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Abstract— **This paper presents a novel wideband sweep non-linearity software correction method for a frequency modulated continuous wave (FMCW) transceiver based on the high-order ambiguity function (HAF) and time resampling. By emphasizing the polynomial-phase nature of the FMCW signal, it is shown that the HAF processing algorithm is well suited for estimating the sweep nonlinearity coefficients. The estimated coefficients are used to build a correction function which is applied by resampling the beat signal on each sweep interval. The sweep nonlinearity correction procedure is validated on real data acquired with a low-cost X-band T/R module.**

I. INTRODUCTION

The frequency modulated continuous wave (FMCW) transceiver is currently used in applications such as radio-altimeters, anti-collision systems, navigation radars [1] or as sensors for inhomogeneity identification on transmission lines [2]. A system based on a FMCW transceiver continuously transmits a chirp signal and uses deramp processing of the received waveform which encodes the range information in the beat signal's frequency. An essential problem of this type of transceiver is that the voltage controlled oscillator (VCO) adds a certain degree of nonlinearity which leads to a deteriorated resolution by spreading the target energy through different frequencies [3]. This problem is usually solved either by hardware [4], [5], [6] or software [7], [8], [9] approaches. The simplest hardware correction is the use of a predistorted VCO control voltage to have a linear frequency modulation output [4]. However, this approach does not work when the external conditions (e.g. temperature, supply voltage) change. Software solutions typically use an additional path in the transceiver with a known propagation delay (usually a surface acoustic wave delay line) for estimating the nonlinearities. The estimation is done either by breaking the beat signal into several sub-bands and computing the frequency peaks [9] or by extracting the phase information from the analytical signal [7], [8]. In [7] the correction is done by assuming that the phase error is proportional with the propagation delay. The correction method proposed in [8] needs to up-sample the beat signal in order to satisfy the Nyquist condition for the nonlinearity bandwidth and consequently this method is not well suited for large bandwidth nonlinearities (up to gigahertz).

This paper proposes a novel nonlinearity software correction method for large bandwidth nonlinearities. The method is designed for nonlinearities that can be described by a polynomial expression. This assumption leads to a polynomial-phase FMCW signal. The coefficients of the polynomial-phase signal (PPS) are estimated using the high-order ambiguity function (HAF) [10] on a reference response which can be either a delay line or a high reflectivity target whose propagation delay should be roughly known. Afterwards, with the estimated coefficients the nonlinearity correction function is built and applied through a time resampling procedure. The processing is applied on the beat frequency signal, so the bandwidth of the nonlinear term from the transmitted signal doesn't impose the sampling rate. The correction algorithm is tested on real data acquired with a demonstrator system based on an X-band low-cost FMCW transceiver.

The remaining of the paper is organized as follows. Section II presents the nonlinearity correction algorithm and is divided in three parts. The nonlinear FMCW signal model is discussed first. Then, the estimation method of the FMCW signal coefficients is described. The time resampling based correction procedure is exposed in the last part of the second section. Section III shows results of the developed nonlinearity correction algorithm applied to real data collected with a FMCW transceiver demonstrator system. Finally, the conclusions are stated in Section IV.

II. NONLINEARITY CORRECTION ALGORITHM

A. NONLINEAR FMCW SIGNAL MODEL

The slope of the frequency-voltage characteristic for some radio frequency VCOs may be reasonably approximated by a quadratic curve [11]. However, a more general approach is to assume a polynomial frequency-voltage dependence. With this assumption, for a linear tuning voltage sweep, the transmitted analytical signal in a sweep period T_p can be written as:

$$s_T(t) = \exp\left[j2\pi\left(f_0 t + \frac{1}{2}\alpha_0 t^2 + \sum_{k=2}^{K}\frac{\beta_k}{k+1}t^{k+1}\right)\right], \quad (1)$$

where f_0 is the initial frequency, α_0 is the linear chirp rate in the origin and β_k with $k = \overline{2, K}$ are the nonlinearity

coefficients. In the following, it is considered that the reflected signal comes from N different targets. The signal received from a target can be expressed as a delayed and attenuated version of the transmitted signal $s_T(t)$:

$$s_R(t) = \sum_{i=1}^{N} A_i s_T(t - \tau_i), \qquad (2)$$

where τ_i and A_i are the propagation delay and amplitude of the signal received from target i. By mixing the transmitted and received signals, the beat signal is:

$$s_b(t) = \sum_{i=1}^{N} A_i s_T(t) s_T^*(t - \tau_i), \qquad (3)$$

Due to the fact that the propagation delay is typically a few orders of magnitude smaller than the sweep period, the other higher-order terms can be neglected. In consequence the beat signal can be written in the form of a polynomial-phase signal:

$$s_b(t) = \sum_{i=1}^{N} A_i \exp\left[j2\pi \left(f_0 + \alpha_0 t + \sum_{k=2}^{K} \beta_k t^k \right) \tau_i \right]. \quad (4)$$

If the range profile is computed as the Fourier transform of the multi-component PPS signal in (4), the energy of each target would be spread and the resolution deteriorated.

The correction procedure proposed in this paper aims to eliminate this effect by turning the multi-component PPS into a sum of N complex sinusoids. In this way each target appears as a *sinc* function in the range profile. The correction procedure consists of two steps: an estimation of the FMCW signal coefficients (linear chirp rate α_0 and nonlinearity coefficients β_k) using the high-order ambiguity function and a correction of the beat signal by time resampling.

B. ESTIMATION OF THE FMCW SIGNAL COEFFICIENTS

The estimation is based on the presence of a reference target response (with amplitude A_{ref} and propagation delay τ_{ref}) in the beat signal. This particular PPS component can be extracted by bandpass filtering the beat signal around the beat frequency corresponding to τ_{ref}. The reference signal can be written as:

$$s_b(t, \tau_{ref}) = \exp\left[j2\pi \left(f_0 + \alpha_0 t + \sum_{k=2}^{K} \beta_k t^k \right) \tau_{ref} \right]. \tag{5}$$

Based on the obtained reference signal, the FMCW signal coefficients can be estimated using the HAF. The starting point is the high-order instantaneous momentum (HIM), which can be defined for a signal $s(t)$ as [12]:

$$HIM_k[s(t); \tau] = \prod_{i=0}^{k-1} \left[s^{(*i)}(t - i\tau) \right]^{\binom{k-1}{i}}, \qquad (6)$$

where k is the HIM order, τ is the lag and $s^{(*i)}$ is an operator defined as:

$$s^{(*i)}(t) = \begin{cases} s(t) & \text{if } i \text{ is even,} \\ s^*(t) & \text{if } i \text{ is odd,} \end{cases} \qquad (7)$$

where i is the number of conjugate operator "*" applications. The high-order ambiguity function (HAF) is defined as the Fourier transform of the HIM.

If we assume a PPS model for the analyzed signal, i.e.:

$$s_{PPS}(t) = A \exp\left[j2\pi \sum_{m=0}^{k} a_m t^m \right], \qquad (8)$$

the essential property of the HIM is that, the k-th order HIM is reduced to a harmonic with amplitude $A^{2^{k-1}}$, frequency f_k and phase Φ_k:

$$HIM_k[s_{PPS}(t); \tau] = A^{2^{k-1}} \exp\left[j \left(2\pi \tilde{f}_k t + \Phi_k \right) \right], \quad (9)$$

where

$$\tilde{f}_k = k! \tau^{k-1} a_k. \qquad (10)$$

So the HAF of this HIM should have a spectral peak at the frequency f_k. Based on this result, an algorithm that estimates sequentially the coefficients a_k was proposed in [10]. Starting with the highest order coefficient, at each step, the spectral peak is determined, and an estimation value $\hat{a}_k$ of a_k is computed from (10). With this value, the phase term of the higher order is removed:

$$s_{PPS}^{(k-1)}(t) = s_{PPS}^{(k)}(t) \exp\left(-j2\pi \hat{a}_k t^k \right) \qquad (11)$$

and the procedure repeats iteratively. A typical problem of this nonlinear method is the propagation of the approximation error from one higher order to the lower ones, but the effect is not critical if only a small approximation order (3 or 4) is required [13], which is the case of typical frequency-voltage VCO characteristics.

After applying this iterative algorithm to the FMCW reference signal and obtaining the polynomial phase coefficients, the linear chirp rate and the nonlinearity coefficients can be computed as:

$$\begin{aligned} \alpha_0 &= \frac{a_1}{\tau_{ref}} \\ \beta_k &= \frac{a_k}{\tau_{ref}}, k = \overline{2, K}. \end{aligned} \qquad (12)$$

C. TIME RESAMPLING

Due to the fact that the frequency-voltage characteristic of a VCO is monotonous, for a linear voltage sweep the resulting polynomial phases of the beat signal components are monotonous functions for $t \in [0, T_p]$. Therefore, the beat signal in (4) can be rewritten as:

$$s_b(t) = \sum_{i=1}^{N} A_i \exp\left\{ j2\pi \left[f_0 + \alpha_0 \theta(t) \right] \tau_i \right\}, t \in [0, T_p], \quad (13)$$

where

$$\theta(t) = t \left(1 + \sum_{k=2}^{K} \frac{\beta_k}{\alpha_0} t^{k-1} \right) \qquad (14)$$

is a monotonous function of time t, which can be interpreted as a new time axis. Hence, if the time axis is changed to θ, the beat signal becomes a sum of N complex sinusoids, which was the purpose of the correction algorithm. Notice that in the definition of θ the nonlinearity coefficients β_k are normalized to the linear chirp rate α_0 which means that the exact value of the reference propagation delay is not needed. However, a rough value is required for the estimation part in order to extract the reference response.

From the implementation point of view, the beat signal is a digital signal $s_b[n]$ uniformly sampled at the moments t_n, $n = \overline{0, N_s - 1}$ where N_s is the number of samples. However, the samples $s_b[n]$ of the beat signal related to the moments $\theta_n = \theta(t_n)$ of the θ time axis lead to a non-uniformly sampled signal. It can be shown that the average sampling interval of θ is:

$$\overline{\theta_S} = \frac{\overline{\alpha}}{\alpha_0} T_s, \qquad (15)$$

where $\overline{\alpha}$ is the mean chirp rate and T_s the uniform sampling interval. According to [14], for a nonuniformly sampled signal, the average sampling rate must respect the Nyquist condition. For $\overline{\alpha} > \alpha_0$, this condition can be fulfilled if the beat signal is oversampled (the chirp rate in the origin and the average chirp rate typically have the same order of magnitude, so an oversampling of at most 10 is enough). If the signal is alias-free it can be resampled with an interpolation procedure (e.g. with spline functions) in order to obtain a uniformly sampled signal in relation with the θ time axis. Afterwards, the range profile is computed by applying the discrete Fourier transform to the resampled signal.

III. RESULTS AND DISCUSSION

The nonlinearity correction procedure was tested on a FMCW transceiver based on a RFVC1800 X-band VCO having 15% linearity according to the linearity definition given in [15]. The tuning voltage versus frequency calibration curve of the VCO was measured. The range profiles obtained with a predistorted command signal based on the calibration curve were considered as reference. A few data sets were collected under the same external conditions as for the calibration curve measurement. Two delay lines having air-equivalent lengths of $30cm$ (short path) and respectively $240cm$ (long path) were used as targets. The chirp bandwidth was $4GHz$ and the sweep interval $100ms$. The correction was done using both lines in order to analyze the influence of the reference response delay on the algorithm performance. For a 4th order polynomial approximation the HAFs plots of the FMCW coefficients in both cases are shown in Fig. 1. The estimated coefficients were used to generate nonlinearity corrected beat signals by employing the time resampling procedure. The range profiles
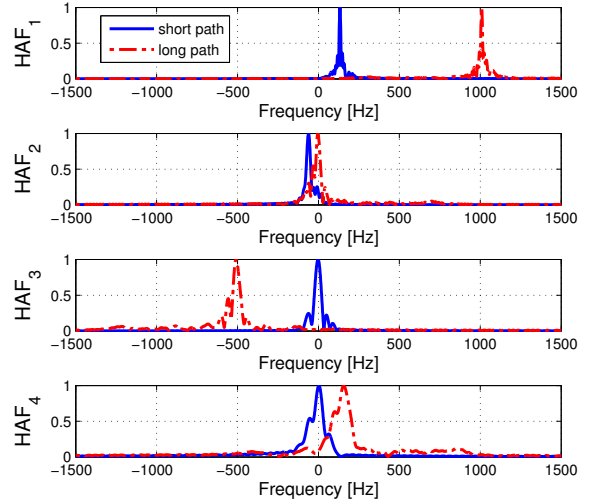

Fig. 1. High-order ambiguity functions of the FMCW signal for two delay lines. The continuous plots show the functions for a $30cm$ air-equivalent length delay line (short path) and the dotted ones for a $240cm$ air-equivalent delay line (long path).

were obtained by applying a Hamming window before the Fourier transform.

Fig. 2 shows a comparison between the range profiles obtained for the two delay lines in different cases and the range profile obtained for the predistorted sweep. The range profile for a linear sweep is presented in Fig. 2(a). The energy of both targets is highly spread in frequency and the main lobe for the long path occupies more than 40 resolution cells which was expected in view of the high degree of nonlinearity of the VCO. The range profile corrected using the short path coefficients shown in Fig. 2(b) is similar to the predistorted sweep range profile for the short path response, but the energy of the long path is still spread and there are two main lobes for the same target. This effect is linked with the HAFs for the high-order nonlinearity terms (3 and 4) in the short reference path case which are hardly noticeable and can't be estimated properly. However, the long path calibration range profile from Fig. 2(c) is very similar to the one obtained with the predistorted sweep, so this software nonlinearity correction method provides good results if the calibration path is long enough to emphasize the nonlinearities (the highest order term to be highlighted and properly estimated). A clear advantage of the proposed correction algorithm compared to the predistorted sweep technique is that the FMCW signal coefficients used for correction are computed for each sweep and can include various frequency drifts (due to temperature, frequency pushing, etc.). The results of the range profiles comparison are summarized in Table I where the $-10dB$ resolutions are computed for both targets. Although the resolution for the long path corrected range profile is better than for the predistorted sweep range profile there are still present some residual nonlinearities (deterministic as well as random) which have small bandwidths and whose effect increases with range. However, they can be further mitigated with methods like those
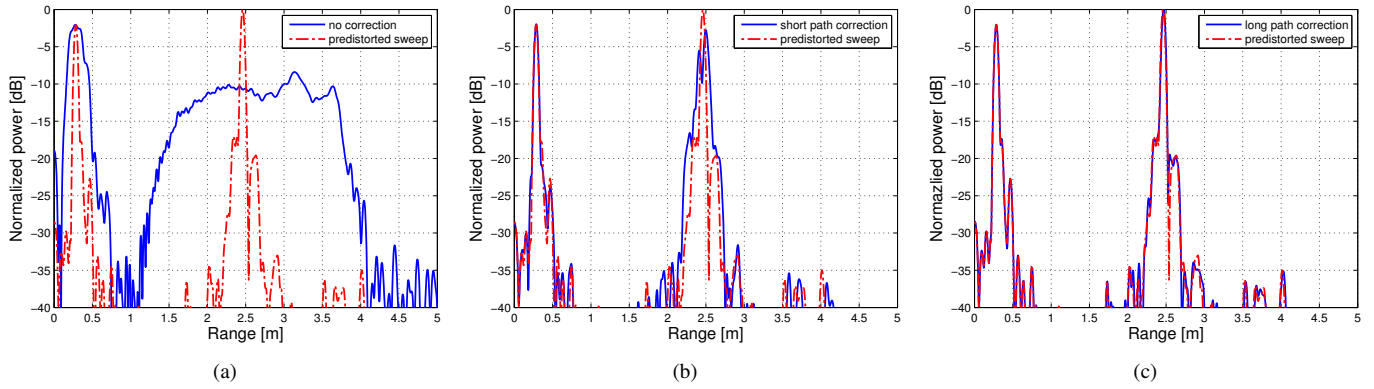
Fig. 2. Range profiles for two delay lines. The profiles obtained with a linear sweep and software nonlinearity correction are compared with the profile resulted for a predistorted sweep (shown with dotted line) computed from the measured frequency-voltage calibration curve. There are three cases considered: (a) no correction, (b) short path correction, (c) long path correction.

TABLE I

NONLINEARITY CORRECTION ALGORITHM -10dB RESOLUTION

	Short Path Resolution (cm)	Long Path Resolution (cm)
Predistorted sweep	8.18	8.47
Short path correction	8.23	19.87
Long path correction	8.17	8.45

presented in [7], [8], [16].

In the following part some results of the software nonlinearity correction procedure applied to FMCW radar data are presented. The data sets were acquired for a frequency sweep from 8 to 11 GHz.

In Fig. 3 are shown the range profiles obtained before and after applying the correction algorithm for a scene containing a corner reflector (situated at $2.5m$ from the radar) which was used as reference target for the nonlinearity estimation. In the corrected range profile the main lobe of the corner reflector target reaches the theoretical $-3dB$ resolution for a $3GHz$ bandwidth and Hamming window (around $6.5cm$).

Fig. 4 presents the results obtained from applying the nonlinearity correction for a synthetic aperture image acquired by moving the FMCW radar on a $30cm$ rail. The scene in the synthesized image contained some metal bars and one highly-reflective metal disc. The software nonlinearity correction was employed by resampling each line of the initial image before applying the matched filter algorithm [17] to obtain the synthetic aperture radar (SAR) image. Notice that in the image from Fig. 4(a) the targets can't be distinguished while in the corrected version they are clearly range focused (Fig. 4(b)).

IV. CONCLUSION

A novel wideband nonlinearity software correction method for a FMCW transceiver is proposed. The method estimates the nonlinearity coefficients using the HAF processing algorithm
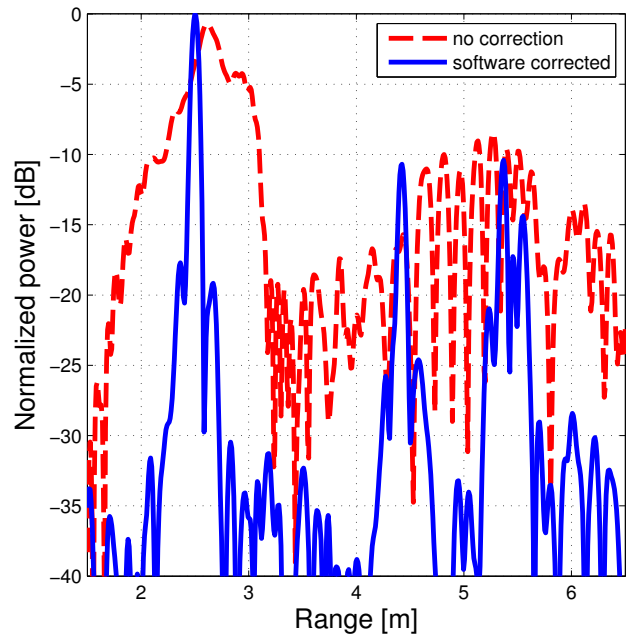

Fig. 3. Range profile for an indoor scene containing a corner reflector (at $2.5m$) used as reference target for the software nonlinearity correction method. The chirp bandwidth was 3 GHz and the sweep interval 100 ms.

on the beat signal corresponding to a certain reference path which can be a delay line or a highly reflective target. Afterwards, a nonlinearity correction function is built and applied to the beat signal by time resampling. The procedure was validated on real data acquired with a T/R system based on an X-band low-cost FMCW transceiver. The improvements due to the software nonlinearity correction are clearly highlighted in the range profiles and the SAR images obtained with the FMCW radar demonstrator system.

SAR Image [dB] – No correction
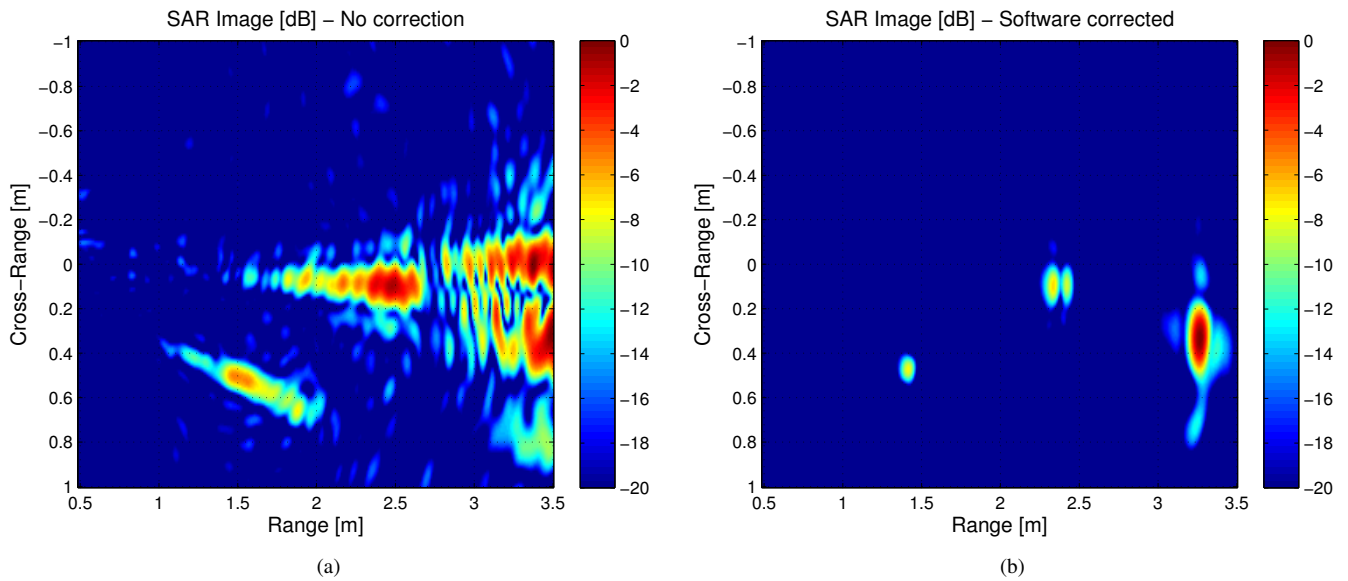
SAR Image [dB] – Software corrected

(a)

(b)

Fig. 4. Results of the software nonlinearity correction applied to the FMCW image before focusing. In the SAR image affected by the VCO nonlinearity (a) the targets are almost undistiguishable, while in the corrected image (b) the targets are clearly separated.

REFERENCES

[1] A. Stove, "Linear FMCW radar techniques," *IEE Proc. Part F Radar and Signal Processing*, vol. 139, no. 5, pp. 283–296, Oct. 1998.

[2] F. Bao, L. Arend, S. Bertl, and J. Detlefsen, "Application of FMCW radar principle for fast inhomogeneity identification on transmission lines," in *Proc. GeMIC'11*, Darmstadt, Germany, Mar. 2011, pp. 1–4.

[3] A. Meta, P. Hoogeboom, and L. P. Ligthart, "Signal processing for FMCW SAR," *IEEE Trans. Geosci. Remote Sensing*, vol. 45, no. 11, pp. 3519–3532, Nov. 2007.

[4] L. Li, J. Yu, and J. Krolik, "Software-defined calibration for FMCW phased-array radar," in *Proc. IEEE Radar Conference*, vol. 20, Washington DC, USA, May 2010, pp. 877–881.

[5] P. Lowbridge, "A low cost mm-wave cruise control system for automotive applications," *Microwave Journal*, pp. 24–36, Oct. 1993.

[6] N. Pohl, T. Jacschke, and M. Vogt, "Ultra high resolution SAR imaging using an 80 ghz FMCW-Radar with 25 ghz bandwidth," in *Proc. EuSAR'12*, Nuremberg, Germany, Apr. 2012, pp. 189–192.

[7] M. Vossiek, P. Heide, M. Nalezinski, and V. Magori, "Novel FMCW radar system concept with adaptive compensation of phase errors," in *Proc. EuMC'96*, vol. 1, Prague, Czech Republic, Sept. 1996, pp. 135–138.

[8] A. Meta, P. Hoogeboom, and L. Ligthart, "Range nonlinearities correction in FMCW SAR," in *Proc. IGARSS'06*, Denver, Denver, USA, July 2006, pp. 403–406.

[9] W. Ying, L. Qingshan, G. Deyun, and Z. Guangyong, "Research on nonlinearity correction imaging algorithm of FMCW SAR," in *Proc. ICSPS'10*, vol. 1, Dalian, China, July 2010, pp. 425–429.

[10] S. Peleg and B. Porat, "Estimation and classification of signals with polynomial phase," *IEEE Trans. Inform. Theory*, vol. 37, no. 2, pp. 422–430, 1991.

[11] P. Brennan, