Simple Tracking Output Feedback H ∞ Control for Switched Linear Systems: Lateral Vehicle Control Application

L Menhour, Damien Koenig, B D 'Andréa-Novel

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L. Menhour * D. Koenig ** B. d'Andréa-Novel ***

* *Centre de Recherche STIC, IUT de Troyes, 9, Rue du Québec, 10000 Troyes, France (e-mail: lghani.menhour@univ-reims.fr).*
** *Laboratoire de Grenoble Images Parole Signal et Automatique, UMR CNRS-INPG-UJF, 38402 Saint Martin d'Hres, France (e-mail: Damien.Koenig@gispa-lab.grenoble-inp.fr)*
*** *Mines-ParisTech, CAOR-Centre de Robotique, Mathématiques et systèmes, 60 boulevard Saint-Michel, 75272 Paris cedex 06, France (e-mail: brigitte.dandrea-novel@mines-paristech.fr)*

Abstract: In this paper, the problem of the switched H_∞ tracking output feedback control problem is studied. The control design problem is addressed in the context of discrete-time switched linear systems. Then, the design of continuous-time case becomes trivial. Linear Matrix Inequality (LMI) and Linear Matrix Equality (LME) representations are used to express all sufficient conditions to solve the control problem. Some transformations leading to sufficient conditions for the control problem are also used. All conditions are established for any switching using a switched Lyapunov function and a common Lyapunov function. The effectiveness of the proposed control approach is shown through a steering vehicle control implementation. Interesting simulation results are obtained using real data acquired by an instrumented car.

Keywords: Switched linear systems, H_∞ norm, poly-quadratic stability, steering control.

1. INTRODUCTION

Many real complex systems have several operating modes, each of them corresponds to a local dynamical behavior. In many cases, all operating modes can be described by nonlinear mathematical models. Unfortunately, the use of such models becomes a very hard task. One solution used for this problem, is the multi-model representation. In practice, this approach is used to design gain-scheduled methods (Stilwell and Rugh [1999], Apkarian and Gahinet [1995]), linear parameter varying control (Apkarian et al. [1997, 1995]), fuzzy systems (Castillo and Melin [2008], Mendel [2004], Sugeno and Kang [1988]). In same reasoning, the switched systems have been developed (Branicky [1998], Liberzon and Morse [1999], Daafouz et al. [2002], Sun and Ge [2005], Lin and Antsaklis [2007], Koenig et al. [2008], Lin and Antsaklis [2009], Koenig and Marx [2009]). Such systems are defined by a finite number of subsystems and switching rules. In particular, the switched linear systems are obtained by linearization of nonlinear systems in the vicinity of some operating modes (or operating points).

The design of switched systems remains an attractive problem and widely addressed for linear systems by using common Lyapunov function (Liberzon and Morse [1999], Sun and Ge [2005], Lin and Antsaklis [2007, 2009]) and switched Lyapunov functions (Branicky [1998], Daafouz et al. [2002], Du et al. [2007]). The stability of other switched systems is also addressed like switched linear systems with state delays (Du et al. [2007]), switched linear and nonlinear descriptor systems (Koenig et al. [2008], Koenig and Marx [2009]), switched linear systems with singular perturbation (Hachemi et al. [2012], Deaecto et al. [2012]). It should be pointed out that the literature on the design of such systems is abundant.

In this paper, we investigate the switched control problem for discrete-time switched linear systems. All sufficient conditions are established and demonstrated in Lyapunov sens using switched Lyapunov functions approach. The design method uses also some matrix transformations leading to the sufficient conditions and H_∞ norm. All conditions are expressed in term of linear matrix inequality (LMI) and linear matrix equality (LME). In addition, the output feedback control and tracking reference model methods are also used here. The model reference control (Crusius and Trofino [1999], Bartolini et al. [1997], Wolovich and Ferreira [1979]) has been widely used to control several real systems like electrical devices (Hu et al. [1995]), mechanical systems (Qu and Dorsey [1991], Liao et al. [2002]), networked systems (Gao and Chen [2008]).

The vehicle dynamics control problem is used to show the effectiveness of the proposed controllers. The lateral control seems to be a good example for the switching control application. Many vehicle controllers have been reported (see, Cerone et al. [2009], Plochl and Edelmann [2009] and the references therein) and some of them assume that the vehicle model is exactly known, and such

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an assumption is generally not satisfied. Consequently, the switched systems can be successfully used.

This paper is organized as follows. Section 2 describes the Linear bicycle vehicle model and the problem statement. Section 3 presents the design method of switched control. The simulation results using the real data are given in Section 4. Section 5 summarizes conclusions and perspectives.

