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Abstract - *Delivery lead-time has become a factor of competitiveness for companies and an important criterion of purchase for the customers today. Thus, in order to increase their profit, companies must not focus only on price but also need to quote the right delivery lead time to their customers. Some authors to find a way in quoting the right delivery lead-time while considering an M/M/1 system. In M/M/1, all customers are accepted. This can lead to longer lead times in the queue. Firms can react by quoting longer lead times in order to cope with this situation. However, this leads to lower demand and revenue. Starting from this observation, we investigate in this paper whether a customer rejection policy can be more beneficial for the firm than an all-customers' acceptance policy. Indeed, our idea is based on the fact that rejecting some customers might help to quote shorter lead time for the accepted customers, which might lead to higher demand and profit. We model this rejection policy based on an M/M/1/K system. We analytically determine the optimal firm's policy (optimal price and quoted lead time) in case of M/M/1/1 system. Then, we compare the optimal firm's profit under M/M/1/1 with the optimal profit obtained by M/M/1. Two situations are considered: a system without holding and penalty costs and a system where these costs are included.*

Keywords: *Lead-time quotation, Pricing, M/M/1/K, M/M/1.*

1. Introduction

The delivery lead-time, which represents the elapsed time between the placement of the order by the customer and the receipt of this order, has become a factor of competitiveness for companies and an important purchase criterion for many customers. Geary and Zonnenberg (2000) reported that top performers among 110 organizations conducted initiatives not only to reduce costs and maintain reliability, but also to improve delivery speed and flexibility. Baker et al. (2001) found that less than 10% of end consumers and less than 30% of corporate customers base their purchasing decisions on price only; for a substantial majority of purchasers both price and delivery lead time are crucial factors that determine their purchase decisions. Thus, in order to increase their profit, companies must not focus only on price but also need to quote the right delivery lead time to their customers. A short quoted lead time can lead to higher demand but can also result in late delivery, which affects the firm's reputation for on-time delivery and deters future customers (Slotnick, 2014). In addition, companies risk to lose markets if they are not capable of respecting the quoted delivery lead-times (Kapuscinski and Tayur, 2007). A long quoted lead time can reduce the risk of late delivery but leads to lower demand. This raises the following relevant question: What is the best lead time that must be quoted by a company when customers are not only sensitive to price but also to lead time? Some authors tried to answer this question while considering an M/M/1 system (Palaka at al., 1998, So and Song, 1998, and Pekgün et al., 2008). As stated by Gross et al. (2008); Kleinrock (1975); Thomopoulos (2012), one of the characteristics of M/M/1 is the infinite system capacity. Thus, the M/M/1 accepts all customers, which can lead to longer lead times in the queue. In order to cope with this situation, firms can react by quoting longer lead times in order to maintain the desired service level. However, this leads to lower demand and revenue. Starting from this observation, we investigate in this paper whether a customer rejection policy can be more beneficial for the firm than an all-customers acceptance policy. Indeed, our idea is based on the fact that rejecting some customers might help to quote shorter lead time for the accepted customers, which might lead to higher demand and profit. We model this rejection policy based on an M/M/1/K system. We analytically determine the optimal firm's policy (optimal price and quoted lead time) in case of M/M/1/1 system. Then, we compare the optimal firm's profit under M/M/1/1 with the optimal profit obtained by M/M/1. Two situations are considered: a system without holding and penalty costs and a system where these costs are included.

The rest of this paper is organized as follows. A literature review on M/M/1 systems with leadtime-dependent demand is presented in the next section. Then, we develop in section 3 the formulation of the M/M/1/K system with price-and leadtime-sensitive demand. In Section 4, we analytically solve the M/M/1/K system for K=1 without holding and penalty cost and compare the results to those obtained with M/M/1. We dedicate section 5 to the case with holding and penalty cost. We finally conclude and give future work directions.

2. Literature Review

In the past decades, a considerable number of researchers in economics and operations management have studied: price-, rebate-, space-, quality, and advertising-dependent demand. Huang et al. (2013) suggested that there may exist further research opportunities for using lead time–dependent demand as they had found only few publications belonging to lead time–dependent categories.

A highlighted by Huang et al. (2013), the M/M/1 model is widely used in the literature to incorporate a price- and lead-time sensitive demand in a make-to-order system. In what follows, we review such models.

Palaka et al. (1998) studied the lead-time setting, pricing decisions, and capacity utilization of a profit maximizing firm that faces a linear price- and leadtime-sensitive demand. Costs related to congestion (holding cost) and late deliveries (penalty cost) are considered in Palaka et al.'s model. So and Song (1998) developed an analytical framework for a firm to understand the strong interrelationships among pricing, delivery time guarantees, demand, and the overall profitability of offering the services. The authors used a log linear function to model the demand as a function of price, delivery time, and delivery reliability level. Pekgün et al. (2008) studied centralization and decentralization of pricing and lead-time decisions of a Make-To-Order (MTO) firm, while using the same setting of Palaka et al. (1998) for their decentralized model but without holding and penalty cost.

There are some other research that used the M/M/1 to model their MTO system in order to model the lead-time- and price-dependent demand (Ho and Zheng, 2004; Liu et al., 2007; Ray and Jewkes, 2004; Zhao et al., 2012). Ray and Jewkes(2004) conduct a research about costumer lead-time management with demand and price sensitive to lead-time. In their research, the price is sensitive to lead-time. Demand is modeled as linear function, so is the price sensitive lead-time. Ho and Zheng (2004), they model the demand in MNL model. They conduct research in single firm and competitive multi-firm. Ho and Zheng (2004) discuss about the competitive market that the demand sensitive to lead-time. They use game theory approach to show how a firms should react to the market and another firm's lead-time strategy. The demand is modeled as costumer utility (satisfactory). Liu et al. (2007) conduct research about the decentralized supply chain in single firm. They mainly use Stackelberg game with supplier and retailer to quote the lead time and price. Zhao et al. (2012) discuss about the lead time and pricing decision for two types of costumer: lead-time sensitive or price sensitive. They use two policies which are uniform or differentiated model. Zhao et al. (2012) model the demand in the willingness-to-pay model for single firm problem.

