation power at the output of DAC, a reduction in PAPR and DC bias, and moreover, a reduction in quantization noise.

A. Gain in Modulation Power

In this section, we provide an analytical analysis of the gain in modulation power that can be achieved at the output of the DAC, if the time-domain DCO-OFDM is deliberately clipped at small clipping ratios, i.e., $\gamma < \gamma_{\text{opt}}$. The required DC bias is a function of the PAPR and can mathematically be expressed as $\beta_{\text{DC,req}} = \gamma_{\text{req}}\sigma_x$. Consider an optimal DC bias of $\beta_{\text{DC,opt}}$ which corresponds to previously defined optimized clipping ratio, γ_{opt}. Using γ_{opt}, the power at output of DAC, modulating the LED can be given as

$$P_{\text{opt}} = \alpha_{\text{opt}}^2 \sigma_x^2 \frac{\mathcal{D}^2}{\xi_{\text{opt}}^2} = \frac{\alpha_{\text{opt}}^2 \mathcal{D}^2}{\gamma_{\text{opt}}^2} \tag{18}$$

where $\xi_{\text{opt}} = \gamma_{\text{opt}}\sigma_x$, α_{opt} is the attenuation factor because of clipping using optimized clipping ratio, γ_{opt}, and $\mathcal{D}$ defines the input limits of the DAC. Now consider if the signal is deliberately clipped using a much smaller clipping ratio, i.e., $\gamma < \gamma_{\text{opt}}$, with same DAC input limits, the power at the output of DAC output can be given as

$$P_{\text{clip}} = \alpha^2 \sigma_x^2 \frac{\mathcal{D}^2}{\xi_{\text{upper}}^2} = \frac{\alpha^2 \mathcal{D}^2}{\gamma^2} \tag{19}$$

where $\xi_{\text{upper}} = \gamma\sigma_x$. Then the gain in modulation power over the optimized power at the output of DAC can thus be defined as

$$\mathcal{G}(\gamma) = \frac{P_{\text{clip}}}{P_{\text{opt}}} = \frac{\alpha^2/\gamma^2}{\alpha_{\text{opt}}^2/\gamma_{\text{opt}}^2}. \tag{20}$$

Fig. 4 evaluates the gain, $\mathcal{G}(\gamma)$ (in decibels) computed using (6) and (20) for 16-QAM constellation, and different number of DAC bits, Q_b. Moreover, the optimized DC bias considered is given as $\beta_{\text{DC,opt}} = \gamma_{\text{opt}}\sigma_x$. It is evident that if the signal is deliberately clipped at smaller clipping ratios, then a gain in modulation power at the output of DAC can be achieved. However, the clipping distortions in these cases, i.e., $\gamma < \gamma_{\text{opt}}$ will be dominant (Fig. 1). Therefore, in the following section, we present a method to mitigate the clipping distortions and results in an improvement in system performance.

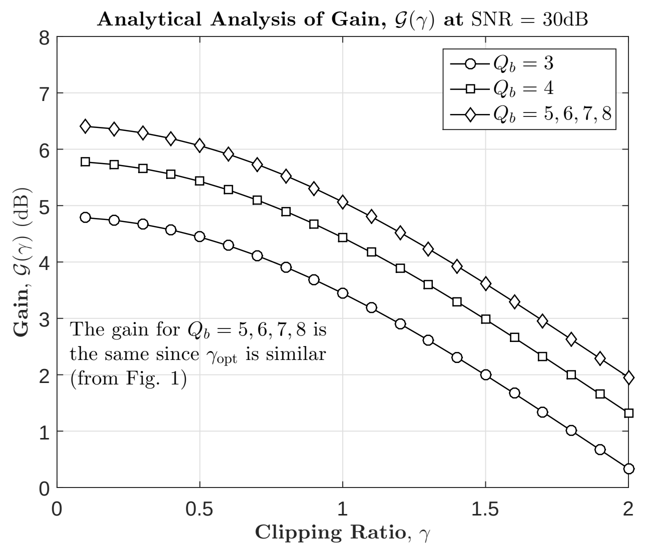

Fig. 4: Gain $\mathcal{G}$ (in decibels) in modulation power achieved by clipping the signal at the transmitter versus clipping ratio for 16-QAM DCO-OFDM.

B. Clipping Mitigation

In this section, we present an iterative method for clipping mitigation under severe clipping conditions, i.e., $\gamma < \gamma_{\text{opt}}$. We have presented a theoretically similar method for clipping mitigation for DCO-OFDM has been presented in [14]. Clipping mitigation is achieved by means of clipping distortion cancellation in the frequency-domain. This method is a simplified version of the one proposed in [6] and is explained as follows:

1) The time-domain DCO-OFDM signal after the equalization process at the receiver is evaluated, and the DC bias that has been added at the transmitter is subtracted, i.e., $\hat{\boldsymbol{s}}$ = IDFT$\{\hat{\boldsymbol{\mathcal{Y}}}\} - \beta_{\text{DC}}$, where $\hat{\boldsymbol{\mathcal{Y}}}$ is the signal after equalization.
2) (Initialization) set $i \leftarrow 1$, for the first iteration, the distortion contribution is neglected, i.e., $\boldsymbol{\mathcal{D}}^{(1)} = 0$. Moreover, the method is input as $\boldsymbol{\theta}^{(1)}$=DFT$\{\hat{\boldsymbol{s}}\}$.
3) Retrieved set of symbols, $\hat{\boldsymbol{\theta}}^{(i)}$, are obtained as $\hat{\boldsymbol{\theta}}^{(i)}$ = Slicer$\{\boldsymbol{\theta}^{(i)}\}$, where Slicer$\{.\}$ represents hard symbol estimator.

