MRL-based importance measures

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In this paper, we propose a novel importance measure, namely LIM, which is defined as the improvement ability in the system residual life when replacing a component/group of components at a given time. LIM measure allows considering the current condition (state or degradation level) of all components at given time and the system structure into a single metric to rank a component/group of components regarding to the system life time improvement ability. Moreover, to take into account economic aspects (e.g., preventive maintenance costs, benefit gained by preventive maintenance and economic dependence between components), an extension of LIM measure is then investigated. Thanks to LIM measure and its extension, a component/group of components can be "optimally" selected for preventive maintenance regarding to the technical criterion (residual life of the system) and/or the economic issues (benefit and cost). A numerical example of a 4-component system is introduced to illustrate the use and the advantages of the proposed importance measures.

Keywords: Importance measure, residual life, reliability, condition-based maintenance, economic dependence, multi-component system

1. Introduction

Importance measures providing information about the importance of a component or a group of components on the system performance (reliability/availability, productivity, safety, or any performance metrics of interest) can help to identify design weakness or operation bottlenecks and to suggest optimal modifications for system upgrades. A large number of importance measures have been developed and successfully applied for various purposes, see [8] for an overview about recent advances on importance measures. In risk analyses, importance measures are used in risk-informed decision-making, [4, 5]. In reliability engineering, they are used to prioritize components in a system for reliability improvement, [1, 2, 3]. Recently, importance measures have been applied for maintenance optimization and spare parts management [9, 13, 15]. More recently, the link between component importance and preventive maintenance decision making has been discussed in [15].

In the framework of condition-based maintenance optimization, the monitoring information on the current condition (state or degradation level) of components can be crucial for decision-making process. However, very few existing importance measures allow incorporating the condition of components at a given time. Moreover, in practice, positive economic dependence, which implies that joint maintenance of several components is cheaper than performing maintenance on components separately, often exists and should be integrated in maintenance decision-making. To the best of our knowledge, no existing importance measure allows taking into account this kind of interaction between components. To face this issue, we propose here a novel importance measure based on the system residual life, namely LIM measure, that can be used to rank the components/groups of components with respect to their improvement ability in the system residual life, given the current condition (state or degradation level) of all components at given time. Moreover, to take into account economic aspects (e.g., preventive maintenance costs, benefit gained by preventive maintenance and economic dependence

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between components), an extension of LIM measure is then investigated. Thanks to LIM measure and its extension, a component/group of components can be "optimally" selected for preventive maintenance regarding to the technical criterion (residual life of the system) and/or the financial issues (benefit and cost).

This paper is organized as follows. Section 2 is devoted to the description of the system modelling and related assumptions. The mean residual life of the system is also discussed. The proposed LIM measure is proposed in Section 3. The influence of the components' information level at a given time on the LIM measure and LIM's importance ranking is studied. In addition, an extension of LIM measure is also herein given. To illustrate the uses of LIM measure and its extension, a numerical example of a 4-component system is introduced in Section 4. In addition, some numerical results are herein analyzed and discussed. Finally, the last section presents the conclusions drawn from this work.

2. System modeling & residual useful life

2.1. *System description and assumptions*

We consider a coherent system composed of n non-identical components which are interconnected according to a structure in terms of reliability block diagram (RBD). Each component is subject to an underlying deterioration process X_t^i which can cause random failures. It is assumed that the degradation evolution of component i ($i = 1, ..., n$) can be assumed to be described by a stochastic process X_t^i. Component i is considered as failed when its degradation level searches a critical threshold Z_i, $X_t^i \geq Z_i$. Z_i is also called the failure threshold. In that way, the reliability of component i can be expressed as follows:

$$R^i(t) = \mathbb{P}(X_t^i < Z_i) = \int_0^{Z_i} f_D^i(x)dx, \quad (1)$$

where $f_D^i(.)$ is the pdf describing the deterioration process of component i. It is shown in the literature that Gamma stochastic processes is widely used for modelling the degradation process of components [12]. A detailed description is given in Appendix.

Based on the components' reliability, the system reliability

Let $R(t)$ be the system reliability. $R(t)$ is a function of all component reliability $R^i(t)$ [11].

$$R(t) = \varphi(R^1(t), R^2(t), ..., R^n(t)). \quad (2)$$

$\varphi(.)$ can be obtained by using the minimal path set concept or the minimal cut set one [11].

2.2. *Mean residual life*

Let $\mathfrak{F}_t$ be the current state of the system at time t: $\mathfrak{F}_t = 1$ if the system is working at time t, $\mathfrak{F}_t = 0$ for otherwise. Assume now the system is functioning at time t ($\mathfrak{F}_t = 1$), the predictive reliability of the system within the interval horizon $(t, t+u)$ (with $u > 0$) can be obtained from Equation (2) by replacing the components reliability $R^i(t)$ by the components' conditional reliability $R^i(t+u|\mathfrak{F}_t^i)$ with $i = 1, 2, ..., n$. $\mathfrak{F}_t^i$ is the available information on the component i at time t.

It should be noticed that if the system is not working at time t ($\mathfrak{F}_t = 0$): $R(t+u|\mathfrak{F}_t = 0) = 0$. For the evaluation of the conditional reliability for component i, four cases are herein specified according to the type of available information $\mathfrak{F}_t^i$:

- If component i is not working at time t, $\mathfrak{F}_t^i = 0$ and $R^i(t+u|\mathfrak{F}_t^i = 0) = 0$;
- If component i is working but its deterioration level is not measured, $\mathfrak{F}_t^i = 1$, the conditional reliability of component i is then calculated as

$$R^i(t+u|\mathfrak{F}_t^i = 1) = \frac{R^i(t+u)}{R^i(t)}. \quad (3)$$

  It is important to note that if the failure behaviour of component i follows an exponential distribution, then $R^i(t+u|\mathfrak{F}_t^i = 1) = R^i(u)$. This means that component i is considered as new one if it survives at time t, consequently, there is no need for preventive maintenance if this is known.
- If component i is working and its deterioration level is measured at time t, $\mathfrak{F}_t^i = 2$, the conditional reliability of component i is then calculated as

$$\begin{aligned} R^i(t+u|\mathfrak{F}_t^i = 2) &= \mathbb{P}(X_{t+u}^i < Z_i | X_t^i = x_t^i) \\ &= \int_{x_t^i}^{Z_i} f_D^i(x)dx, \end{aligned} \quad (4)$$

  with $X_t^i = x_t^i$ is the deterioration level of component i at time t.
- If component i is replaced by a new one at time t, $\mathfrak{F}_t^i = 3$, its conditional reliability is evaluated as: $R^i(t+u|\mathfrak{F}_t^i = 3) = R^i(u)$.

The mean residual life of a system at time t is defined as the average duration left before the system fails. Mathematically, it can be written as follows

$$MRL(t) = \int_0^\infty R(t+u|\mathfrak{F}_t = 1)dx. \quad (5)$$

It is clear that $MRL(t)$ depends not only on the time t but also on the components' information level given at time t. As an example, if at time t the system is still working and a component i is instantaneously replaced, the mean residual life of a system is then:

$$MRL(t|\mathfrak{F}_t^i = 3) = \int_0^\infty R(t+u|\mathfrak{F}_t = 1, \mathfrak{F}_t^i = 3)dx. \quad (6)$$

3. Mean residual life-based importance measures

3.1. *Definition of LIM measure*

Mean residual life-based importance measure for component i is defined as follows:

$$LIM^i(t) = MRL(t|\mathfrak{F}_t^i = 3) - MRL(t) \quad (7)$$

By definition, this importance measure provides the potential improvement in the system residual life when component i is replaced at time t given the current condition/information of all components of the system. The LIM measure has the following properties:

- For a coherent system $MRL(t|\mathfrak{F}_t^i = 3) \geq MRL(t)$, consequently $LIM^i(t)$ is non negative;
- $LIM^i(0) = 0$;
- $LIM^i(t) = 0$ if component i is still working at time t and its failure rate is time-independent;
- $LIM^i(t)$ depends on the current information (state or degradation level) of all components given at time t;
- In addition, $LIM^i(t)$ is time-dependent if the degradation process of one or several components is time-dependent.

