# Linear Parameter Varying systems: from modelling to control

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# LPV systems

## Definition of an Linear Parameter Varying system

$$\Sigma(\rho):\left[\begin{array}{c}\dot{x}\\ \hline z\\ y\end{array}\right]=\left[\begin{array}{c|cc}A(\rho) & B_1(\rho) & B_2(\rho)\\ \hline C_1(\rho) & D_{11}(\rho) & D_{12}(\rho)\\ C_2(\rho) & D_{21}(\rho) & D_{22}(\rho)\end{array}\right]\left[\begin{array}{c}x\\ \hline w\\ u\end{array}\right]$$

$x(t)\in\mathbb{R}^n$, ...., $\rho=(\rho_1(t),\rho_2(t),\ldots,\rho_N(t))\in\Omega$, is a vector of time-varying parameters ($\Omega$ convex set), assumed to be **known** $\forall t$

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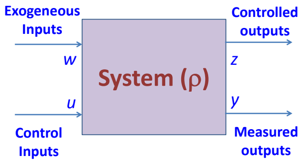
(Scherer, ACC Tutorial 2012)




Dampened mass-spring system:

$$\ddot{p}+c\,\dot{p}+k(t)\,p=u,\quad y=x$$

First-order state-space representation:

$$\frac{d}{dt}\begin{pmatrix}p\\ \dot{p}\end{pmatrix}=\begin{pmatrix}0 & 1\\ -k(t) & -c\end{pmatrix}\begin{pmatrix}p\\ \dot{p}\end{pmatrix}+\begin{pmatrix}0\\ 1\end{pmatrix}u,$$

$$y=\begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}p\\ \dot{p}\end{pmatrix}$$

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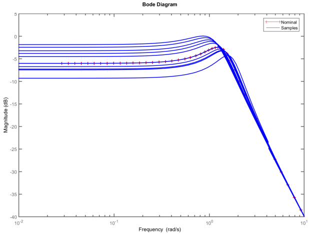
$x(t) \in \mathbb{R}^n$, ...., $\rho = (\rho_1(t), \rho_2(t), \dots, \rho_N(t)) \in \Omega$, is a vector of time-varying parameters ($\Omega$ convex set), assumed to be **known** $\forall t$

The frozen Bode plots for $c = 1$ and $k \in [1,3]$




(Scherer, ACC Tutorial 2012)

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$$y = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} p \\ \dot{p} \end{pmatrix}$$

# LPV systems (2)

Let the LPV system be:

$$\Sigma(\rho):\left[\begin{array}{c}\dot{x}\\ \hline z\\ y\end{array}\right]=\left[\begin{array}{c|cc}A(\rho) & B_1(\rho) & B_2(\rho)\\ \hline C_1(\rho) & D_{11}(\rho) & D_{12}(\rho)\\ C_2(\rho) & D_{21}(\rho) & D_{22}(\rho)\end{array}\right]\left[\begin{array}{c}x\\ \hline w\\ u\end{array}\right]$$

$x(t)\in\mathbb{R}^n$, ...., $\rho=(\rho_1(t),\rho_2(t),\ldots,\rho_N(t))\in U_\rho$, is a vector of time-varying parameters ($U_\rho$ convex set)

- $\rho(.)$ varies in the set of continously differentiable parameter curves $\rho:[0,\infty)\rightarrow\mathbb{R}^N$. It is assumed to be **known or measurable**.
- The parameters $\rho$ are always assumed to be **bounded**:

$$\rho\in U_\rho\subset\mathbb{R}^N \text{ and } U_\rho \text{ compact} \tag{1}$$

defined by the minimal $\underline{\rho_i}$, and maximal $\overline{\rho_i}$ values of $\rho_i(t)$

$$\rho_i(t)\in[\underline{\rho_i},\ \overline{\rho_i}],\quad\forall i$$

- The system matrices $A(.)$ .... are continuous on $U_\rho$

# LPV systems (3): about the parameters

- Parameters are **exogenous** if they are external variables. The system is in that case *non stationary*.
  See the previous damped mass-sping system.
- Parameters are **endogenous** if they are function of the state variables, $\rho = \rho(x(t), t)$, and, in that case, the LPV system is referred to as a **quasi-LPV system**.
  This case is encountered when approximating Nonlinear systems.
  For instance:
  $$\dot{x}(t) = x^2(t) = \rho(t)x(t)$$
  with $\rho(t) = x(t)$.
- It is sometimes required that the derivative of the parameters are bounded, i.e:
  $$\dot{\rho} \in U_{\dot{\rho}} \subset \mathbb{R}^N \text{ and } U_{\dot{\rho}} \text{ compact} \qquad (2)$$
  defined by the minimal $\underline{v_i}$, and maximal $\overline{v_i}$ values of $\dot{\rho}_i(t)$
  $$\dot{\rho}_i(t) \in [\underline{v_i}, \ \overline{v_i}], \ \ \forall i$$
  This corresponds to the case of *slow varying parameters*
- Other representations can be considered if $\rho$ is piecewise-constant, or varies in a finite set of elements ($\rho(t) \in \{0, 1\}$ for switching systems)

Next, several classes of LPV models are presented, and some ways to go from one class to another are given.

# Some comments

- LPV systems can model uncertain systems ($\rho$ fixed but unknown) or parameter-varying models ($\rho(t)$)

## LPV=linear or nonlinear?

- What is often referred to as gain-scheduling control, corresponds to Jacobian linearization of the nonlinear plant about a family of equilibrium points Shamma (90), Rugh & Shamma (2000)
  In terms of control design this means, lineraization around operating conditiosn, design (at each operating points) of a LTI controller, and interpolation of the LTI controllers in between operating conditions (often used in Aerospace and Automotive industries).
  **Pros**: Simplicity of design for a non linear system
  **Cons**: No *a priori* guarantee of stability nor robustness
- **But**: this differs from quasi-LPV representations where nonlinearities are hiddden in some parameter descriptions (as seen later in the course)

## LPV=LTV

- Theoretical analysis of LPV system properties (stability, controllability, observability), often falls into the framework of LTV systems or of nonlinear ones (for quasi-LPV representations), see (Blanchini).