3. Proposed Model (The M/M/1/K)

We consider a make-to-order firm operating under the following M/M/1/K setting. Customers are served in first-come, first-served basis (FCFS). The arrival process is assumed to be Poisson process. The processing time of customers in the system is assumed to be exponentially distributed. Contrary to the assumptions of M/M/1 model where all customers are accepted (as in Palaka et al., 1998 and Pekgun et al., 2008), we reject clients when there is already K clients in the system. The capacity is assumed to be constant while price, quoted lead-time and demand are decision variables.

Similarly to Liu et al. (2007); Palaka et al. (1998); and Pekgün et al. (2008), the demand is assumed to be a linear decreasing function in price and quoted lead-time.

$$\Lambda(p,l) = a - b_1 p - b_2 l, \tag{1}$$

where:

p = price of the good/service set by the firm,
l = quoted lead-time,
$\Lambda(p,l)$ = expected demand for the good/service with price p and quoted lead-time l,
a = market potential,
b_1 = price sensitivity of demand,
b_2 = lead-time sensitivity of demand,

Since the demand is downward sloping in both price and quoted lead-time, b_1 and b_2 are restricted to be non-negative

According to Palaka et al. (1998), this linear demand function is tractable and has several desirable properties. For instance, with such a linear demand, the price elasticity is increasing in both price and quoted lead-time. Customers would be more sensitive to long lead-times when they are paying more for the goods or service. Similarly, customers would be more sensitive to high prices when they also have longer waiting times.

In order to prevent the firms from quoting unrealistically short lead-times, we assume that the firm maintains a certain minimum service level. The service level is defined as the probability of meeting the quoted lead-time ($P(W \leq l) \geq s$).

Since we assume an M/M/l/K queueing system with mean service rate, μ, mean arrival rate (or, demand), λ, and throughput rate (effective demand), $\bar{\lambda}$,the expected number of customers in the system is given by L_s (see eq. (2)), and the actual lead-time (time in the system) is exponentially distributed with mean $L_s / \bar{\lambda}$ (see eq. (3)). The probability that the firm is able to meet the quoted lead-time l ($P(W \leq l)$) and the probability that a job is late ($P(W > l)$) are given in eq. (4). Equations (2) and (3) are based on Gross et al. (2008) and equation (4) is based on Sztrik (2011).

$$L_s = \frac{\rho}{1-\rho} - \frac{(K+1)\rho^{K+1}}{1-\rho^{K+1}} \text{ with } \rho = \frac{\lambda}{\mu} \tag{2}$$

$$W = \frac{L_s}{\bar{\lambda}} \text{ with } \bar{\lambda} = \lambda(1-P_K) \text{ and } P_K = \frac{1-\rho}{1-\rho^{K+1}}\rho^K \text{ if } \rho \neq 1 \text{ or } P_K = \frac{1}{K+1} \text{ if } \rho = 1 \tag{3}$$

$$P(W \leq l) = 1 - \sum_{k=0}^{K-1}\left(\sum_{i=0}^{k}\frac{(\mu \cdot l)^i}{i!}e^{-\mu \cdot l}\right)\frac{P_k}{1-P_K} \text{ and } P(W > l) = \sum_{k=0}^{K-1}\left(\sum_{i=0}^{k}\frac{(\mu \cdot l)^i}{i!}e^{-\mu \cdot l}\right)\frac{P_k}{1-P_K} \tag{4}$$

The objective of the firm is to maximize its revenue, which includes the following three main parts:

(1) **Expected revenue (net of direct costs)** is represented by $\bar{\lambda}(p-m)$, where m is the unit direct variable cost.
(2) **Total Congestion costs** is expressed as the mean number of jobs in the system multiplied by the unit holding cost ($L_s \times F$). This cost typically represents the in-process inventory holding cost.
(3) **Total Lateness penalty cost** is expressed as (penalty per job per unit lateness) × (number of overdue clients) × (expected lateness given that a job is late). The number of overdue clients is equal to: (throughput rate) × (probability that a job is late). The penalty cost per job per unit lateness (denoted by c) reflects the direct compensation paid to customers for not meeting the quoted lead-time. Mathematically, this total Lateness penalty cost is given by $\left(c \times \bar{\lambda} \times P(W \geq l) \times W\right)$.

Thus, the firm's optimization problem can be modeled as follows:

(P0) $$\underset{l,p,\lambda}{\text{Maximize}} \quad \bar{\lambda}(p-m) - (L_s \times F) - \left(c \times \bar{\lambda} \times P(W \geq l) \times W\right) \tag{5}$$

$$\text{Subject to} \quad \lambda \leq a - b_1 p - b_2 l \tag{6}$$

$$1-\sum_{k=0}^{K-1}\left(\sum_{i=0}^{k}\frac{(\mu\cdot l)^i}{i!}e^{-\mu\cdot l}\right)\frac{P_k}{1-P_K}\geq s \tag{7}$$

$$\rho=\frac{\lambda}{\mu} \tag{8}$$

$$P_K=\frac{1-\rho}{1-\rho^{K+1}}\rho^K \text{ if } \rho\neq 1 \text{ and } P_K=\frac{1}{K+1} \text{ if } \rho=1 \tag{9}$$

$$\bar{\lambda}=\lambda(1-P_K) \tag{10}$$

$$\lambda,p,l,\bar{\lambda}\geq 0 \tag{11}$$

where,

Decision Variables			Parameters		
p	=	price of the good/service set by the firm,	a	=	market potential,
l	=	quoted lead-time,	b_1	=	price sensitivity of demand,
λ	=	mean arrival rate (demand),	b_2	=	lead-time sensitivity of demand,
			μ	=	mean service rate (production capacity),
			m	=	unit direct variable cost,
			s	=	service level defined by company,
			F	=	unit holding cost
			c	=	penalty cost per job per unit lateness
			P_K	=	probability of rejected customer,
			K	=	system capacity.

In this formulation, constraint (6) imposes that the mean demand (λ) cannot be greater than the demand obtained with price (p) and quoted lead-time (l). Constraint (7) expresses the service level constraint. Constraint (9) calculates the probability of rejecting customer. Constraint (10) gives the number of customers that are served and exit the system. Constraint (11) is the non-negativity constraint.

Solving the general problem analytically seems difficult. Hence, we focus in this paper on the particular case of K=1. In section 4, we start by studying the system without penalty and holding costs. Then, in section 5, we investigate whether the (M/M/1/1) model can give a higher profit than the widely considered M/M/1 model. In section 6, we solve the (M/M/1/1) model while considering the penalty and holding costs. We dedicate section 7 to the comparison between this model and the (M/M/1).