For decision-making, since LIM measure allows integrating the current condition (state or degradation level) of all components, it seems to be an effective indicator in finding the most important component which should be replaced, at given time t, to improve the system residual life.

As an example, we consider a series structure of two components $C1$ and $C2$. It is assumed that the degradation process of each component is described by a Gamma process with shape and scale parameter (α^i, β^i) with $i = 1, 2$. The failure threshold of C1 and C2 is $Z_1 = Z_2 = 100$. The reliability block diagram (RBD) of the system and the degradation parameters are shown in Figure 1.

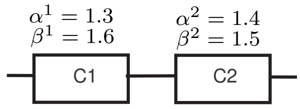

Fig. 1. A 2-component system.

The reliability of the system is expressed as:

$$R(t) = R^1(t).R^2(t).$$

Assume that the system is still functioning at time t, i.e., the two components are still functioning, $\mathfrak{F}_t^i \neq 0$ with $i = 1, 2$. The mean residual life of the system without any maintenance action at time t is calculated by:

$$MLR(t) = \int_0^\infty R^1(t+u|\mathfrak{F}_t^i \neq 0).R^2(t+u|\mathfrak{F}_t^i \neq 0)du.$$

The LIM measure of component i ($i = 1, 2$) can be evaluated as follows:

$$LIM^i(t) = \int_0^\infty R^{3-i}(t+u|\mathfrak{F}_t^{3-i} \neq 0)[R^i(u) - R^i(t+u|\mathfrak{F}_t^i \neq 0)]du. \quad (8)$$

Regarding to the components information level available at time t, two cases are considered:

(1) The deterioration level of the two components is unknown, i.e., $\mathfrak{F}_t^1 = \mathfrak{F}_t^2 = 1$. LIM measure of each component is evaluated by using Equations (8) and (3) and the obtained results for $t = 10$ are shown in Table 1. According to LIM's value, component C2 is more important than C1. As a consequence, the replacement of C2 is more effective in improving the system residual life time than replacement of C1.

(2) The degradation level of each component is measured, i.e., $\mathfrak{F}_t^1 = \mathfrak{F}_t^2 = 2$. Since two components are subject to a stochastic degradation process, the degradation level of each component at time can be random. As an example, two experimentals, namely cases 2a et 2b, are realized to simulate the degradation evolution of the two components. For each component, its degradation level at time t given by two experimentals are different and reported in Table 1. Based on the components' degradation level at time t, LIM measure of each component is evaluated by using Equations (8) and (4). In the first experimental (case 2a), $LIM^1(t) > LIM^2(t)$, C1 is thus more important than C2 in improving the system residual life time. It should be noticed that this importance ranking is not the same the one given when the components' degradation levels are unknown. In addition, in the

second experimental (case 2b), $LIM^1(t) < LIM^2(t)$, i.e., C2 becomes more important than C1.

Table 1. LIM measure and components ranking.

	Information at $t = 10$				LIM measure		Ranking	
	$\mathfrak{F}_t^1$	X_t^1	$\mathfrak{F}_t^2$	X_t^2	$LIM^1(t)$	$LIM^2(t)$	C1	C2
Case 1	1	-	1	-	2.69	3.06	2	1
Case 2a	2	18.03	2	14.99	2.85	1.93	1	2
Case 2b	2	15.91	2	24.16	1.10	4.83	2	1

These experimental results show that both the information level and the degradation level of the components have an important impacts on the LIM value and the associated components importance ranking.

3.2. *LIM measure for a group of components*

The LIM measure can be applied to a group of components. Indeed, LIM measure of a group G containing k components $\{j_1, ..j_k\}$ with $k = 2, 3, ...$ at time t can be written as follows:

$$LIM^{\{j_1,..jk\}}(t)$$
$$= MRL(t + u|\{\mathfrak{F}_t^{j_1} = 3, ..., \mathfrak{F}_t^{j_k} = 3\}) - MRL(t), \quad (9)$$

where $MRL(t+u|\{\mathfrak{F}_t^{j_1} = 3, ..., \mathfrak{F}_t^{j_k} = 3\})$ is the system residual life when k components $\{j_1, ..j_k\}$ are jointly replaced at time t.