# Some references

## Those not to be ignored

- Modelling, identification : (Bruzelius, Bamieh, Lovera, Toth) + 2011 TCST Special Issue on "Applied LPV modelling and identification"
- Control (Shamma, Apkarian & Gahinet, Adams, Packard, Beker, Seiler, Grigoriadis ...)
- Stability, stabilization (Scherer, Wu, Blanchini ...)
- Geometric analysis (Bokor & Balas)
- Survey paper: Hoffmann & Werner, 2015
- Fault tolerant control: special issues by
  - Balas, 2012: in *International Journal of Adaptive Control and Signal Processing*
  - Casavola, Rodrigues & Theilliol, 2015: in *International Journal of Robust and Nonlinear Control*

## Some recent books

- R. Toth, Modeling and identification of linear parameter-varying systems, Springer 2010
- J. Mohammadpour, C. Scherer, (Eds), Control of Linear Parameter Varying Systems with Applications, Springer-Verlag New York, 2012
- O. Sename, P. Gaspar, J. Bokor (Eds), Robust Control and Linear Parameter Varying Approaches: Application to Vehicle Dynamics, Springer, 2013

# Outline

# Different Models

According to the dependency on the parameter set, we may have several classes of models:

1. Affine parameter dependency: $A(\rho) = A_0 + A_1\rho_1 + ... + A_N\rho_N$
2. Polynomial dependency: $A(\rho) = A_0 + A_1\rho + A_2\rho^2 + ... + A_S\rho^S$
3. Rational dependency:$A(\rho) = [A_{n0} + A_{n1}\rho_{n1} + ... + A_{nN}\rho_{nN}][I + A_{d1}\rho_{d1} + ... + A_{dN}\rho_{dN}]^{-1}$

## Brief insight in LFR Models

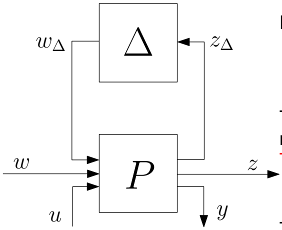



Figure: System under LFT form

$\Delta$ is defined such as: $z_\Delta = \Delta(.)w_\Delta$. It represents the parameter variations. $\Delta(.)$ is a linear function of the parameter vector.

Denoting the transfer matrix $N(s)$ as:

$$\left[\begin{array}{c} z_\Delta \\ \hline z \end{array}\right] = \left[\begin{array}{c|c} N_{11}(s) & N_{12}(s) \\ \hline N_{21}(s) & N_{22}(s) \end{array}\right] \left[\begin{array}{c} w_\Delta \\ \hline w \end{array}\right]$$

The Linear Fractional Representation (LFR) gives then the transfer matrix from $w$ to $z$, and is referred to as the upper Linear Fractional Transformation (LFT) :

$$F_u(N,\Delta) = N_{22} + N_{21}\Delta(I - N_{11}\Delta)^{-1}N_{12}$$

This LFT exists and is well-posed if $(I - N_{11}\Delta)^{-1}$ is invertible.

This will not be presented in the course. Please refer to (Apkarian & Gahinet; Scherer, Rantzer) for the use of LFT for robust analysis and design. This many need to study Integral Quadratic Constraints (IQCs).

# Polytopic models

A polytopic system is represented as

$$\Sigma(\rho) = \sum_{k=1}^{Z} \alpha_k(\rho) \left[ \begin{array}{cc} A_k & B_k \\ C_k & D_k \end{array} \right], \quad \text{with} \sum_{k=1}^{2^N} \alpha_k(\rho) = 1 \ , \ \alpha_k(\rho) > 0$$

where $\left[ \begin{array}{cc} A_k & B_k \\ C_k & D_k \end{array} \right]$ are LTI systems.

This representation is often used to rewrite an affine LPV system. Indeed, assuming that the parameters are bounded ($\rho_i \in \left[ \begin{array}{cc} \underline{\rho}_i & \overline{\rho}_i \end{array} \right]$), the vector of parameters evolves inside a polytope represented by $Z = 2^N$ vertices $\omega_i$, as

$$\rho \in \textbf{Co}\{\omega_1, \ldots, \omega_Z\} \tag{3}$$

It is then written as the convex combination:

$$\rho = \sum_{i=1}^{Z} \alpha_i \omega_i, \ \alpha_i \geq 0, \ \sum_{i=1}^{Z} \alpha_i = 1 \tag{4}$$

where the vertices are defined by a vector $\omega_i = [v_{i1}, \ldots, v_{iN}]$ where $v_{ij}$ equals $\underline{\rho_j}$ or $\overline{\rho_j}$.

The LTI system $\left[ \begin{array}{cc} A_k & B_k \\ C_k & D_k \end{array} \right]$ here corresponds to the LPV system frozen at the vertex $k$.

# From a generic affine LPV systems to a poytopic model

For a LPV system with 2 parameters, boundend $\left[\begin{array}{cc} \underline{\rho}_{1,2} & \overline{\rho}_{1,2} \end{array}\right]$, the corresponding polytope onws 4 vertices as:

$$\mathscr{P}_\rho = \left\{ (\underline{\rho}_1, \underline{\rho}_2), (\underline{\rho}_1, \overline{\rho}_2), (\overline{\rho}_1, \underline{\rho}_2), (\overline{\rho}_1, \overline{\rho}_2) \right\} \tag{5}$$

The polytopic coordinates are ($\alpha_i$) are obtained as:

$$\begin{array}{ll}
\omega_1 = (\underline{\rho}_1, \underline{\rho}_2), & \alpha_1 = \left(\dfrac{\overline{\rho}_1 - \rho_1}{\overline{\rho}_1 - \underline{\rho}_1}\right) \times \left(\dfrac{\overline{\rho}_2 - \rho_2}{\overline{\rho}_2 - \underline{\rho}_2}\right) \\
\omega_2 = (\underline{\rho}_1, \overline{\rho}_2), & \alpha_2 = \left(\dfrac{\overline{\rho}_1 - \rho_1}{\overline{\rho}_1 - \underline{\rho}_1}\right) \times \left(\dfrac{\rho_2 - \underline{\rho}_2}{\overline{\rho}_2 - \underline{\rho}_2}\right) \\
\omega_3 = (\overline{\rho}_1, \underline{\rho}_2), & \alpha_3 = \left(\dfrac{\rho_1 - \underline{\rho}_1}{\overline{\rho}_1 - \underline{\rho}_1}\right) \times \left(\dfrac{\overline{\rho}_2 - \rho_2}{\overline{\rho}_2 - \underline{\rho}_2}\right) \\
\omega_4 = (\overline{\rho}_1, \overline{\rho}_2), & \alpha_4 = \left(\dfrac{\rho_1 - \underline{\rho}_1}{\overline{\rho}_1 - \underline{\rho}_1}\right) \times \left(\dfrac{\rho_2 - \underline{\rho}_2}{\overline{\rho}_2 - \underline{\rho}_2}\right)
\end{array} \tag{6}$$