4) An estimated time-domain DCO-OFDM signal, $\hat{\boldsymbol{r}}^{(i)}$, is obtained via $\hat{\boldsymbol{r}}^{(i)}$ = IDFT$\{\hat{\boldsymbol{\theta}}^{(i)}\}$.
5) $\hat{\boldsymbol{r}}^{(i)}$ is subject to a similar clipping process as at the transmitter, i.e., $\hat{\boldsymbol{c}}^{(i)} = \mathrm{clip}\{\hat{\boldsymbol{r}}^{(i)}, \gamma\}$ and symbols are evaluated as $\hat{\boldsymbol{\theta}}_c^{(i)}$ = DFT$\{\hat{\boldsymbol{c}}^{(i)}\}$. Note that a distortion, $\boldsymbol{\mathcal{D}}^{(i)}$, similar to the one introduced at the transmitter is now introduced in $\hat{\boldsymbol{\theta}}_c^{(i)}$, where $\hat{\boldsymbol{\theta}}_c^{(i)} = \hat{\boldsymbol{\theta}}^{(i)} + \boldsymbol{\mathcal{D}}^{(i)}$.
6) An estimate of the clipping distortion component, $\boldsymbol{\mathcal{D}}^{(i)}$, is made as $\boldsymbol{\mathcal{D}}^{(i)} = \hat{\boldsymbol{\theta}}_c^{(i)} - \hat{\boldsymbol{\theta}}^{(i)}$.
7) (Update) $i \leftarrow i + 1$ and $\boldsymbol{\theta}^{(i)} = \boldsymbol{\theta}^{(i-1)} - \boldsymbol{\mathcal{D}}^{(i-1)}$.
8) If $\boldsymbol{\theta}^{(i)} = \boldsymbol{\theta}^{(i-1)}$ and $i \neq 1$, return, else go to step 3).

V. Simulation Results and Discussion

In this section, simulation results are presented to demonstrate and compare the performance of clipping mitigation method considering 16-QAM DCO-OFDM, 3 iterations of iterative method and Q_b of 4 and 5. Note that the minimum clipping ratio for 4-, 16-, 64- and 256-QAM is recommended as 1, 1.4, 1.9 and 2.5, respectively to achieve a target BER of 10^{-3} and with 3 iterations of the iterative method. DCO-OFDM with 1024 sub-carriers is simulated, and the results are averaged over 2000 independent OFDM realizations. Ideal DCO-OFDM with $\beta_{\mathrm{DC}} = 7$dB is presented as a benchmark. The BER performances of quantized DCO-OFDM (without clipping) and DCO-OFDM with optimum clipping ratio γ_{opt} are also provided as reference. The idea here is to use a low bit resolution DAC and clip the signal at a low clipping ratio, thus, reducing the quantization noise and increasing the modulation power. And finally using clipping mitigation method at the receiver to mitigate clipping distortion.

Results depicted in Fig. 5 demonstrate that if the signal is clipped at a smaller clipping ratio compared to the optimal one, a low bit resolution DAC can be used, and with clipping mitigation at the receiver, good BER performance can be achieved. It has been demonstrated that if $Q_b = 4$ and clipping ratio $\gamma = 1.5$ is used in conjunction with clipping mitigation, the BER performance is comparable with the ideal DCO-OFDM (infinite DAC resolution), and is better than the BER performance when γ_{opt} is used. Furthermore, if the bit resolution is increased from $Q_b = 4$ to $Q_b = 5$, the performance of clipped DCO-OFDM with clipping mitigation is better than the ideal DCO-OFDM, because due to clipping, the required DC bias has been reduced from 7dB to 5.1dB, which results in power efficiency. Refer to Fig. 3, which shows that at least $Q_b = 6$ is required to achieve a comparable performance with ideal DCO-OFDM if γ_{opt} is used, whereas, if time-domain DCO-OFDM signal is deliberately clipped at lower clipping ratio, and clipping mitigation is applied at the receiver, a similar performance can be achieved with $Q_b = 4$, thus, reducing the number of bit of DAC by 2.

VI. Conclusions

In this paper, we analytically model the non-linear distortions in OWC systems such as clipping distortion and quantization noise. Bit resolution required for DAC/ADC with respect to different constellation sizes, as a function of clipping ratio has been evaluated. Analytical expression of the gain in modulation power at the output of DAC is also computed, which shows that reducing clipping ratio can have added benefits for IM-DD. We also propose an iterative method for clipping mitigation, which can be used in conjunction with clipping to relax the bit resolution required for the DAC.

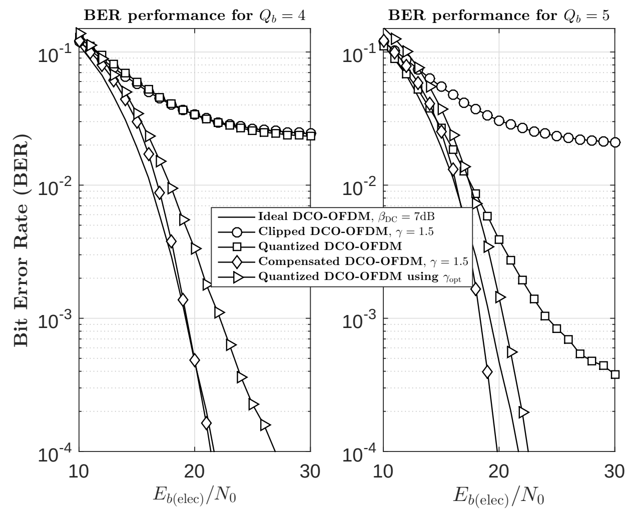

Fig. 5: Clipping mitigation using the proposed compensation method for 16-QAM DCO-OFDM.

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