It is clear that $LIM^{\{j_1,..jk\}}(t)$ provides the potential improvement in the system residual life time thanks to the replacement of a group of components at time t. Therefore, LIM measure can help to select a group of components to be preventively replaced. The use of LIM measure in raking a component/group of components is illustrated in Section 4.

3.3. *An extension of LIM measure*

Maintenance cost and benefit often take an important role in maintenance decision-making and should be integrated in the decision-making. To this end, LIM measure is extended to incorporate both the benefit gained by the improvement ability in the system residual life time, thanks to a maintenance action, and the associated maintenance cost.

The extension of LIM measure for a component i at time t is defined as follows:

$$LIM_c^i(t) = \frac{h(LIM(t))}{C_P^i}, \quad (10)$$

where:

- C_P^i is the preventive maintenance cost of component i;
- $h(LIM(t))$ is a function of the system life improvement thanks to the replacement of component i at time t.

$h(LIM(t))$ may be linear or non-linear function. As an example, a linear function is herein used:

$$h(LIM(t)) = B.LIM(t), \quad (11)$$

with B is positive real number. The factor B can be seen as the benefit rate regarding to the system operating time. The two following cases are specified:

- $B = 1$, i.e., $h(LIM(t)) = LIM(t)$, $LIM_c^i(t)$ represents the ratio of the improvement ability in the system life time, thanks to the replacement of component i at time t, to its replacement cost;
- $B > 1$, $h(LIM(t))$ can be also expressed as a benefit gained from the system life time improvement resulting from the replacement of component i at time t. $LIM_c^i(t)$ can help to find the most cost-effective component to be preventively maintained according to a benefit-cost threshold K ($K \geq 1$). More precisely:
  - if $LIM_c^i(t) \geq K$, component i is then called cost-effective one at time t, i.e., component i could be an admissible component for preventive maintenance;
  - $0 \leq LIM_c^i(t) < K$ means that the cost benefit resulting from the replacement of component i is not enough. As a consequence, component i is not cost-effective at time t, i.e., it should not be selected for preventive maintenance at least from a financial point of view.

In the same sprit of LIM measure, LIM_c can be applied for a group of several components as follows:

$$LIM_c^{\{j_1,..j_k\}}(t, u) = \frac{h(LIM^{\{j_1,..j_k\}}(t, u))}{C_P^{\{j_1,..j_k\}}}, \quad (12)$$

where $C_P^{\{j_1,..j_k\}}$ is the total maintenance cost when replacing the group components $(j_1, ..j_k)$ together. It is important to note that in a multi-component system, economic dependence often exists between components [7, 10]. In fact, economic dependence means that the joint maintenance of several components is cheaper than when these components are separately maintained. It is also pointed out in the literature that the economic dependence between components has a significant impact on total maintenance cost and should be integrated in maintenance decision-making process [6, 14].

From an economical point of view, the LIM_c measure seems to interesting indicator for decision-making since it allows considering not only the benefit associated with the system life time improvement, the maintenance cost but also the economic dependence between components. An illustration on the use of LIM_c measure for importance ranking and maintenance decision-making is presented in the next section.

4. Numerical example

The objective of this section is to show how LIM measure and its extension LIM_c can be used for ranking components/groups of components under given information level about the components' condition at a given time t. To this end, we consider a 4-non repairable component system whose RBD is given in Figure 2. It should be noticed that the proposed importance measures (LIM and LIM_c) can be applied for any kind of systems represented by a reliability bloc diagrams (RBD).

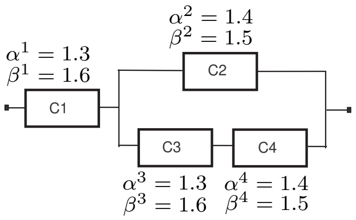

Fig. 2. An example of a 4-component system.

It is assumed that the components' reliability behavior are described by homogeneous gamma stochastic processes (see Appendix) with shape and scale parameters α^i, β^i ($i = 1, .., 4$) which are shown in Figure 2. The failure threshold associated to the components are $Z_1 = Z_2 = Z_3 = Z_4 = 100$.

The reliability of the system is expressed as:

$$R(t) = R^1(t).R^2(t) + R^1(t).R^3(t).R^4(t) - R^1(t).R^2(t).R^3(t).R^4(t).$$

Assume that the system is still functioning at time t atu (arbitrary time unit). In order to improve the system life time, once or several components should be maintained at time t. The later raises the interesting and challenging question of selective maintenance, i.e. which component(s) should be chosen for preventive replacement ? We propose here an heuristic decision rule based on LIM and/or LIM_c measure(s), i.e. the component having the highest importance ranking, regarding to LIM or/and LIM_c criterion, should be selected. To illustrate the use of LIM and LIM_c, we consider in this study that only one component can be replaced at given time t, even if, of course, LIM and its extension LIM_c can be applied and used to select any group of components.

It is also assumed that the degradation level of all components is measured at time t. It should be noticed that the impacts of information level on LIM measure and the associated importance ranking is already discussed in Section 3.1.

4.1. *LIM measure and importance ranking*

Suppose that the maintenance costs are not considered due to whatever reason, e.g. they are not available. Based on LIM, an importance ranking is provided which help to select an "optimal" component for preventive maintenance. Table 2 illustrates the use of LIM importance measure for components ranking and maintenance prioritization. At each given time point ($t = 10, 30$), LIM of each component is first evaluated, the associated components ranking is then determined. The results show that the LIM value of each component depends on the degradation level of all components measured at time t. As a consequence, the LIM importance of a component may change with time. For example, when $t = 10$ the most important component for the system life time improvement is C1 which should be selected to be preventively replaced to improve the system residual life time. However, C1 may be no longer be the best choice at other time points, i.e., C2 becomes the most important one when $t = 30$.

Table 2. LIM measure and importance ranking.

	$t = 10$			$t = 30$		
Unit	X_t^i	LIM^i	Ranking	X_t^i	LIM^i	Ranking
C1	27.46	2.42	1	52.28	0.45	3
C2	21.07	1.85	2	81.35	6.42	1
C3	27.53	1.62	3	61.71	3.90	2
C4	17.09	0.33	4	49.71	0.32	4

4.2. *LIM_c for ranking components and selective maintenance*

In this section, the costs are assumed to be known: $C_p^1 = 250$, $C_p^2 = 200$, $C_p^3 = 100$, $C_p^4 = 110$ acu (arbitrary cost unit). The system life time improvement resulting from the replacement of a such component i, $LIM^i(t)$, is herein converted into an economic benefit with benefit rate $B = 500$ acu. To integrate the benefit and maintenance costs in component's importance ranking, LIM_c is herein used and the obtained results are reported in Table 3.

It is shown that the importance raking based on LIM_c is not the same as the one given by LIM (see again Table 2). This can be explained by the fact that LIM focuses only on the system

Table 3. LIM_c measure and importance ranking.

Unit	$t=20$			$t=30$		
	X_t^i	LIM_c^i	Ranking	X_t^i	LIM_c^i	Ranking
C1	27.46	4.84	2	52.28	0.89	4
C2	21.07	4.63	3	81.35	**16.04**	2
C3	27.53	8.08	1	61.71	**19.50**	1
C4	17.09	1.52	4	49.71	1.46	3

residual life improvement, while LIM_c takes into consideration both the benefit associated with the system life time improvement and the corresponding maintenance cost. For example, when $t = 10$, C1 is the most important component according to LIM, whereas C3 is the most important one according to LIM_c.

Now, for maintenance decision-making, we assume that a component is considered as cost-effective if its LIM_c value is not lower than a benefit-cost threshold $K = 10$. In that way, at time $t = 20$, any component is cost-effective. This means that any replacement action should be carried at time $t = 20$. However, C2 and C3 become cost-effective at time $t = 30$. This implies that both C2 and C3 are admissible components for preventive maintenance at time t. Of course, according to LIM_c criterion, C3 is the most important component. Note that C2 is the most important regarding to the LIM criterion (see again Section 4.1).