where $\rho_1$ and $\rho_2$ are the instantaneous values of the parameters ($\rho_i^{(k)}$ in the implementation step). The LPV system is then rewritten under the polytopic representation:

$$\begin{aligned}
\begin{pmatrix} A(\rho_{1,2}) & B(\rho_{1,2}) \\ C(\rho_{1,2}) & D(\rho_{1,2}) \end{pmatrix} = \alpha_1 \begin{pmatrix} A(\omega_1) & B(\omega_1) \\ C(\omega_1) & D(\omega_1) \end{pmatrix} + \alpha_2 \begin{pmatrix} A(\omega_2) & B(\omega_2) \\ C(\omega_2) & D(\omega_2) \end{pmatrix} \\
+ \alpha_3 \begin{pmatrix} A(\omega_3) & B(\omega_3) \\ C(\omega_3) & D(\omega_3) \end{pmatrix} + \alpha_4 \begin{pmatrix} A(\omega_4) & B(\omega_4) \\ C(\omega_4) & D(\omega_4) \end{pmatrix}
\end{aligned} \tag{7}$$

# From nonlinear to LPV using Linear Differential Inclusion

See (Boyd et al, 1994).
Let consider the nonlinear system

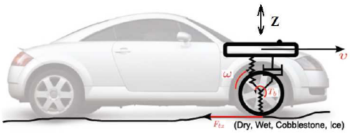
$$\Sigma_{\mathcal{N}\mathcal{L}} : \begin{cases} \dot{x} = f(x(t), w(t)) \\ z = g(x(t), w(t)) \end{cases} \tag{8}$$

Suppose that, for each $x$, $w$ and $t$, there is a matrix $G(x,w,t) \in \Omega$ s.t.:

$$\begin{bmatrix} f(x,w) \\ g(x,w) \end{bmatrix} = G(x,w,t) \begin{bmatrix} x \\ w \end{bmatrix} \tag{9}$$

where $\Omega \in \mathbb{R}^{(n_x+n_z)\times(n_x+n_u)}$.
As said in (Boyd et al, 1994):
"*Then of course every trajectory of the nonlinear system* (8) *is also a trajectory of the LDI defined by* (9)*. If we can prove that every trajectory of the LDI defined by* (9) *has some property (e.g., converges to zero), then a fortiori we have proved that every trajectory of the nonlinear system* (8) *has this property.*"

# LPV modelling of a quarter car vehicle suspension model



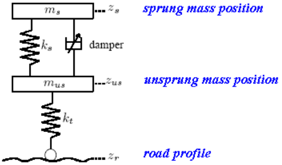




Figure: Simple quarter vehicle model for semi-active suspension control

## Quarter vehicle dynamics

$$\begin{cases} m_s\ddot{z}_s &= -k_s z_{def} - F_{damper} \\ m_{us}\ddot{z}_{us} &= k_s z_{def} + F_{damper} - k_t\left(z_{us} - z_r\right) \end{cases} \quad (10)$$

$z_{def} = z_s - z_{us}$ : damper deflection, $\dot{z}_{def} = \dot{z}_s - \dot{z}_{us}$ : deflection velocity.

- The damper's characteristics : Force-Deflection-Deflection Velocity relation

$$F_{damper} = g\left(z_{def}, \dot{z}_{def}\right) \quad (11)$$

where $g$ can be linear or nonlinear.

# LPV modelling of a quarter car vehicle suspension model (cont.)

## A Semi-active nonlinear MR damper model [Gu et al., 2006, Nino-Juarez et al., 2008]

$$F_{damper} = c_0\dot{z}_{def} + k_0 z_{def} + f_I \tanh\left(c_1\dot{z}_{def} + k_1 z_{def}\right) \tag{12}$$

- The $\tanh$ function allows to model the bi-viscous behavior.
- 5$c_0$, $k_0$, $c_1$, $k_1$): constant parameters. $k_0$, $k_1$ dedicated to the hysteresis behavior.
- $f_I$ is a controllable force and depends on input current $I$.

## LPV model

Choosing $\rho = \tanh\left(c_1\dot{z}_{def} + k_1 z_{def}\right)$, and denoting $u = f_I$ the control input, the quarter car model can be represented as:

$$\begin{cases} \dot{x}(t) = Ax(t) + B(\rho)u(t), \\ y(t) = Cx(t) + Du(t) \end{cases} \tag{13}$$

# A brief insight in identification of LPV models

Global approaches - input/output models $(y(k) = -\sum_{i=1}^{n_a} a_i(\rho(k))y(k-i) + \sum_{j=1}^{n_b} b_i(\rho(k))u(k-j))$

(Bamieh & Giarre 99, 02): characterisation of persistency of excitation conditions for input-output LPV models

Previdi & Lovera 03, 04): NLPV model class (LFT feeded by a neural network model for the scheduling policy)

(Toth, 07 + book 2010): An LPV system can be viewed as a collection of "local" behaviours (associated with constant parameter values

Global approaches - state space models

(Lee & Poolla): maximum likelihood (ML) algorithm for the identification of MIMO LPV-LFT models (PEM algorithm)

(Verhaegen et al, 02, 07, 09...): Supspace methods

Local approaches

Interpolation of locally identified LTI models... need to pay attention to:

- Input/output form (Toth, 07 + book 2010): interpolating transfer function coefficients
- State space form (Steinbuch et al, 03): consistency of state space basis

# Outline

# LPV systems properties

Let consider the LPV syetsm

$$\Sigma_{\rho} \left\{ \begin{array}{l} \dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t), \quad x(0) = x_0 \\ y(t) = C(\rho(t))x(t) + D(\rho(t))u(t) \end{array} \right. \tag{14}$$

### What kind of properties we should pay attention to?

When $\rho$ is fixed (constant) the previous system is LTI and

- controllability, observability, stability, are uniquely defined
- controllability $\Leftrightarrow$ reachabillity, observability $\Leftrightarrow$ reconstructibility
- these properties are equivalent by a state change of basis.

### But when $\rho(t)$ is time varying .....

- these facts may not be true (asymptotic and exponential stabilty may differ)
- need to study properies of Linear Time-Varying systems.
- A generalization of the $\exp(At)$ is needed, defining the state transition matrix $\Phi(t, t_0, \rho(t))$
- For a change of basis $T(t)$ with $x(t) = T(t)x_{new}(t)$ then, $\dot{x}(t) = \dot{T}(t)x_{new}(t) + T(t)\dot{x}_{new}(t)$

# Illustration for observability

In an analogous way the unobservability property is defined as : a state $x(t)$ is not observable if the corresponding output vanishes, i.e. if the following holds: $y(t) = \dot{y}(t) = \ddot{y}(t) = \ldots = 0$. In the case of LTV systems it corresponds to:

### Definition

*The LPV system* (14) *is completely observable if* $rank\mathscr{O} = n \; \forall t$*, where*

$$\mathscr{O} = \begin{bmatrix} o_1^T & o_2^T & \ldots & o_n^T \end{bmatrix}^{\mathscr{T}}$$

*where* $o_1 = C(\rho)$ *and* $o_{i+1} = o_i A + \dot{o}_i,\; i > 1$ *(for instance* $o_2 = \dot{\rho}\frac{\partial C(\rho)}{\partial \rho} + C(\rho)A(\rho)$*).*

A weaker notion of observability can be defined for the LPV systems (14) in the functional sense $\mathscr{O}$ function of $\rho(t)$.

### Definition

*The LPV system* (14) *is structurally observable if* $rank\mathscr{O} = n$

This does not guarantee that $\mathscr{O}$ is invertible $\forall t$ and for all parameter values.

Finally the above notion differ from the direct extension of the observability matrix for LTI systems, i.e $\mathscr{O} = \begin{bmatrix} C(\rho) \\ C(\rho)A(\rho) \\ \vdots \\ C(\rho)A^{n-1}(\rho) \end{bmatrix}$.

This definition is ONLY valid if $\rho$ is constant, i.e. it corresponds to the observability of the LTI systems frozen at the values of the constant parameter vector $\rho$.

# Example

$$\Sigma_1(\rho) : \begin{cases} \dot{x}(t) & = & A(\rho)x(t) \\ y(t) & = & C(\rho)x(t) \end{cases}$$

with

$$A = \begin{pmatrix} 1 & 1 \\ \rho(t) & 2 \end{pmatrix}, C = \begin{pmatrix} \rho(t) & 1 \end{pmatrix}$$

Observability matrix with $\rho_f$ is a frozen value of $\rho(t)$:

$$\mathscr{O} = \begin{pmatrix} \rho_f & 1 \\ 2\rho_f & \rho_f + 2 \end{pmatrix}$$

which is of rank 2 apart for $\rho_f = 0$. Therefore the LTI frozen systems are observable.

However the observaility matrix of the considered time-varying system is given by:

$$\mathscr{O} = \begin{pmatrix} \rho(t) & 1 \\ \dot{\rho}(t) + 2\rho(t) & \rho(t) + 2 \end{pmatrix}$$

which is of rank 2 in the functional sense. Therefore the structural rank of $\Sigma_1(\rho)$ is 2. However it is of rank 1 if $\rho$ satisfies $\dot{\rho}(t) = \rho(t)^2$. The system is then not completely observable.
Therefore, for some specific parameter definitions, the parameter variations may therefore induce a loss of observability.

# Outline

# Problem statement and facts

### Recall

For LTI systems all notions of stability are equivalent: global/local, asymptotic/exponential, time-domain (Lyapunov)/frequency-domain (Bode, poles...).

### Why stability analysis fo LPV systems is not an easy task?

Let consider $\dot{x} = A(\rho(t))x$. Stability analysis is more involved (as for LTV systems) since:

- there is a set of solutions for a given $x_0$ (family of systems from $\rho$ variations)
- the system may be stable for frozen parameter values and unstable for varying parameters (as for switching systems)
- asymptotic and exponential stability are no more equivalent and cannot be characterized by the eigenvalues of $A(\rho(t))$.
- In term of design, we will often rely on the notion of quadratic stability (using quadratic Lyapunov function $V(x) = x^T PX$) which is stronger but easier to check for stability and simpler to use for control and observer design, see (Wu, PhD 95)

### Robust or LPV? (Blanchini,00 & 07)

- Robust analysis and control: dedicated to LTI systems subject to time-varying uncertainties
- LPV (or gain-scheduling) analysis and control: dedicated to LTV systems or to linearizations of non linear systems along the trajectory of $\rho$

# Recall: robust stability with time-invariant uncertainties

This concept is very useful for the stability analysis of uncertain systems.
Let us consider an uncertain system

$$\dot{x} = A(\delta)x$$

where $\delta$ is an parameter vector that belongs to an uncertainty set $\Delta$.

### Problem statement

Is the system asymptotically stable for all $\delta$ in $\Delta$?

### Definition

*The considered system is said to be quadratically stable for all uncertainties $\delta \in \Delta$ if there exists a (single) Lyapunov function $V(x) = x^T \boldsymbol{P} x$ with $\boldsymbol{P} = \boldsymbol{P}^T > 0$ s.t*

$$A(\delta)^T \boldsymbol{P} + \boldsymbol{P} A(\delta) < 0, \textit{for all } \delta \in \Delta \tag{15}$$

### Computation

For polytopic uncertaities (convex set), i.e. if $\rho \in \mathbf{Co}\{\omega_1, \ldots, \omega_Z\}$, then, the problem becomes feasible since it remains to find $\mathbf{P} = \mathbf{P}^T > 0$ such that:

$$A(\omega_i)^T \mathbf{P} + \mathbf{P} A(\omega_i) < 0, \quad i = 1, \ldots, Z$$

# Quadratic stability for time-varying parameters

Let us consider the LPV system

$$\dot{x} = A(\rho(t))x$$

where $\rho(t)$ is an time-varying parameter vector that belongs to an uncertainty set $\Omega$.

### Use of a single Lyapunov function

If there exists $\mathbf{P} = \mathbf{P}^T > 0$ such that:

$$A(\rho(t))^T \mathbf{P} + \mathbf{P} A(\rho(t)) < 0, \forall \rho(t) \in \Omega$$

then the system is stable for arbitrarily fast time-varying uncertainties

### Remarks

- Quadractic stability imples exponential stability (Wu, 95)
- It is an infinite dimension problem (can be relaxed for polytopic uncertainties)
- It could be conservative since stability is checked for any variation of the parameters !

Pay attentation in what follows: LPV system means TIME-VARYING parameters so a polytopic LPV system is not an uncertain polytopic system (in the latter case the coefficient $\alpha_i$ of the polytopic description are constant even if unknown)

# Parameter Varying Lyapunov functions

Let consider now a parameter dependent Lyapunov function $V_\rho(x(t)) = x(t)^T P(\rho) x(t) > 0$ for every $x \neq 0$ and $V(0) = 0$.

## Uncertain systems ($\rho$ is time-invariant)

The uncertain system $\dot{x} = A(\rho)x$ is exponentially stable if there exists $V_\rho$ such that (classical approach for polytopic uncertain systems):

$$A(\rho)^T P(\rho) + P(\rho)A(\rho) < 0, \; \forall \; \rho \in \blacksquare$$

## LPV systems ($\rho$ is time-varying)

The uncertain system $\dot{x} = A(\rho(t))x$ is exponentially stable if there exists $V_\rho$ such that:

$$A(\rho)^T P(\rho) + P(\rho)A(\rho) + \sum_{i=1}^{N} \dot{\rho}_i \frac{\partial P(\rho)}{\partial \rho_i} < 0 \; \forall \; \rho(t) \in \blacksquare$$

which, in addition to bounded parameters, needs to consider rate-bounded parameter variations. Such a condition is more complex since:

- It involves the partial differentiation of $P$
- it has to be checked for all $\rho(t) \in \blacksquare$
- It implies to choose a parametrization of $P(\rho)$: from affine to polynomial

# $\mathscr{L}_2$ stability of LPV systems (Wu, 95)

## Definition

*Given a parametrically dependent stable LPV system $\Sigma_\rho = (A(\rho), B(\rho), C(\rho), D(\rho))$ for zero initial conditions $x_0$. The induced $\mathscr{L}_2$ norm is defined as:*

$$||\Sigma_\rho||_{i,2} = \sup_{\rho(t)\in\Omega} \sup_{w(t)\neq 0 \in \mathscr{L}_2} \frac{\|y\|_2}{\|u\|_2}$$

*which is often referred to as (by abuse of langage) the $H_\infty$ gain $||\Sigma_\rho||_\infty$ of the LPV system.*

## Theorem

*A sufficient condition for the $\mathscr{L}_2$ stability of system $\Sigma_\rho$ is the generalized BRL, using parameter dependent Lyapunov functions, i.e assuming $|\dot{\rho}_i| < \nu_i,\ \forall i$, if there exists $P(\rho) > 0,\ \forall \rho$ s.t*

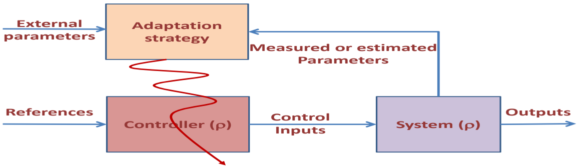
$$\begin{bmatrix} A(\rho)^T P(\rho) + P(\rho)A(\rho) + \sum_{i=1}^N \nu_i \frac{\partial P(\rho)}{\partial \rho_i} & P(\rho)\ B(\rho) & C(\rho)^T \\ B(\rho)^T\ P(\rho) & -\gamma I & D(\rho)^T \\ C(\rho) & D(\rho) & -\gamma I \end{bmatrix} < 0, \qquad \forall i. \qquad (16)$$

*then $||\Sigma_\rho||_{i,2} \leq \gamma$*

# Outline

# Towards LPV control

## The "gain scheduling" approach




## Some references

- Modelling, identification : (Bruzelius, Bamieh, Lovera, Toth)
- Control (Shamma, Apkarian & Gahinet, Adams, Packard, Beker ...)
- Stability, stabilization (Scherer, Wu, Blanchini ...)
- Geometric analysis (Bokor & Balas)

# The $H_\infty/LPV$ control problem

### Definition

Find a LPV controller $C(\rho)$ s.t the closed-loop system is stable and for $\gamma_\infty > 0$, $sup \frac{\|z\|_2}{\|w\|_2} < \gamma_\infty$,

- Unbounded set of LMIs (Linear Matrix Inequalities) to be solved ($\rho \in \Omega$)
- Some approaches: polytopic, LFT, gridding. See Arzelier [HDR, 2005], Bruzelius [Thesis, 2004], Apkarian et al. [TAC, 1995]...

### A solution: *The "polytopic" approach* [C. Scherer et al. 1997]

- Problem solved off line for each vertex of a polytope (convex optimisation) (using here a single Lyapunov function i.e. quadratic stabilization).
- On-line the controller is computed as the convex combination of local linear controllers

$$C(\rho) = \sum_{k=1}^{2^N} \alpha_k(\rho) \begin{bmatrix} A_c(\omega_k) & B_c(\omega_k) \\ C_c(\omega_k) & D_c(\omega_k) \end{bmatrix}, \sum_{k=1}^{2^N} \alpha_k(\rho) = 1 \, , \, \alpha_k(\rho) > 0$$




- Easy implementation !!

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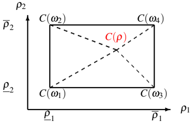
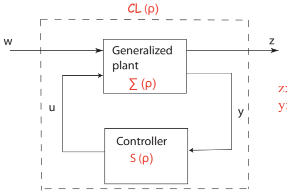



- Easy implementation !!