2. SWITCHED CONTROL: PROBLEM STATEMENT

For switched control design, let us recall the single track vehicle model commonly used for lateral control.

2.1 Single track vehicle model

The lateral model used here is composed of lateral and yaw motions which are described by:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ff(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where

$$A = \begin{bmatrix} -\dfrac{2C_f + 2C_r}{mV_x} & -V_x - \dfrac{2C_fL_f - 2C_rL_r}{mV_x} & 0 \\ -\dfrac{2C_fL_f - 2C_rL_r}{I_zV_x} & -\dfrac{2C_fL_f^2 + 2C_rL_r^2}{mV_x} & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$B = \begin{bmatrix} \dfrac{C_f}{m} & \dfrac{2L_fC_f}{I_z} & 0 \end{bmatrix}^T$, $x = \begin{bmatrix} V_y & \dot{\psi} & \psi \end{bmatrix}^T$, $C = [\,1\ 0\ V_x\,]$, $F = [\,-g\ 0\ 0\,]^T$, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) = \delta(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^p$ is the output, and $f(t) = \phi_r(t) \in \mathbb{R}^{n_f}$ is the disturbance input that satisfies $f \in L_2[0, \infty)$. A, B, C and F are system matrices with appropriate size. Let us recall that to obtain the model (1), the following linear tire force model is used:

$$F_{yf} = C_f\left(\delta - \frac{V_y + \dot{\psi}L_f}{V_x}\right),\ F_{yr} = -C_r\left(\frac{V_y - \dot{\psi}L_r}{V_x}\right)$$

2.2 Problem formulation

In model (1), several parameter variations can be listed: cornering stiffnesses C_f and C_r, longitudinal speed Vx. In (1), all parameters are considered constant. Unfortunately, many driving actions act on these parameters. The characteristics of Figure 1 highlight the braking maneuver in which C_f and C_r have two different modes. We can observe that the lateral tire forces operate in linear region with small sideslip angle (less than $2\,deg$). According to the characteristic of Figure 1 and model (1), the following switching rules can be considered:

1- First rule on C_f and C_r:

$$\begin{cases} C_{(f,r)} = C_{(f,r)1} \text{ If } \beta < \beta^* \\ C_{(f,r)} = C_{(f,r)2} \text{ If } \beta > \beta^* \end{cases} \tag{2}$$

2- Second rule on V_x:

$$\begin{cases} V_x = V_{x_1} & \text{If} & V_x \in [V_{x_1} - \Delta,\, V_{x_2} - \Delta[\\ V_x = V_{x_2} & \text{If} & V_x \in [V_{x_2} - \Delta,\, V_{x_3} - \Delta[\\ \quad\vdots & \vdots & \quad\vdots \\ V_x = V_{x_M} & \text{If} & V_x \in [V_{x_{M-1}} - \Delta,\, V_{x_M} - \Delta[\end{cases} \tag{3}$$

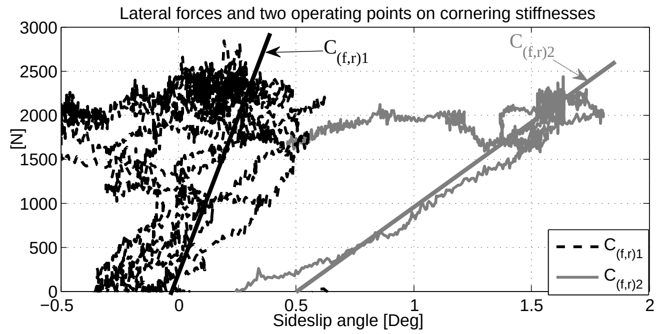

Fig. 1. Experimental braking maneuver: characteristic of the lateral tire force with two slopes of cornering stiffnesses ($C_{(f,r)_1}$ and $C_{(f,r)_2}$)

with β^* is the switching threshold on the sideslip angle and $\Delta = \frac{V_{x_2} - V_{x_1}}{2}$. However, the cornering stiffnesses coefficients (C_f and C_r) are measured by expensive sensors (around 100 K€). For this, a switched controller can then be designed with unmeasurable premise variable. Moreover, the system must be robust again the premise variables (Kiss et al. [2011], Ichalal et al. [2010]). In our case, the premise variables are β and V_x which can be estimated online (see Villagra et al. [2009]). Since, the LTI model (1) can be viewed as a switched one:

$$\begin{cases} \dot{x}(t) = \displaystyle\sum_{i=1}^{M} \alpha_i(t)\left[A_ix(t) + B_iu(t) + F_i\omega(t)\right] \\ y(t) = \displaystyle\sum_{i=1}^{M} \alpha_i(t)C_ix(t) \end{cases} \tag{4}$$