4.3. *Joint consideration of LIM and LIM_c*

Both LIM and its extension, LIM_c, are jointly considered to find the most appropriate component for maintenance decision-making. Figure 3 shows the LIM vs LIM_c at two different time points (t=20 and t=30).

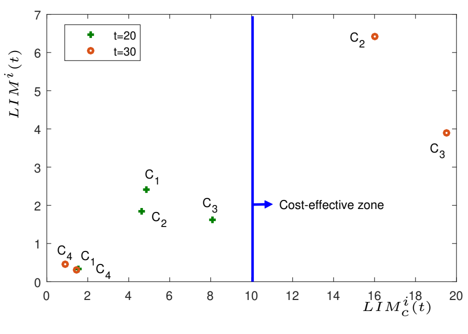

Fig. 3. Joint consideration of LIM and LIM_c measure.

The most important components are given in the top right corner of the figure and the components in the bottom left corner are the less important. It is clear that each criterion provides an importance ranking, and it is thus difficult to find the most important component. However, if the decision maker judges that the system life time improvement is a priority criterion, C2 should be then selected for preventive maintenance at time $t = 30$. Otherwise, if an improvement of 3.90 atu in the system life time is enough (e.g., due to a technical reason), C3 should be then the best choice.

5. Conclusions

In this work, a novel importance measure, namely LIM which is defined as the improvement ability in the system residual life when replacing a component/group of components, is introduced. LIM measure allows ranking a component/group of components regarding to the system life time improvement ability by considering both the current condition (state or degradation level) of all components at given time and the system structure into a singe technical metric. Moreover, to take into account economic aspects (e.g., maintenance costs, economic dependence between components and the benefit given by maintenance operations), an extension of LIM measure is then developed. In that way, LIM_c can help to find the most cost-effective component/group of components. The use and advantages of both LIM and LIM_c measure are then illustrated through an numerical example of a 4-non repairable component system. The numerical results show that, at given time, LIM and LIM_c depend strongly on the components condition and may provide two different importance rankings. However, from a practical point of view, LIM and LIM_c measures are complementary and should be jointly considered in order to find the most appropriate component/group of components to maintain.

Our future research works will focus on the investigation of the proposed importance measures, LIM and its extension, with consideration of economic and stochastic dependence. Another perspective should be a comparison study with others importance measures in ranking components and decision-making.

Appendix: Reliability evaluation with Gamma deterioration process

Assume that the degradation process of component i is described by a Gamma processes $(\tilde{X}_t)_{t\geq 0}$ which has the following characteristics:

- $(\tilde{X}_t)_{t\geq 0}$ has independent increments;
- for all $0 \leq l < t$, the random increment $\tilde{X}_t - \tilde{X}_l$ follows a Gamma probability density (pdf) with shape parameter $\alpha^i(t-l)$ and scale parameter β^i:

$$f_D^j(x) = \frac{1}{\Gamma(\alpha^i(t-l))}(\beta^i)^{\alpha^i(t-l)} x^{\alpha^i(t-l)-1} e^{-\beta^i x} \mathcal{I}_{\{x\geq 0\}},$$

where, $\mathcal{I}_{\{x\geq 0\}}$ is an indicator function. $\mathcal{I}_{\{x\geq 0\}} = 1$ if $x \geq 0$, $\mathcal{I}_{\{x\geq 0\}} = 0$ and otherwise;

The mean deterioration speed and its variance are α^i/η_i and $\alpha^i/(\beta^i)^2$ respectively. Various deterioration behaviors can be modeled by changing the couple of parameters α^i, β^i.

The reliability of component i can be evaluated as follows

$$R^i(t) = \int_{x_0^i}^{Z_i} \frac{1}{\Gamma(\alpha^i t)} (\beta^i)^{\alpha^i t} x^{\alpha^i t - 1} e^{-\beta^i x} dx,$$

where x_0^i is the degradation level of component i at time 0.

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