4. M/M/1/1 without Penalty and Holding Costs

In this section, we solve the system with K = 1 and without holding and penalty costs. In this case, the objective function consists only in maximizing the expected revenue. With K=1, the Probability $P(W>l)$ becomes equal to $e^{-\mu l}$. Hence, the service level constraint can be written as $1-e^{-\mu l}\geq s$. The formulation of the problem becomes:

(P1) $$\underset{\lambda,l,p,\bar{\lambda}}{\text{Maximize}}\quad \bar{\lambda}(p-m) \tag{12}$$

Subject to $$\lambda\leq a-b_1p-b_2l \tag{13}$$

$$1-e^{-\mu l}\geq s \tag{14}$$

$$\rho=\frac{\lambda}{\mu} \tag{15}$$

$$P_1=\frac{\rho}{1+\rho} \tag{16}$$

$$\bar{\lambda} = \lambda(1 - P_1) \tag{17}$$

$$\lambda, p, l, \bar{\lambda} \geq 0 \tag{18}$$

Eq. (14) can be rewritten as $\mu l \geq \ln(1/(1-s))$. Then, by integrating the equality constraints (eq. (15-18)) into the objective function, we can simplify the formulation as:

$$\text{(P1') } \underset{\lambda, l, p}{\text{Maximize}} \quad \left(\frac{\lambda\mu}{\mu+\lambda}\right) \times (p - m) \tag{19}$$

$$\text{Subject to } \lambda \leq a - b_1 p - b_2 l \tag{20}$$

$$\mu l \geq \ln(1/(1-s)) \tag{21}$$

$$\lambda, p, l \geq 0 \tag{22}$$

Using the new formulation (P1'), we firstly transform the problem into a single variable optimization model.

Suppose that the optimal solution is given by price, p^*, quoted lead-time l^*, and demand rate λ^*, and that $\lambda^* < \Lambda(p^*,1^*.)$. Since the revenues are increasing in p, one could increase the price to p' (while holding the demand rate and quoted lead-time constant) until $\lambda^* = \Lambda(p',1')$. This change will increase revenues without increasing costs. Therefore, (p^*, l^*, and λ^*) cannot be an optimal solution. Hence, the demand constraint eq. (20) is binding at optimality in our new problem. Thus:

$$\lambda = a - b_1 p - b_2 l \Leftrightarrow p = \frac{a - b_2 l - \lambda}{b_1} \tag{23}$$

To analyze the service level constraint, we applying the Lagrangian multiplier method:

$$L(\lambda, l, \gamma) = \left(\frac{\lambda\mu}{\mu+\lambda}\right) \times (p - m) + \gamma\left\{\mu l - \ln\left(\frac{1}{1-s}\right)\right\} \text{ with } p = \frac{a - b_2 l - \lambda}{b_1}$$

$$\Leftrightarrow L(\lambda, l, \gamma) = \left(\frac{\lambda\mu}{\mu+\lambda}\right) \times \left(\frac{a - b_2 l - \lambda - m b_1}{b_1}\right) + \gamma\left\{\mu l - \ln\left(\frac{1}{1-s}\right)\right\} \tag{24}$$

We see that a stationary point to problem (P1') must satisfy:

$$\frac{\partial L}{\partial \lambda} = 0, \frac{\partial L}{\partial l} = 0, \ \gamma\frac{\partial L}{\partial \gamma} = 0, \gamma \geq 0 \text{ and } \mu l \geq \ln(1/(1-s))$$

Hence:

$$\frac{\partial L}{\partial l} = 0 \quad \Leftrightarrow \gamma = \frac{\lambda b_2}{b_1(\mu+\lambda)} \tag{25}$$

$$\gamma\frac{\partial L}{\partial \gamma} = 0 \Leftrightarrow \gamma\left\{\mu l - \ln\left(\frac{1}{1-s}\right)\right\} = 0 \tag{26}$$

Given that $\gamma = \lambda b_2 / (b_1(\mu+\lambda))$ thus, $\gamma \neq 0$. Thus, $\mu l - \ln(1/(1-s)) = 0$ which means service level constraint (eq. (21)) is binding at optimality in our new problem. Hence, $l = \ln(1/(1-s))/\mu$. We denote $\ln(1/(1-s))$ by z, and get:

$$l = z/\mu \text{ with } z = \ln(1/(1-s)) \tag{27}$$

Substitute l by z/μ into the equations of p at optimality conditions, we obtain

$$p = \frac{a\mu - b_2 z - \lambda\mu}{\mu b_1} \tag{28}$$

However, we also have three conditions which are:

1. Demand is positive ($\lambda \geq 0$)
2. Price is positive ($p \geq 0$)

$$\frac{a\mu - b_2 z - \lambda\mu}{\mu b_1} \geq 0 \Leftrightarrow \lambda \leq a - \frac{b_2 z}{\mu}$$

3. Profit is positive where we must satisfy condition: $p \geq m$.

$$\frac{a\mu - b_2 z - \lambda\mu}{\mu b_1} \geq m \Leftrightarrow \lambda \leq a - \frac{b_2 z}{\mu} - b_1 m$$

Those three conditions imply that $0 \leq \lambda \leq a - (b_2 z/\mu) - b_1 m$. Then, substituting p by its value in the objective function, we get a new formulation of the problem with a single variable (λ):

$$\text{(P1''): } \underset{0 \leq \lambda \leq a - \frac{b_2 z}{\mu} - b_1 m}{\text{Maximize}} \ f(\lambda) = \left(\frac{\lambda a\mu - \lambda b_2 z - \lambda^2 \mu - \lambda m\mu b_1}{\mu b_1 + \lambda b_1} \right) \quad (29)$$

Now, we can solve analytically the (M/M/1/1) model without penalty and holding costs.

Proposition 1. Problem (P1'') is relevant ($\lambda, p \geq 0$ and $f(\lambda^*) \geq 0$) if and only if $(a\mu - b_2 z)/\mu b_1 \geq m$

We assume that the condition of proposition 1 holds. We use the first derivative conditions in order to identify the stationary points of function $f(\lambda)$.

$$\frac{df(\lambda)}{d\lambda} = 0 \Leftrightarrow a\mu - b_2 z - m\mu b_1 - 2\lambda\mu - \lambda^2 = 0 \quad (30)$$

The discriminant of this quadratic equation is

$$\Delta = 4\mu^2 + 4a\mu - 4b_2 z - 4m\mu b_1 \text{ with } z = \mu l$$

$$\Leftrightarrow \Delta = 4\mu(\mu + a - b_2 l - mb_1)$$

The $a - b_2 l - mb_1$ is equivalent to λ with $p = m$ and λ is non-negative. Hence, it is proven that $\Delta \geq 0$. Hence, the quadratic equation (30) has two real roots, which are:

$$\lambda_1 = -\mu - \sqrt{\mu^2 + a\mu - b_2 z - m\mu b_1} \text{ and } \lambda_2 = -\mu + \sqrt{\mu^2 + a\mu - b_2 z - m\mu b_1} \quad (31)$$

Given that λ_1 is negative, there is only one feasible stationary point λ_2. Suppose $\lambda_2 \geq 0$:

$$-\mu + \sqrt{\mu^2 + a\mu - b_2 z - m\mu b_1} \geq 0 \Leftrightarrow \frac{a\mu - b_2 z}{\mu b_1} \geq m \quad (32)$$

This result corresponds to proposition 1. Hence, under proposition 1, λ_2 is positive. When $\lambda_2 \to 0$, then objective function become 0. And when $\lambda_2 \to \infty$, then the objective function becomes $-\infty$. Hence, λ_2 is the optimal solution as given in proposition 2.

Proposition 2.
The optimal solution of the (M/M/1/1) model without penalty and holding costs is:
1. (optimal demand) $\lambda^* = -\mu + \sqrt{\mu^2 + a\mu - b_2 z - m\mu b_1}$ with $z = \ln(1/(1-s))$,
2. (optimal lead-time) $l^* = \ln(1/(1-s))/\mu$,
3. (optimal price) $p^* = (a - b_2 l^* - \lambda^*)/b_1$, and
4. (optimal profit) = $(\lambda^* \mu/(\mu + \lambda^*)) \times (p^* - m)$.

5. Comparison between M/M/1/1 and M/M/1 without Penalty and Holding Costs

In this section, we compare our model M/M/1/1 with the M/M/1 model developed in Pekgün et al. (2008). Recall that Pekgün et al. (2008) consider the same demand function used in this paper and do not include holding and penalty costs. We use a base case with parameters: lead-time sensitivity (b_2) = 6, price sensitivity (b_1) = 4; Production capacity (μ) = 10; service level (s) = 0.95; unit direct variable cost (m) = 5. We consider different scenario by varying the market potential (a) and one of the above parameters. For each pair of value, for example (a, b_2), we determine the optimal profits obtained from the M/M/1 and M/M/1/1, and deduce the relative gain resulting from using the M/M/1/1 setting

This gain is calculated as follows $\dfrac{\text{Profit}^{\text{M/M/1/1}} - \text{Profit}^{\text{M/M/1}}}{\text{Profit}^{\text{M/M/1}}} \times 100\%$ (33)

Clearly, a positive gain means that our M/M/1/1 is better while a negative gain indicates that the M/M/1 performs better.

First, we study the impacts of market potential (a) and lead-time sensitivity (b_2). As one can observe in table 1, the M/M/M/1 can be better than M/M/1. When the lead-time sensitivity is high, the rejection policy is always better. The rejection policy became better than all acceptance policy when the market potential is smaller.

Table 1 - Comparison based on a and b_2

b_2	M/M/1 vs M/M/1/1					
20	-	40.87%	17.94%	8.29%	3.42%	0.66%
19	-	37.10%	15.27%	6.12%	1.57%	-0.99%
18	-	33.39%	12.61%	3.96%	-0.30%	-2.65%
17	-	29.73%	9.96%	1.79%	-2.18%	-4.33%
16	-	26.13%	7.31%	-0.40%	-4.07%	-6.03%
15	-	22.57%	4.66%	-2.59%	-5.99%	-7.74%
14	-	19.05%	2.01%	-4.80%	-7.92%	-9.48%
13	-	15.57%	-0.65%	-7.03%	-9.87%	-11.24%
12	-	12.11%	-3.32%	-9.29%	-11.86%	-13.04%
11	-	8.68%	-6.02%	-11.57%	-13.88%	-14.87%
10	-	5.26%	-8.74%	-13.90%	-15.94%	-16.74%
9	-	1.85%	-11.49%	-16.28%	-18.06%	-18.67%
8	-	-1.56%	-14.30%	-18.71%	-20.23%	-20.65%
7	-	-4.98%	-17.17%	-21.22%	-22.48%	-22.71%
6	-	-8.43%	-20.13%	-23.83%	-24.83%	-24.86%
5	-	-11.92%	-23.20%	-26.56%	-27.30%	-27.14%
	20	30	40	50	60	70
	a					

Note:

-	Problem is infeasible for both M/M/1 and M/M/1/1

Second, we vary the market potential (a) and price sensitivity (b_1). We can deduce that when the costumer is highly sensitive to price, the M/M/M/1 performs better than the M/M/1 (see table 2).

Table 2 - Comparison based on a and b_1

b_1	M/M/1 vs M/M/1/1					
14	-	-	-	-	-	-
13	-	-	-	-	-	4.54%
12	-	-	-	-	-	-8.43%
11	-	-	-	-	4.54%	-15.87%
10	-	-	-	-	-8.43%	-20.13%
9	-	-	-	4.54%	-15.87%	-22.51%
8	-	-	-	-8.43%	-20.13%	-23.83%
7	-	-	4.54%	-15.87%	-22.51%	-24.51%
6	-	-	-8.43%	-20.13%	-23.83%	-24.83%
5	-	4.54%	-15.87%	-22.51%	-24.51%	-24.91%
4	-	-8.43%	-20.13%	-23.83%	-24.83%	-24.86%

b_1	M/M/1 vs M/M/1/1					
3	4.54%	-15.87%	-22.51%	-24.51%	-24.91%	-24.72%
2	-8.43%	-20.13%	-23.83%	-24.83%	-24.86%	-24.53%
1	-15.87%	-22.51%	-24.51%	-24.91%	-24.72%	-24.30%
	20	30	40	50	60	70
	a					

Third, we vary the market potential (*a*) and service level (*s*) (see Table 3). To satisfy the service level, the firm can quote any lead-time under the M/M/1 setting as there is no penalty associated with overdue clients. However, if the service level is very high (close to 1), it requires quoting a very long lead-time. This leads to a great decrease in demand and, consequently, in profit. In this case, the M/M/1/1 can be better.

Table 3 - Comparison based on a and s

s	M/M/1 vs M/M/1/1					
0.999	-	18,48%	1,57%	-5,17%	-8,24%	-9,77%
0.995	-	7,35%	-7,07%	-12,47%	-14,68%	-15,59%
0.99	-	2.61%	-10.87%	-15.74%	-17.58%	-18.23%
0.98	-	-2.12%	-14.77%	-19.12%	-20.59%	-20.98%
0.97	-	-4.90%	-17.10%	-21.16%	-22.43%	-22.66%
0.96	-	-6.89%	-18.79%	-22.65%	-23.76%	-23.89%
0.95	-	-8.43%	-20.13%	-23.83%	-24.83%	-24.86%
0.94	-	-9.70%	-21.23%	-24.81%	-25.71%	-25.68%
	20	30	40	50	60	70
	a					

Finally, we vary the market potential (*a*) and production capacity (*μ*) (see Table 4). A small production capacity impacts on the service time as the firm will require longer lead-time to serve the clients. Hence, the policy to reject some of the clients is better in this situation.

Table 4 - Comparison based on a and μ

μ	M/M/1 vs M/M/1/1					
10	-	-8.43%	-20.13%	-23.83%	-24.83%	-24.86%
9	-	-5.72%	-17.35%	-20.83%	-21.77%	-21.82%
8	-	-1.70%	-13.51%	-16.94%	-17.94%	-18.09%
7	-	4.32%	-8.15%	-11.82%	-13.05%	-13.42%
6	-	13.62%	-0.53%	-4.90%	-6.63%	-7.40%
5	-	28.77%	10.79%	4.86%	2.15%	0.67%
4	-	56.01%	28.86%	19.59%	14.97%	12.21%
3	-	116.15%	61.96%	44.62%	35.87%	30.51%
2	-	359.61%	144.13%	98.95%	78.23%	66.03%
1	-	-	962.02%	359.37%	240.29%	186.72%
	20	30	40	50	60	70
	a					

It is shown that the use of the client rejection policy can be better in some cases even when the penalty and holding cost are removed. Therefore, we expect that when penalty and holding cost are added, the rejection policy will even perform better. We investigate this problem in the next section

6. M/M/1/1 with Penalty Cost and Holding Cost

With the addition of penalty and holding costs, the objective function will be composed by three parts: expected revenue, total congestion costs, and total lateness penalty costs. The formulation of this objective function has been presented earlier. The service level constraint in this case is similar to the previous case: $1-e^{-\mu l} \geq s$ as the Probability $P(W > l) = e^{-\mu l}$. Thus the formulation of this problem is:

(P2) $$\underset{\lambda,p,l,\bar{\lambda}}{Maximize} \quad \bar{\lambda}(p-m) - \frac{F\lambda}{\mu+\lambda} - \frac{c\bar{\lambda}}{\mu}e^{-\mu l} \tag{34}$$

Subject to
$$\lambda \leq a - b_1 p - b_2 l \tag{35}$$
$$1-e^{-\mu l} \geq s \tag{36}$$
$$\rho = \frac{\lambda}{\mu} \tag{37}$$
$$P_1 = \frac{\rho}{1+\rho} \tag{38}$$
$$\bar{\lambda} = \lambda(1-P_1) \tag{39}$$
$$\bar{\lambda}, \lambda, l, p \geq 0 \tag{40}$$

Integrating the equality constraints (eq. (36 – 39)) into the objective function and rewriting $1-e^{-\mu l} \geq s$ as $\mu l \geq \ln(1/(1-s))$, we get as the following formulation of the problem:

(P2') $$\underset{\lambda,l,p}{\text{Maximize}} \quad \frac{\lambda\left(p\mu - m\mu - F - ce^{-\mu l}\right)}{\mu+\lambda} \tag{41}$$

Subject to
$$\lambda \leq a - b_1 p - b_2 l \tag{42}$$
$$\mu l \geq \ln(1/(1-s)) \tag{43}$$
$$\lambda, p, l \geq 0 \tag{44}$$

Suppose that the optimal solution is given by price, p^*, quoted lead-time l^*, and demand rate λ^*, and that $\lambda^* < \Lambda(p^*, 1^*.)$. Since the revenues are increasing in p, one could increase the price to p' (while holding the demand rate and quoted lead-time constant) until $\lambda^* = \Lambda(p', 1')$. This change will increase revenues without increasing the costs. Therefore, (p^*, l^*, and λ^*) cannot be an optimal solution. Thus, the demand constraint (eq. (42)) is binding at optimality. However, we also have conditions where: demand is positive ($\lambda \geq 0$) and price is positive ($p \geq 0$):

$$\frac{a\mu - b_2 z - \lambda\mu}{\mu b_1} \geq 0 \Leftrightarrow \lambda \leq a - \frac{b_2 z}{\mu}$$

Then, we substitute price p by its value and obtain the following formulation:

(P2") $$\underset{0 \leq \lambda \leq a - \frac{b_2 z}{\mu}; l \geq 0}{Maximize} \quad \frac{\lambda\left[\left(\mu(a - b_2 l - \lambda)/b_1\right) - m\mu - F - ce^{-\mu l}\right]}{\mu+\lambda} \tag{45}$$

Subject to
$$\mu l \geq \ln(1/(1-s)) \tag{46}$$
$$\lambda, l \geq 0 \tag{47}$$

Unlike the first case where the penalty and holding costs are not considered, the service level constraint is not necessarily binding in this case, which complicates the solving approach. Indeed, for large values of unit penalty cost c, the real service level has to be very high (close to 1) to avoid a high penalty cost. This indicates that the real service level can be greater than the imposed service level (s).

The detailed proof is given in appendix I. We now present the main steps to get the optimal solution given in proposition 4.

To solve the problem, we apply the Lagrangian multiplier method. The stationary points of problem (P2") must satisfy:

$$\frac{\partial L}{\partial \lambda}=0, \frac{\partial L}{\partial l}=0, \gamma\frac{\partial L}{\partial \gamma}=0, \gamma \geq 0 \text{ and } \mu l \geq \ln\left(\frac{1}{1-s}\right)$$

where,

$$L(\lambda,l,\gamma)=\frac{\lambda\left[\left(\mu\left(a-b_2 l-\lambda\right)/b_1\right)-m\mu-F-ce^{-\mu l}\right]}{\mu+\lambda}+\gamma\left\{\mu l-\ln\left(\frac{1}{1-s}\right)\right\} \tag{48}$$

$$\frac{\partial L}{\partial l}=0 \iff 1-\frac{b_2}{cb_1}+\frac{\gamma(\mu+\lambda)}{\lambda c}=1-e^{-\mu l}$$

Substitute $1-\frac{b_2}{cb_1}=s_c$, we have:

$$\iff s_c+\frac{\gamma(\mu+\lambda)}{\lambda c}=1-e^{-\mu l} \tag{49}$$

$$\gamma\frac{\partial L}{\partial \gamma}=0 \iff \gamma\left\{\mu l-\ln\left(\frac{1}{1-s}\right)\right\}=0 \tag{50}$$

From $\gamma(\partial L/\partial\gamma)=0$, we know that there are two situations ($\gamma=0$ or $\mu l-\ln(1/(1-s))=0$). Suppose that $\gamma=0$, it implies that $\mu l-\ln(1/(1-s))>0$ which also imply that the service level constraint is non-binding. Since $\gamma=0$, equation (49) implies that $s_c=1-e^{-\mu l}$. In addition, if the service level is non-binding, then $1-e^{-\mu l}>s$. It implies that $s_c>s$.

Next, suppose $\gamma\neq 0$, thus it implies $\mu l-\ln(1/(1-s))=0$ which also means that the service level constraint is binding. Hence $1-e^{-\mu l}=s$. Combining with the eq. (49), it implies that: $s_c+\frac{\gamma(\mu+\lambda)}{\lambda c}=s$. This imply that $s_c\leq s$.

In the non-binding situation, the service level will be $s_c=1-e^{-\mu l}$ with $s_c>s$. And in the binding situation, the service level will be $1-e^{-\mu l}=s$ with $s_c\leq s$. Hence combining the both case, the service level will be: $1-e^{-\mu l}=Max\{s,s_c\}$.

As we have already explained, we have two situations:

(1) the service level constraint (46) is non-binding: $s<s_c$,

(2) the service level constraint (46) is binding: $s\geq s_c$,

where the critical value for the service level (s_c) equals to $1-b_2/cb_1$. With this critical service level (s_c), the lead-time can be found based on the two mutually exclusive cases of the service level:

$$1-e^{-\mu l}=Max\{s_c,s\} \iff l^*=\frac{1}{\mu}\ln\left(\frac{1}{1-Max\{s_c,s\}}\right)$$

$$\iff l^*=\frac{1}{\mu}\ln x \text{ where, } x=Max\{1/(1-s),b_1c/b_2\} \tag{51}$$

Before we move to find the optimal demand, we will discuss about the condition to have positive profit. To have the non-negative objective function, we have conditions:

1. $p\geq m$:

$$\frac{(a-b_2 l)}{b_1}\geq m \text{, with } l^*=\frac{1}{\mu}\ln x \text{ and } x=Max\{1/(1-s),b_1c/b_2\}. \tag{52}$$

2. $\bar{\lambda}(p-m)-\frac{F\lambda}{\mu+\lambda}-\frac{c\bar{\lambda}}{\mu}e^{-\mu l}\geq 0 \Leftrightarrow \frac{\lambda\left(p\mu-m\mu-F-ce^{-\mu l}\right)}{\mu+\lambda}\geq 0$

It is obvious that $\mu+\lambda\geq 0$. Hence, the numerator should be:

$\lambda\left(p\mu-m\mu-F-ce^{-\mu l}\right)\geq 0$

$\Leftrightarrow p\geq m+F/\mu+ce^{-\mu l}/\mu$

$\Leftrightarrow \frac{(a-b_2 l)}{b_1}\geq m+F/\mu+ce^{-\mu l}/\mu$

$$\Leftrightarrow a\mu-\mu b_2 l-\mu m b_1-Fb_1-cb_1e^{-\mu l}\geq 0 \tag{53}$$

Equation (53) should be bigger than equal to zero to have a non-negative objective function. We determine the conditions to have positive profit in proposition 3.

Proposition 3. The problem P2" is relevant (the optimal profit is positive) if and only if $\frac{a-b_2 l}{b_1}\geq m$ and $a\mu-\mu b_2 l-\mu m b_1-Fb_1-cb_1e^{-\mu l}\geq 0$.

We now show how to find the optimal demand. Indeed,

$$\frac{\partial L}{\partial\lambda}=0\Leftrightarrow\frac{\mu\left(a\mu-\mu b_2 l-2\mu\lambda-\mu m b_1-Fb_1-b_1ce^{-\mu l}-\lambda^2\right)}{b_1(\mu+\lambda)^2}=0 \tag{54}$$

Numerator of eq. (54) should be equal to zero.

$$\mu\left(a\mu-\mu b_2 l-2\mu\lambda-\mu m b_1-Fb_1-b_1ce^{-\mu l}-\lambda^2\right)=0 \tag{55}$$

The discriminant (Δ) of eq. (55) should be greater than 0 to have real roots.

$$\Delta \quad =4\mu^2+4a\mu-4\mu b_2 l-4\mu m b_1-4Fb_1-4b_1ce^{-\mu l} \tag{56}$$

$\Delta=(-2\mu)^2-4(-1)\left(a\mu-\mu b_2 l-\mu m b_1-Fb_1-b_1ce^{-\mu l}\right)\geq 0$

$$\Leftrightarrow \mu^2+a\mu-\mu b_2 l-\mu m b_1-Fb_1-b_1ce^{-\mu l}\geq 0 \tag{57}$$

The $a\mu-\mu b_2 l-\mu m b_1-Fb_1-b_1ce^{-\mu l}$ corresponds to proposition 3. This imply that to have a non-negative objective functions, we have to have $a\mu-\mu b_2 l-\mu m b_1-Fb_1-b_1ce^{-\mu l}\geq 0$. With the $\mu\geq 0$, we can say that $\mu^2+a\mu-\mu b_2 l-\mu m b_1-Fb_1-b_1ce^{-\mu l}\geq 0$ and $\Delta\geq 0$ are proven. Because it is proven that the discriminant (Δ) is bigger than zero thus eq. (55) has two roots which are:

$\lambda_1 \quad =-\mu-\sqrt{\mu^2+a\mu-\mu b_2 l-\mu b_1 m-Fb_1-b_1ce^{-\mu l}}$ and

$$\lambda_2 \quad =-\mu+\sqrt{\mu^2+a\mu-\mu b_2 l-\mu b_1 m-Fb_1-b_1ce^{-\mu l}} \tag{58}$$

λ_1 is negative. Then, there is only one stationary point λ_2. Suppose $\lambda_2\geq 0$:

$-\mu+\sqrt{\mu^2+a\mu-\mu b_2 l-\mu b_1 m-Fb_1-b_1ce^{-\mu l}}\geq 0$

$$a\mu-\mu b_2 l-\mu b_1 m-Fb_1-b_1ce^{-\mu l}\geq 0 \tag{59}$$

This final equation corresponds to the proposition 3. Under the conditions of proposition 3, λ_2 has a value greater than zero.

When $\lambda_2\to 0$ and $l\to 0$, then objective function become 0. When $\lambda_2\to\infty$ and $l\to 0$, then the objective function becomes $-\infty$. When $\lambda_2\to 0$ and $l\to\infty$, then the objective function becomes 0. And when $\lambda_2\to\infty$ and $l\to\infty$, then the objective function becomes $-\infty$. Thus, the lead-time (l^*) and demand (λ_2) provide the optimal solution. As a summary, the optimum point of this problem can be found based on the proposition 4.

Proposition 4. For problem P2:

1. The optimal lead-time $l^* = \ln x/\mu$ with $x = Max\{1/(1-s), b_1c/b_2\}$.
2. The optimal demand can be found by using equation $\lambda^* = -\mu + \sqrt{\mu^2 + a\mu - \mu b_2 l^* - \mu b_1 m - Fb_1 - b_1 ce^{-\mu l^*}}$,
3. the optimal price is $p^* = (a - b_2 l^* - \lambda^*)/b_1$.
4. The optimal profit $= \lambda^*(p^*\mu - m\mu - F - ce^{-\mu l^*})/(\mu + \lambda^*)$.

Based on the result found in this section, we will compare the results of the rejection policy with penalty and holding cost (modeled by M/M/1/1) to the results obtained with M/M/1 under the same setting.

7. Comparison between M/M/1/1 and M/M/1 With Penalty and Holding Costs

In this section, we compare our model (M/M/1/1) with the existing M/M/1 taken from Palaka et al. (1998). We vary the market potential (a) and other parameters. For each pair of value, for example (a, b_2), we calculate the relative gain obtained by using M/M/1/1 instead of M/M/1. This relative is calculated as (33) We use the same parameters setting as in the previous comparison (lead-time sensitivity (b_2) = 6, price sensitivity (b_1) = 4; Production capacity (μ) = 10; service level (s) = 0.95; unit direct variable cost (m) = 5) with additions of F = 2 and c = 10.

First, we compare the result based on the variation of market potential (a) and lead-time sensitivity (b_2). As expected, there are more cases where M/M/1/1 is better than M/M/1 (see Table 5) compared to first case without penalty and holding. In the M/M/1 the holding cost can be very high because all clients are accepted. As observed in table 5, the M/M/1/1 is better when dealing with customers that are very sensitive to lead-time with low the market potential.

Table 5 - Comparison based on a and b_2

b_2	M/M/1 vs M/M/1/1					
20	-	53.96%	26.95%	15.50%	9.58%	6.11%
19	-	49.95%	24.23%	13.33%	7.74%	4.49%
18	-	46.02%	21.53%	11.17%	5.90%	2.86%
17	-	42.16%	18.84%	9.02%	4.05%	1.22%
16	-	38.37%	16.17%	6.86%	2.19%	-0.43%
15	-	34.64%	13.51%	4.69%	0.33%	-2.09%
14	-	30.96%	10.85%	2.52%	-1.54%	-3.76%
13	-	27.34%	8.20%	0.34%	-3.43%	-5.45%
12	-	23.77%	5.56%	-1.85%	-5.34%	-7.16%
11	-	20.24%	2.91%	-4.05%	-7.26%	-8.89%
10	-	16.74%	0.25%	-6.27%	-9.21%	-10.65%
9	-	13.28%	-2.42%	-8.52%	-11.19%	-12.43%
8	-	9.84%	-5.10%	-10.80%	-13.19%	-14.25%
7	-	6.42%	-7.81%	-13.11%	-15.24%	-16.10%
6	-	3.01%	-10.56%	-15.47%	-17.33%	-18.01%
5	-	-0.40%	-13.35%	-17.88%	-19.49%	-19.97%
	20	30	40	50	60	70
	a					

Second, we vary the market potential (a) and price sensitivity (b_1) (see Table 6). In M/M/1, the lead-time can become very long because we accept all clients. With the addition of the penalty cost for overdue clients, the profit will be worse with high value of price sensitivity of clients. Hence, for a firm facing a demand that is very sensitive to price, the M/M/1/1 could be better policy.

Table 6 - Comparison based on a and b_1

b_1	M/M/1 vs M/M/1/1					
13	-	-	-	-	-	-
12	-	-	-	-	-	26.18%
11	-	-	-	-	42.97%	11.99%
10	-	-	-	-	20.26%	2.93%
9	-	-	-	35.50%	7.00%	-3.27%
8	-	-	-	14.45%	-1.51%	-7.76%
7	-	-	28.29%	2.01%	-7.34%	-11.16%
6	-	-	8.70%	-6.00%	-11.56%	-13.87%
5	-	21.30%	-2.99%	-11.49%	-14.77%	-16.10%
4	-	3.01%	-10.56%	-15.47%	-17.33%	-18.01%
3	14.50%	-8.05%	-15.75%	-18.51%	-19.47%	-19.70%
2	-2.69%	-15.24%	-19.53%	-20.96%	-21.32%	-21.23%
1	-13.22%	-20.20%	-22.45%	-23.04%	-22.98%	-22.67%
	20	30	40	50	60	70
	a					

Third, we compare based on the variation of market potential (a) and service level (s) (see Table 7). If the firms set the service level close to 1, it will cause very long lead-time in M/M/1 model. The long lead-time will cause high holding cost which affect the demand and ruin the profit. Thus, rejecting some clients could be an alternative to keep the high profit. The M/M/1/1could be better in such situations.

Table 7 - Comparison based on a and s

s	M/M/1 vs M/M/1/1					
0.999	-	27,99%	8,68%	0,74%	-3,09%	-5,15%
0.995	-	16,78%	0,28%	-6,25%	-9,19%	-10,63%
0.99	-	12.21%	-3.25%	-9.22%	-11.81%	-12.99%
0.98	-	7.90%	-6.63%	-12.10%	-14.35%	-15.29%
0.97	-	5.58%	-8.48%	-13.68%	-15.75%	-16.56%
0.96	-	4.07%	-9.69%	-14.72%	-16.67%	-17.41%
0.95	-	3.01%	-10.56%	-15.47%	-17.33%	-18.01%
0.94	-	2.22%	-11.20%	-16.02%	-17.83%	-18.46%
	20	30	40	50	60	70
	a					

Fourth, we compare based on the variation of market potential (a) and production rate (μ) (see Table 8). The production capacity affects the service time offered by the firms. If the production capacity is small; the service time for each client will be very long (service time = $1/\mu$). This long service will increase the waiting time, hene implying a higher holding cost. Thus, for firms with small production capacity, it can be better to reject some costumers.

Table 8 - Comparison based on a and μ

μ	M/M/1 vs M/M/1/1					
12	-	-3,15%	-16,06%	-21,16%	-23,10%	-23,74%
11	-	-0,55%	-13,70%	-18,65%	-20,51%	-21,14%
10	-	3,01%	-10,56%	-15,47%	-17,33%	-18,01%
9	-	7,95%	-6,36%	-11,41%	-13,40%	-14,21%
8	-	14,97%	-0,67%	-6,15%	-8,45%	-9,51%
7	-	25,30%	7,18%	0,79%	-2,08%	-3,57%
6	-	41,28%	18,43%	10,30%	6,40%	4,19%
5	-	68,27%	35,52%	24,01%	18,26%	14,82%
4	-	121,78%	64,17%	45,51%	36,16%	30,47%
3	-	277,90%	122,19%	84,63%	66,97%	56,50%
2	-	-	312,26%	183,40%	136,67%	111,73%
1	-	-	-	1496,14%	550,83%	363,51%
	20	30	40	50	60	70
	a					

Fifth, we vary the market potential (a) and holding cost (F) (see Table 9). The holding cost affects the total profit. In M/M/1, there is a possibility that a client has a very long lead-time. This long holding period will lead to an expensive holding cost. Thus, it decreases the profit. This condition explains that reject some clients could be a better policy.

Table 9 - Comparison based on a and F

F	M/M/1 vs M/M/1/1					
11	-	46,09%	21,57%	11,21%	5,93%	2,89%
10	-	40,94%	17,99%	8,33%	3,46%	0,70%
9	-	35,92%	14,43%	5,44%	0,98%	-1,51%
8	-	31,00%	10,88%	2,55%	-1,52%	-3,74%
7	-	26,18%	7,35%	-0,36%	-4,05%	-6,00%
6	-	21,44%	3,81%	-3,30%	-6,60%	-8,30%
5	-	16,76%	0,27%	-6,26%	-9,20%	-10,63%
4	-	12,14%	-3,30%	-9,27%	-11,84%	-13,02%
3	-	7,56%	-6,90%	-12,33%	-14,55%	-15,48%
2	-	3,01%	-10,56%	-15,47%	-17,33%	-18,01%
1	-	-1,55%	-14,29%	-18,70%	-20,22%	-20,64%
0	-	-6,13%	-18,14%	-22,07%	-23,25%	-23,41%
	20	30	40	50	60	70
	a					

Sixth, we vary the market potential (a) and penalty cost (c) (see Table 10). Because we set the service level to (95%), it means that there are only 5% of overdue clients. In high market potential and all client's acceptance policy, penalty cost is negligible with comparison to revenue. However, it can be seen that there is a decrease in the superiority of M/M/1 to M/M/1/1 in function of the penalty cost. It can be concluded that in big market potential, if we continue to increase the penalty cost, there will be a situation where the M/M/1/1 is superior. In small market potential, the 5% of overdue clients is significant. The penalty cost is affecting the profit. Hence, reject some client is better as there isn't any penalty in rejecting clients.

Table 10 - Comparison based on a and c

c	M/M/1 vs M/M/1/1					
10	-	3,01%	-10,56%	-15,47%	-17,33%	-18,01%
9	-	2,78%	-10,74%	-15,63%	-17,48%	-18,14%
8	-	2,55%	-10,93%	-15,79%	-17,62%	-18,27%
7	-	2,32%	-11,11%	-15,95%	-17,76%	-18,40%
6	-	2,09%	-11,30%	-16,11%	-17,90%	-18,53%
5	-	1,87%	-11,48%	-16,27%	-18,05%	-18,66%
4	-	1,64%	-11,67%	-16,43%	-18,19%	-18,79%
3	-	1,41%	-11,85%	-16,59%	-18,33%	-18,92%
2	-	1,18%	-12,04%	-16,75%	-18,48%	-19,05%
1	-	0,96%	-12,23%	-16,91%	-18,62%	-19,18%
0	-	0,73%	-12,41%	-17,07%	-18,76%	-19,31%
	20	30	40	50	60	70
	a					

8. Conclusion

In this paper, we provide the general model of M/M/1/K for case with and without holding and penalty cost. We solve both case analytically for K=1. We compare our M/M/1/1 model with the existing M/M/1 model taken from Pekgün et al. (2008) and Palaka et al. (1998). In the case where the penalty and holding cost aren't considered, the M/M/1/K is better when the customers are lead-time sensitive, the market potential is small, and the firm's production capacity (mean service rate) is small. In the case where the penalty and holding cost are considered, the M/M/1/K is better when the customers are lead-time sensitive, the market potential is small, the firm's production capacity (mean service rate) is small and the holding cost is high. We currently working in an extension of this research which is K > 1. Another possible extension is modeling the system in M/D/1.

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