The function $\alpha_i(t)$ is the switching signal

$$\alpha_i : \mathbb{R}^+ \longrightarrow \{0,\, 1\} \quad \sum_{i=1}^{M} \alpha_i(t) = 1, \quad t \in \mathbb{R}^+ \tag{5}$$

Our aim is to design switched controllers, such that the output y of the closed-loop system tracks any given reference output y_r of the following reference model:

$$\begin{cases} \dot{x}_r(t) = \displaystyle\sum_{i=1}^{M} \alpha_i(t)\left[A_{r_i}x_r(t) + F_{r_i}r(t)\right] \\ y_r(t) = \displaystyle\sum_{i=1}^{M} \alpha_i(t)C_{r_i}x_r(t) \end{cases} \tag{6}$$

where, $y_r(t) \in \mathbb{R}^p$ has the same dimension as $y(t)$. $x_r(t) \in \mathbb{R}^{n_r}$ and $r(t) \in \mathbb{R}^{m_r}$ are respectively the reference state and the bounded reference input. A_{r_i}, C_{r_i} and F_{r_i} are appropriately dimensioned with A_{r_i} Hurwitz. The control design procedure assumes that both $x(t)$ and $x_r(t)$ are measurable outputs. For our purpose, we define the following tracking output error:

$$\tilde{y}(t) = y(t) - y_r(t) \tag{7}$$

Therefore, the following augmented system is obtained:

$$\begin{cases} \dot{\xi}(t) = \displaystyle\sum_{i=1}^{M} \alpha_i(t)\left[A_{ai}\xi(t) + B_{ai}u(t) + F_{ai}\omega(t)\right] \\ \tilde{y}(t) = \displaystyle\sum_{i=1}^{M} \alpha_i(t)C_{ai}\xi(t) \end{cases} \tag{8}$$

where $A_{ai} = \begin{bmatrix} A_i & 0 \\ 0 & A_{r_i} \end{bmatrix}$, $B_{ai} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}$, $F_{ai} = \begin{bmatrix} F_i & 0 \\ 0 & F_{r_i} \end{bmatrix}$, $C_{ai} = [\, C_i \;\; -C_{r_i} \,]$, $\xi(t) = \begin{bmatrix} x(t) \\ x_r(t) \end{bmatrix}$, $\omega(t) = \begin{bmatrix} f(t) \\ r(t) \end{bmatrix}$.

The discrete-time system corresponding to (8) using the first order Euler approximation at frequency $200\,Hz$ is

$$\begin{cases} \xi_{k+1} = \sum_{i=1}^{M} \alpha_i(k) \left[\bar{A}_{ai}\xi_k + \bar{B}_{ai}u_k + F_{ai}\omega_k\right] \\ \tilde{y}_k = \sum_{i=1}^{M} \alpha_i(k)\bar{C}_{ai}\xi_k \end{cases} \tag{9}$$

and the functional switching signal $\alpha_i(k)$ is

$$\alpha_i : Z^+ \longrightarrow \{0,\, 1\}, \quad \sum_{i=1}^{M} \alpha_i(k) = 1, \quad k \in \mathbb{Z}^+ \tag{10}$$

In the sequel, the switched control problem for discrete-time system (9) is given.

Problem: Consider the following H_∞ discrete-time tracking output feedback controller for system (9)

$$u_k = -\sum_{i=1}^{M} \alpha_i(k)\bar{K}_{ai}\tilde{y}_k \tag{11}$$

where the gains $\bar{K}_{ai} \in \mathbb{R}^{p\times(n+n_r)}$ are computed such that:

S1. the closed loop system $\xi_{k+1} = \sum_{i=1}^{M} \alpha_i(k)(\bar{A}_{ai} - \bar{B}_{ai}\bar{K}_{ai}\bar{C}_{ai})\xi_k$ is asymptotically stable when $\omega(k) = 0$;

S2. the transfer function $\bar{H}_{y\omega}(z) = \sum_{i=1}^{M} \bar{C}_{ai}(zI - (\bar{A}_{ai} - \bar{B}_{ai}\bar{K}_{ai}\bar{C}_{ai}))^{-1}\bar{F}_{ai}$, from $\omega(k)$ to $\tilde{y}(k)$ satisfies the H_∞ norm $\left\|\bar{H}_{\tilde{y}\omega}(z)\right\|_\infty < \gamma$ for positive scalar γ.

To deal with the above problem, let us recall the following Schur complement commonly used in literature:

Lemma 1. Let $X = X^T > 0$, $N = N^T > 0$ and W be given matrices. By Schur complement, the following statements are equivalent:

$$X - WN^{-1}W^T > 0 \Leftrightarrow \begin{pmatrix} X & W \\ W^T & N \end{pmatrix} > 0 \tag{12}$$

Assumption 1. We assume that for $i \in \{1, \cdots, M\}$, the pair $(\bar{A}_{ai},\, \bar{B}_{ai})$ is stabilizable.

3. DESIGN OF DISCRETE-TIME SWITCHED H_∞ TRACKING OUTPUT FEEDBACK CONTROL

The main objective of this section is to find some sufficient conditions and compute the gains of (11) for system (9), in order to stabilize the following closed-loop system

$$\begin{cases} \xi_{k+1} = \sum_{i=1}^{M} \alpha_i(k) \left[(\bar{A}_{ai} - \bar{B}_{ai}\bar{K}_{ai}Cai)\xi_k + \bar{F}_{ai}\omega_k\right] \\ \tilde{y}_k = \sum_{i=1}^{M} \alpha_i(k)\bar{C}_{ai}\xi_k \end{cases} \tag{13}$$

and satisfy specification S2. For this, we establish the following theorem.

Theorem 1. Under assumption 1, if there exist a constant $\gamma > 0$, matrices $X_{ai} > 0$, $X_{aj} > 0$, M_{ai} and N_{ai} such that the following LMI and LME are satisfied:

$$\begin{bmatrix} -X_{ai} & 0 & X_{ai}\bar{A}_{ai}^T - \bar{C}_{ai}^T\bar{N}_{ai}^T\bar{B}_{ai}^T & X_{ai}\bar{C}_{ai}^T \\ * & -\gamma^2 I & \bar{F}_{ai}^T & 0 \\ * & * & -X_{aj} & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{14}$$

$$\bar{M}_{ai}\bar{C}_{ai} = \bar{C}_{ai}X_{ai} \tag{15}$$

for $(i, j) \in \{1, \cdots, M\}^2$, then, the stabilizing gains of (11) are given by $\bar{K}_{ai} = \bar{N}_{ai}\bar{M}_{ai}^{-1}$.

Proof 1. To give sufficient conditions for the existence of (11) such that the closed-loop system (13) satisfies specifications S1 and S2, the following inequality should be verified:

$$V_{k+1} - V_k + \tilde{y}_k^T\tilde{y}_k - \gamma^2\omega_k^T\omega_k < 0 \tag{16}$$

where $V_k = \sum_{i=1}^{M} \alpha_i(k)\xi_k^T P_{ai}\xi_k$ is the switched Lyapunov functions and P_i are positive definite matrices. Computing the difference $V_{k+1} - V_k$ along (13), the relation (16) becomes

$$\begin{aligned} &\sum_{j=1}^{M} \alpha_j(k+1)\xi_{k+1}^T P_{aj}\xi_{k+1} - \sum_{i=1}^{M} \alpha_i(k)\xi_k^T P_{ai}\xi_k \\ &\quad + \tilde{y}_k^T\tilde{y}_k - \gamma^2\omega_k^T\omega_k < 0 \end{aligned} \tag{17}$$

To consider all switches, the following particular cases are considered

$$\begin{cases} \alpha_i(k) = 1 & \text{and } \alpha_{l\neq i}(k) = 0 \\ \alpha_j(k+1) = 1 & \text{and } \alpha_{l\neq j}(k+1) = 0 \end{cases} \tag{18}$$

Using (18), (17) becomes

$$\begin{aligned} (17) \Leftrightarrow\ & \xi_k^T \left[(\bar{A}_{ai} - \bar{B}_{ai}\bar{K}_{ai}\bar{C}_{ai})^T P_{aj}(\bar{A}_{ai} - \bar{B}_{ai}\bar{K}_{ai}\bar{C}_{ai}) \right. \\ & \left. + \bar{C}_{ai}^T\bar{C}_{ai} - P_{ai}\right]\xi_k + \omega_k^T\left[\bar{F}_{ai}^T P_{aj}\bar{F}_{ai} - \gamma^2 I\right] \\ & \times\omega_k + \xi_k^T(\bar{A}_{ai} - \bar{B}_{ai}\bar{K}_{ai}\bar{C}_{ai})^T P_{aj}\bar{F}_{ai}\omega_k \\ & + \omega_k^T\bar{F}_{ai}^T P_{aj}(\bar{A}_{ai} - \bar{B}_{ai}\bar{K}_{ai}\bar{C}_{ai})\xi_k < 0 \end{aligned}$$

which can be rewritten as

$$\begin{aligned} & \left[\, \xi_k^T \;\; \omega_k^T \,\right] \times \\ & \begin{bmatrix} \Pi_{i,j} - P_{ai} & (\bar{A}_{ai} - \bar{B}_{ai}\bar{K}_{ai})^T P_{aj}\bar{F}_{ai} \\ \bar{F}_{ai}^T P_{aj}(\bar{A}_{ai} - \bar{B}_{ai}\bar{K}_{ai}) & -\gamma^2 I + \bar{F}_{ai}^T P_{aj}\bar{F}_{ai} \end{bmatrix} \\ & \times \begin{bmatrix} \xi_k \\ \omega_k \end{bmatrix} < 0 \end{aligned} \tag{19}$$

where $\Pi_{i,j} = (\bar{A}_{ai} - \bar{B}_{ai}\bar{K}_{ai}\bar{C}_{ai})^T P_{aj}(\bar{A}_{ai} - \bar{B}_{ai}\bar{K}_{ai}\bar{C}_{ai}) + \bar{C}_{ai}^T\bar{C}_{ai}$. Then, (19) is negative for $[\, \xi_k \;\; \omega_k \,] \neq 0$ if

$$\begin{bmatrix} \Pi_{i,j} - P_{ai} & (\bar{A}_{ai} - \bar{B}_i\bar{K}_{ai}\bar{C}_{ai})^T P_{aj}\bar{F}_{ai} \\ \bar{F}_{ai}^T P_{aj}(\bar{A}_{ai} - \bar{B}_{ai}\bar{K}_{ai}\bar{C}_{ai}) & -\gamma^2 I + \bar{F}_{ai}^T P_{aj}\bar{F}_{ai} \end{bmatrix} < 0 \tag{20}$$

By Schur complement (12), (20) becomes

$$\left[\begin{array}{cc|cc} -P_{ai} & 0 & (\bar{A}_{ai} - \bar{B}_{ai}\bar{K}_{ai}\bar{C}_{ai})^T & \bar{C}_{ai}^T \\ * & -\gamma^2 I & \bar{F}_{ai}^T & 0 \\ \hline * & * & -P_{aj}^{-1} & 0 \\ * & * & * & -I \end{array}\right] < 0 \tag{21}$$

Now, pre- and post-multiplying (21) by $Z_{ai} = diag(X_{ai} = P_{ai}^{-1},\, I,\, I,\, I)$, (21) becomes

$$\begin{bmatrix} -X_{ai} & 0 & X_{ai}(\bar{A}_{ai} - \bar{B}_{ai}\bar{K}_{ai}\bar{C}_{ai})^T & X_{ai}\bar{C}_{ai}^T \\ * & -\gamma^2 I & \bar{F}_{ai}^T & 0 \\ * & * & -X_{aj} & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{22}$$

Now, to avoid the nonlinearities $\bar{B}_{ai}\bar{K}_{ai}\bar{C}_{ai}X_{ai}$ and $X_{ai}\bar{C}_{ai}^T\bar{K}_{ai}^T\bar{B}_{ai}^T$, let us consider the following matrices transformation describing a linear matrix equality:

$$\bar{M}_{ai}\bar{C}_{ai} = \bar{C}_{ai}X_{ai} \tag{23}$$

and substituting $\bar{K}_{ai} = \bar{N}_{ai}\bar{M}_{ai}^{-1}$ into (22), (14) is obtained.

Remark 1. The design of such a controller for continuous-time system (8) becomes trivial and can easily be deduced from the above one. For more details about this problem, some elements are given in Appendix A.

4. SIMULATION RESULTS USING REAL DATA

The simulations are conducted using a full Non Linear Four Wheels Vehicle Model (NLFWVM) (Menhour et al. [2013]) of a Peugeot 406. For our simulations, we suppose that the reference model describing the lateral displacement of the vehicle is given by

$$\begin{cases} \dot{x}\dot{x}_r(t) = -x_r(t) + r(t) \\ y_r(t) = x_r(t) \end{cases} \tag{24}$$

where $r(t) = V_{x_r(t)}\sin(\psi_r(t)) + V_{y_r(t)}\cos(\psi_r(t)) + a_{y_r}(t)$. For our simulation tests, the reference input $r(t)$ is constructed from the data acquired during an experimental tests. These tests are conducted on a race track with a professional driver under good conditions (see grey curves of Figures 3, 4, 6 and 5). For our simulations, the reference output y_r is computed using measured lateral acceleration a_{y_r}, yaw angle ψ_r), longitudinal V_{x_r} and lateral V_{y_r} speeds. Figure 2 shows the road bank angle $f(t) = \phi_r(t)$ used as unknown input for our simulations and its spectral domain is located in a low frequency range. For all trials, the car's acquisition device operates at frequency $200\,Hz$. For our simulations, switching rule (3) is used and two values $V_{x1} = 60km/h$ and $V_{x2} = 90km/h$ are chosen (see also the upper part of Figure 3). Consequently, two local models are obtained $M = 2$ and $(i, j) \in \{1, 2\}^2$. Then, the stability of two controllers (11) and (A) are guaranteed by the resolution of 4 LMIs/LMEs of theorem 1 and of 2 LMIs/LMEs of theorem 2 to obtain two Lyapunov matrices (X_{a1} and X_{a2}) and a common Lyapunov matrix X respectively. The LMIs/LMEs of theorems 1 and 2 are solved using YALMIP software (Löfberg [2004]). The obtained gains of discrete-time controller (11) are:

$$\bar{K}_{a1} = 0.05644 \quad \text{and} \quad \bar{K}_{a2} = 0.04887$$

and the H_∞ performances are obtained for $\gamma^* = 2.6$.

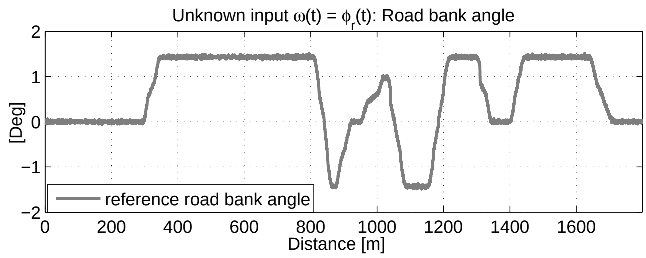

Fig. 2. Unknown input: measured road bank angle

Figures 4, 5, 6 and 7 show that the closed-loop simulation results obtained with two controllers and NLFWVM are similar to the measured ones. The main dynamical variables plotted are the derivative of lateral deviation (Figure 6) and trajectories (Figure 5).

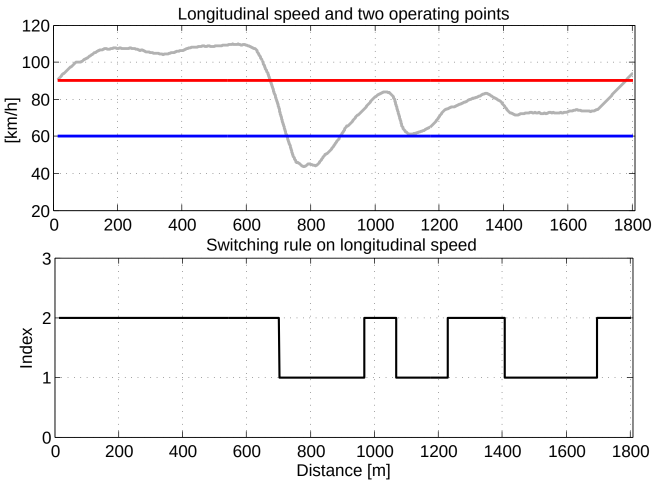

Fig. 3. Measured longitudinal speed with two operating points and switching rule

Figure 7 shows also the performances of two controllers in terms of tracking output errors $\tilde{y}$ which are less than $0.1\,m/s$. These tracking output errors are quite small, but, the controller (11) have less than the controller (A), this may be due to the conservatism of Lyapunov function approach used to design (A).

The steering angles input performances of the two controllers are depicted on Figure 4. The computed steering angles are similar to the measured ones.

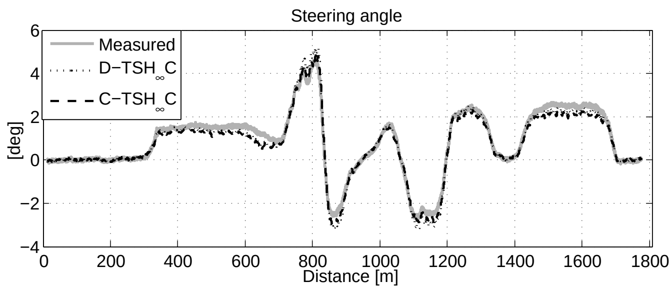

Fig. 4. Steering angles: real and those computed by switched discrete-time and continuous-time controllers

Figures 8 and 9 show the unknown input $f(t) = \phi_r(t)$ attenuation properties of continuous-time subsystems $((A_i - B_iK_iC_i),\ F_i,\ C_i)$ and discrete-time subsystems $((\bar{A}_{ai} - \bar{B}_{ai}\bar{K}_{ai}\bar{C}_{ai}),\ \bar{F}_{ai},\ \bar{C}_{ai})$ from the unknown input to controlled output variable. Moreover, by computing, $\|\omega\|_2 = \|f\|_2 + \|r\|_2$ and $\|\tilde{y}\|_2 = \|y - y_r\|_2$, which gives for continuous-time

$$\frac{\|\tilde{y}\|_2}{\|\omega\|_2} = 0.0021 < \gamma^* = 2.6$$

and for discrete-time

$$\frac{\|\tilde{y}\|_2}{\|\omega\|_2} = 0.0010 < \gamma^* = 2.6$$

this shows the effectiveness of the proposed controllers.

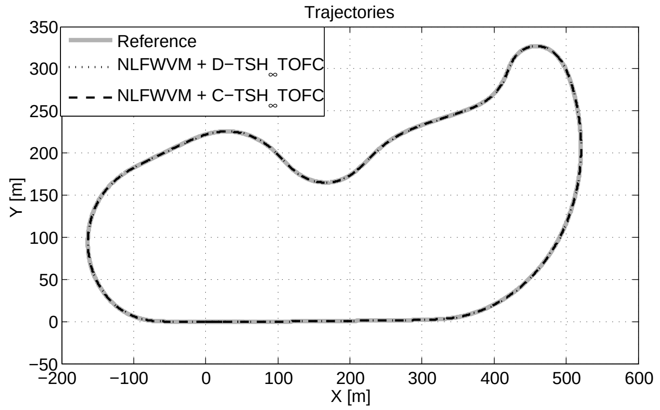

Fig. 5. Trajectories: Reference and controlled model

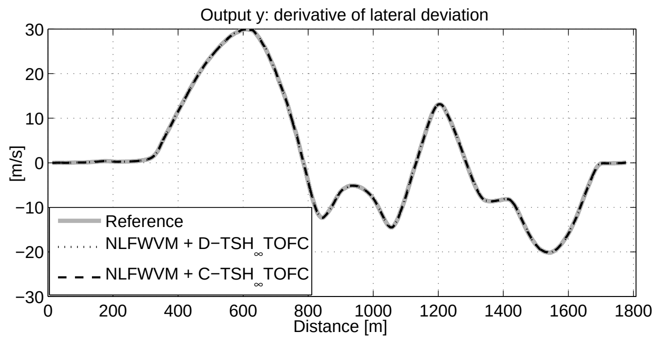

Fig. 6. Tracking output: reference y_r and model y

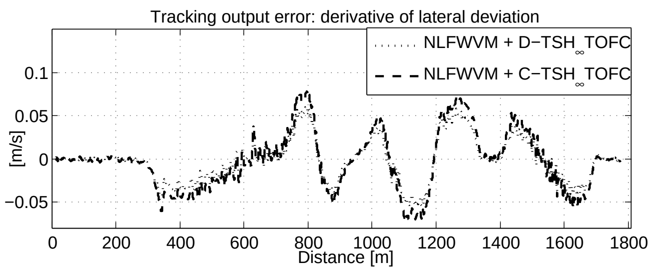

Fig. 7. Tracking output errors $\tilde{y}$

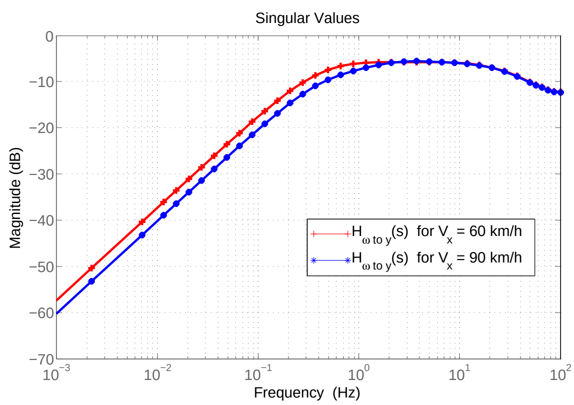

Fig. 8. Singular values of system $((A_i - B_iK_iC_i),\, F_i,\, C_i)$ for $V_x = 60$ and $90\,km/h$, between f to $\tilde{y}$

5. CONCLUSIONS AND FUTURE WORK

Two switched H$_\infty$ tracking output feedback controllers are proposed. Such controllers are developed for continuous-time and discrete-time switched linear systems. Based on a common Lyapunov function and switched Lyapunov functions methods, sufficient conditions for the existence of two controllers are established. All these conditions are expressed in terms of LMIs/LMEs constraints. The design of such controllers gains is reduced to solve a set of LMIs/LMEs using YALMIP software. The effectiveness of the proposed controllers is shown through the steering vehicle control implementation. The simulation tests are conducted using real data.