# LPV control design

w: exagenous input
u: control input




z: output to minimize
y: measurement

Problem on standard form

Dynamical LPV generalized plant:

$$\Sigma(\rho):\begin{bmatrix}\dot{x}\\ \hline z\\ y\end{bmatrix}=\left[\begin{array}{c|cc}A(\rho) & B_1(\rho) & B_2(\rho)\\ \hline C_1(\rho) & D_{11}(\rho) & D_{12}(\rho)\\ C_2(\rho) & D_{21}(\rho) & D_{22}(\rho)\end{array}\right]\begin{bmatrix}x\\ \hline w\\ u\end{bmatrix} \tag{17}$$

LPV controller structure:

$$S(\rho):\begin{bmatrix}\dot{x}_c\\ u\end{bmatrix}=\left[\begin{array}{c|c}A_c(\rho) & B_c(\rho)\\ \hline C_c(\rho) & D_c(\rho)\end{array}\right]\begin{bmatrix}x_c\\ y\end{bmatrix} \tag{18}$$

LPV closed-loop system:

$$\mathscr{CL}(\rho):\begin{bmatrix}\dot{\xi}\\ z\end{bmatrix}=\left[\begin{array}{c|c}\mathscr{A}(\rho) & \mathscr{B}(\rho)\\ \hline \mathscr{C}(\rho) & \mathscr{D}(\rho)\end{array}\right]\begin{bmatrix}\xi\\ w\end{bmatrix} \tag{19}$$

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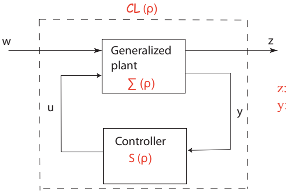


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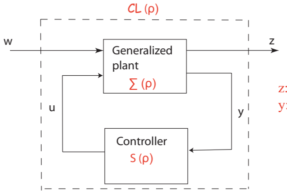


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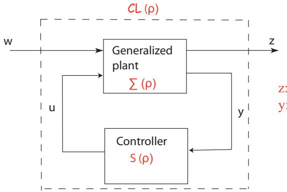


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# LPV control design

$\mathscr{H}_\infty$ criteria Apkarian et al. [TAC, 1995]

Stabilize system $CL(\rho)$ (find $K>0$) while minimizing $\gamma_\infty$.

$$\begin{bmatrix} \mathscr{A}(\rho)^T K + K\mathscr{A}(\rho) & K\mathscr{B}_\infty(\rho) & \mathscr{C}_\infty(\rho)^T \\ \mathscr{B}_\infty(\rho)^T K & -\gamma_\infty^2 I & \mathscr{D}_\infty(\rho)^T \\ \mathscr{C}_\infty(\rho) & \mathscr{D}_\infty(\rho) & -I \end{bmatrix} < 0$$

Infinite set of LMIs to solve ($\rho \in \Omega$) ($\Omega$ is convex)

LPV control designs Arzelier [HDR, 2005], Bruzelius [Thesis, 2004]

LFT, Gridding, Polytopic

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# LPV control design

## Polytopic approach

Solve the LMIs at each vertex of the polytope formed by the extremum values of each varying parameter, with a common $K$ Lyapunov function.

$$C(\rho) = \sum_{k=1}^{2^N} \alpha_k(\rho) \begin{bmatrix} A_c(\omega_k) & B_c(\omega_k) \\ C_c(\omega_k) & D_c(\omega_k) \end{bmatrix}$$

where,

$$\alpha_k(\rho) = \frac{\prod_{j=1}^{N} |\rho_j - \mathscr{C}^c(\omega_k)_j|}{\prod_{j=1}^{N} (\overline{\rho}_j - \underline{\rho}_j)} \ ,$$

where $\mathscr{C}^c(\omega_k)_j = \{\overline{\rho}_j$ if $(\omega_k)_j = \underline{\rho}_j$ or $\underline{\rho}_j\}$ otherwise.

$$\sum_{k=1}^{2^N} \alpha_k(\rho) = 1 \ , \ \alpha_k(\rho) > 0$$

# LPV control design

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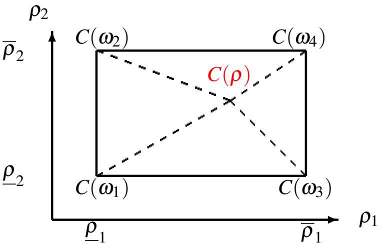

# LPV/$\mathscr{H}_\infty$ control synthesis

**Proposition - feasibility (brief)** Scherer et al. (1997)

Solve the following problem at each vertices of the parametrized points (illustration with 2 parameters):

$$\begin{aligned} \gamma^* = \min \quad & \gamma \\ & \text{s.t. (21) } |_{\underline{\rho_1},\underline{\rho_2}} \\ & \text{s.t. (21) } |_{\underline{\rho_1},\overline{\rho_2}} \\ & \text{s.t. (21) } |_{\overline{\rho_1},\underline{\rho_2}} \\ & \text{s.t. (21) } |_{\overline{\rho_1},\overline{\rho_2}} \end{aligned} \tag{20}$$

$$\begin{bmatrix} A\mathbf{X} + B_2\widetilde{\mathbf{C}}(\rho_1,\rho_2) + (\star)^T & (\star)^T & (\star)^T & (\star)^T \\ \widetilde{\mathbf{A}}(\rho_1,\rho_2) + A^T & \mathbf{Y}A + \widetilde{\mathbf{B}}(\rho_1,\rho_2)C_2 + (\star)^T & (\star)^T & (\star)^T \\ B_1^T & B_1^T\mathbf{Y} + D_{21}^T\widetilde{\mathbf{B}}(\rho_1,\rho_2)^T & -\gamma I & (\star)^T \\ C_1\mathbf{X} + D_{12}\widetilde{\mathbf{C}}(\rho_1,\rho_2) & C_1 & D_{11} & -\gamma I \end{bmatrix} \prec 0$$

$$\begin{bmatrix} \mathbf{X} & I \\ I & \mathbf{Y} \end{bmatrix} \succ 0 \tag{21}$$

# LPV/$\mathscr{H}_\infty$ control synthesis

**Proposition - reconstruction (brief)** Scherer et al. (1997)

Reconstruct the controllers as,

$$\text{solve} \quad \begin{array}{l} (23)\ |_{\underline{\rho_1},\underline{\rho_2}} \\ (23)\ |_{\underline{\rho_1},\overline{\rho_2}} \\ (23)\ |_{\overline{\rho_1},\underline{\rho_2}} \\ (23)\ |_{\overline{\rho_1},\overline{\rho_2}} \end{array} \tag{22}$$

$$\left\{ \begin{array}{rcl} C_c(\rho_1,\rho_2) & = & \widetilde{C}(\rho_1,\rho_2)M^{-T} \\ B_c(\rho_1,\rho_2) & = & N^{-1}\widetilde{B}(\rho_1,\rho_2) \\ A_c(\rho_1,\rho_2) & = & N^{-1}\big(\widetilde{A}(\rho_1,\rho_2) - YAX - NB_c(\rho_1,\rho_2)C_2X \\ & - & YB_2C_c(\rho_1,\rho_2)M^T\big)M^{-T} \end{array} \right. \tag{23}$$

where $M$ and $N$ are defined such that $MN^T = I - XY$ which may be chosen by applying a singular value decomposition and a Cholesky factorization.

# Definition LPV observers

### Definition

*Let consider the LPV system:*

$$\begin{array}{rcl} \dot{x}(t) & = & A(\rho)x(t)+B(\rho)u(t) \\ y(t) & = & C(\rho)x(t) \end{array} \tag{24}$$

*The following LPV state space representation*

$$\begin{array}{rcl} \dot{\hat{x}}(t) & = & A(\rho)\hat{x}(t)+B(\rho)u(t)+L(\rho)(y(t)-C(\rho)\hat{x}(t)) \\ \hat{x}_0 \textit{ to be defined} & & \end{array} \tag{25}$$

*is said to be an observer for* (24) *if*

$$\lim_{t\to\infty}(\hat{x}(t)-x(t)) \to 0 \quad \forall\rho(t)\in\Omega$$

*where* $\hat{x}(t)\in\mathbb{R}^n$ *is the estimated state of* $x(t)$ *and* $L(\rho)$ *is the* $n\times p$ *observer gain matrix to be designed.*

# Some issues for LPV observer design

The estimated error, $e(t) := x(t) - \hat{x}(t)$, satisfies:

$$\dot{e}(t) \quad = \quad (A - LC)(\rho)e(t) \tag{26}$$

The two main problems to be handle are then

- What observability property shall we consider?
- What parameter dependency should we define for $L(\rho)$?

### Quadractic detectability (Wu, 95)

A simple solution is to consider a single Lyapunov function in order to guarantte the quadratic detectability, i.e:

$$(A(\rho) - L(\rho)C(\rho))^T \ \mathbf{P} + \mathbf{P} \ (A(\rho) - L(\rho)C(\rho)) < 0$$

*Some remarks:*

- The previous problem can be solved using a polytopic approach only if $C(\rho) = C$, a constant matrix
- If this is not solvable, one can try using Parameter dependent Lyapunov functions, but the coupling between $L(\rho)$ and $\mathbf{P}(\rho)$ will lead to soved non affine LMIs (a polynomial or a gridding approach is then needed).

# Some issues for LPV observer design (2)

On key issue in observer implementation concerns the knownledge of $\rho(t)$. While previously the result is valid if $\rho(t)$ is perfectly known, such a following observer description must be used if $\rho(t)$ is estimated:

$$\dot{\hat{x}}(t) \quad = \quad A(\hat{\rho})\hat{x}(t) + B(\hat{\rho})u(t) + L(\hat{\rho})(y(t) - C(\hat{\rho})\hat{x}(t)) \tag{27}$$

Denoting $\Delta A = A(\rho) - = A(\hat{\rho})$, $\Delta B = B(\rho) - B(\hat{\rho})$, $\Delta C = C(\rho) - C(\hat{\rho})$, and $\Delta L = L(\rho) - L(\hat{\rho})$, this leads for the estimation error equation:

$$\dot{e}(t) \quad = \quad (A - LC)(\hat{\rho})e(t) + (\Delta A + L(\hat{\rho}).\Delta C)x + \Delta Bu(t) \tag{28}$$

If $C(\rho) = C$ and $B(\rho)$ are constant matrices, then we get the uncertain estimated error system

$$\boxed{\dot{e}(t) = (A(\hat{\rho}) - L(\hat{\rho})C)e(t) + \Delta Ax(t)} \tag{29}$$

The stability analysis is indeed more involved due to the state vector $x$ (see (Daafouz et al, 2010) for the discrete-time case). Either $\Delta Ax(t)$ should be considered as a disturbance, or a state augmentation approach is to be used (which has to be done in closed-loop control).

### Observer-based control

For control design in the latter case, the following state feedback should be used:

$$u(t) = -F(\hat{\rho})\hat{x}(t)$$

# Outline

# Interest of the LPV approach

LPV is a key tool to the control of complex systems.

***Some examples :***

Modelling of complex systems (non linear)

- Use of a quasi-LPV representation to include non linearities in a linear state space model (even delays)
- Transformation of constraints (e.g. saturation) into an 'external' parameter
- Modelling of LTV, hybrid (e.g. switching control)

***BUT :***

A q-LPV system is not equivalent to the non linear one:

- stability: $\rho = \rho(x(t), t)$ is assumed to be bounded... so are the state trajectories
- controllability: some non controllable modes of a non linear system may vanish according to the LPV representation
- observability: unobservability may occur for some specific parameter variations

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# Interest of the LPV approach

**Some of works using LPV approaches - former PhD students**

## Gain-scheduled control

- Account for various operating conditions using a variable "equilibrium point": (Gauthier 2007)
- Control with real-time performance adaptation using parameter dependent weighting functions from endogenous or exogenous parameters (Poussot 2008, Do 2011)
- Analysis and control of LPV Time-Delay Systems: delay-scheduled control Briat 2008
- Control under computation constraints: $H_\infty$ variable sampling rate controller with sampling dependent performances (Robert 2007, Roche 2011, Robert et al., IEEE TCST 2010))

## Coordination of several actuators for MIMO systems

- An LPV structure for control allocation Poussot et al. (CEP 2011)
- Selection of a specific parameter for the control activation (of each actuator) Poussot et al. (VSD 2011), Doumiati et al (EJC 2013), Fergani et al (IEEE TVT 2015)

## Incorporate fault-(diagnosis, accomodation, tolerant control) properties

- LPV fault-scheduling control: see Sename et al (Systol 2013, ICSTCC 2015).

# Some *Grenoble* PhD students on LPV control

- Manh Quan Nguyen, "LPV approaches for modelling and control of vehicle dynamics: application to a small car pilot plant with ER dampers", PhD GIPSA-lab, Univertisté Grenoble Alpes, 2016.
- Waleed Nwesaty, " LPV/$H_\infty$ control design of on-board energy management systems for electrical vehicles", PhD GIPSA-lab, Univertisté Grenoble Alpes, 2015.
- Soheib Fergani, "Robust LPV/$H_\infty$ MIMO control for vehicle dynamics, PhD GIPSA-lab, Univertisté Grenoble Alpes, 2014.
- Maria Rivas, "Modeling and Control of a Spark Ignited Engine for Euro 6 European Normative", PhD, GIPSA-lab / *RENAULT*, Grenoble INP, 2012.
- Ahn-Lam Do, "LPV Approach for Semi-active Suspension Control & Joint Improvement of Comfort and Security", PhD, GIPSA-lab, Grenoble INP, 2011.
- David Hernandez, "Robust control of hybrid electro-chemical generators", PhD, GIPSA-lab / G2Elab, Grenoble INP, 2011.
- Emilie Roche, "Commande Linéaire à Paramètres Variants discrète à échantillonnage variable : application à un sous-marin autonome", PhD, GIPSA-lab, Grenoble INP, 2011.
- Sébastien Aubouet, "Semi-active SOBEN suspensions modelling and control", PhD, GIPSA-lab / *SOBEN*, INP Grenoble, 2010.
- Charles Poussot-Vassal, "Robust LPV Multivariable Global Chassis Control", PhD , GIPSA-lab, INP Grenoble, 2008.
- Corentin Briat, "Robust control and observation of LPV time-delay systems", PhD, GIPSA-lab, INP Grenoble, 2008.
- Christophe Gauthier, "Commande multivariable de la pression d'injection dans un moteur Diesel Common Rail", PhD, LAG / *DELPHI*, Grenoble INP, 2007.
- David Robert, "Contribution à l'interaction commande/ordonnacement", PhD, LAG, Grenoble INP, 2007.
- Alessandro ZIN, "Sur la commande robuste de suspensions automobiles en vue du contrôle global de châssis", PhD, LAG / Grenoble INP, 2005.

# Some references


- W. Nwesaty, A. Iuliana Bratcu and O. Sename, "Power sources coordination through multivariable Linear Parameter-Varying/ $H_{infty}$ control with application to multi-source electric vehicles," in *IET Control Theory & Applications*, vol. 10, no. 16, pp. 2049-2059, 10 31 2016.
- S. Fergani, O. Sename and L. Dugard, "An LPV/$\mathscr{H}_\infty$ Integrated Vehicle Dynamic Controller," in *IEEE Transactions on Vehicular Technology*, vol. 65, no. 4, pp. 1880-1889, April 2016.
- O. Sename, "The LPV approach: the key to controlling vehicle dynamics ?", pleanry talk at *19th International Conference on System Theory, Control and Computing (ICSTCC)*, Cheile Gradistei, Romania, 2015.
- Doumiati, M., Sename, O., Dugard, L., Martinez-Molina, J.-J., Gaspar, P., Szabo, Z., "Integrated vehicle dynamics control via coordination of active front steering and rear braking", (2013) *European Journal of Control*, 19 (2), pp. 121-143.;
*Among the 3 Most Cited Articles published since 2011, extracted from Scopus.*
- O. Sename, J. C. Tudon-Martinez and S. Fergani, "LPV methods for fault-tolerant vehicle dynamic control," plenary paper, *2013 Conference on Control and Fault-Tolerant Systems (SysTol)*, Nice, 2013, pp. 116-130.
- C. Poussot-Vassal, O. Sename, L. Dugard, P. Gaspar, Z. Szabo & J. Bokor, "Attitude and Handling Improvements Through Gain-scheduled Suspensions and Brakes Control", *Control Engineering Practice (CEP)*, Vol. 19(3), March, 2011, pp. 252-263.
- C. Poussot-Vassal, O. Sename, L. Dugard, S.M. Savaresi, "Vehicle Dynamic Stability Improvements Through Gain-Scheduled Steering and Braking Control", *Vehicle System Dynamics (VSD)*, vol 49, Nb 10, pp 1597-1621, 2011
- Briat, C.; Sename, O., "Design of LPV observers for LPV time-delay systems: an algebraic approach", *International Journal of Control*, vol 84, nb 9, pp 1533-1542, 2011
- Robert, D., Sename, O., and Simon, D. (2010). An $H_\infty$ LPV design for sampling varying controllers: experimentation with a T inverted pendulum. *IEEE Transactions on Control Systems Technology*, 18(3):741–749.
- C. Briat, O. Sename, and J.-F. Lafay, ""Memory-resilient gain-scheduled state-feedback control of uncertain LTI/LPV systems with time-varying delays" *Systems & Control Let.*, vol. 59, pp. 451-459, 2010.
- C. Briat, O. Sename, and J. Lafay, "Hinf delay-scheduled control of linear systems with time-varying delays," *IEEE Transactions in Automatic Control*, vol. 42, no. 8, pp. 2255-2260, 2009.
- Poussot-Vassal, C.; Sename, O.; Dugard, L.; Gaspar, P.; Szabo, Z. & Bokor, J. "A New Semi-active Suspension Control Strategy Through LPV Technique", *Control Engineering Practice*, 2008, 16, 1519-1534
- Zin A., Sename O., Gaspar P., Dugard L., Bokor J. "Robust LPV - Hinf Control for Active Suspensions with Performance Adaptation in view of Global Chassis Control", *Vehicle System Dynamics*, Vol. 46, No. 10, 889-912, October 2008
- Gauthier C., Sename O., Dugard L., Meissonnier G. "An LFT approach to Hinf control design for diesel engine common rail injection system", *Oil & Gas Science and Technology*, vol 62, nb 4, pp. 513-522 (2007)