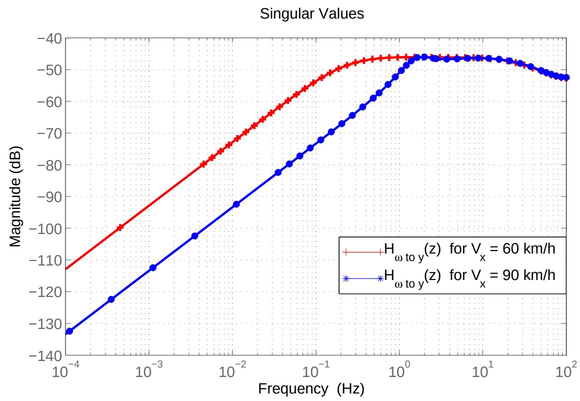

Fig. 9. Singular values of system $((\bar{A}_{ai} - \bar{B}_{ai}\bar{K}_{ai}\bar{C}_{ai}),\, \bar{F}_{ai},\, \bar{C}_{ai})$ for $V_x = 60$ and $90\,km/h$, between f to $\tilde{y}$

The steering vehicle control example assumes the lateral speed to be a measurable state. However, such a variable is measured by an expensive sensor. For this problem, it will be interesting to design a robust H$_\infty$ unknown input observer for system (1) with yaw motion as measurable output.

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Appendix A. SWITCHED H_∞ TRACKING OUTPUT FEEDBACK CONTROL: CONTINUOUS-TIME CASE

Problem: Consider the following switched controller for continuous-time system (8)

$$u(t) = -\sum_{i=1}^{M} \alpha_i(t) K_{ai} \tilde{y}(t) \tag{A.1}$$

where the gains $K_{ai} \in \mathbb{R}^{p\times(n+n_r)}$ are computed such that the specifications S1 and S2 in continuous-time are ensured.

For the above design problem, the following theorem is trivial and can easily be established using common Lyapunov function.

Theorem 2. Suppose that for $i \in \{1, \cdots, M\}$, the pair $(A_{ai},\ B_{ai})$ is stabilizable. If there exist a positive scalar $\gamma > 0$, matrices $X > 0$, M_{ai} and N_{ai} such that the following LMI and LME are satisfied:

$$\begin{bmatrix} \bar{\Phi}_{ai} & F_{ai} & XC_{ai}^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{A.2}$$

$$C_{ai}X = M_{ai}C_{ai} \tag{A.3}$$

for $i \in \{1, \cdots, M\}$, then, the stabilizing gains of (A) are given by $K_{ai} = N_{ai}M_{ai}^{-1}$ with $\bar{\Phi}_{ai} = XA_{ai}^T + A_{ai}X - C_{ai}^T N_{ai}^T B_{ai}^T - B_{ai}N_{ai}C_{ai}$.

Remark 2. The proof of the above theorem can be deduced using the same reasoning of the discrete-time case.

Notation 1. $(.)^T$ stands for the transpose matrix, $(.) > 0$ $(\geqslant 0)$ denotes a symmetric positive definite matrix (semidefinite), we use an asterisk $(*)$ to represent a term that is induced by symmetry, $diag(.)$ stands a diagonal blok matrix. V_x: longitudinal speed $[km/h]$ and V_y: lateral speed $[km/h]$, $\dot{\psi}$:yaw rate $[rad/s]$, ψ: yaw angle $[rad]$, δ: wheel steer angle $[rad]$, ϕ_r: road bank angle $[rad]$, $C_{f,r}$: front and rear cornering stiffnesses $[N/rad]$, β: tire sideslip angle $[rad]$, $L_{f,r}$: distances from the CoG to the front and rear axles $[m]$, I_z: yaw moment of inertia $[Kg.m^{-2}]$, m: vehicle mass $[kg]$, g: acceleration due to gravity $[m/s^2]$, CoG: Center of Gravity.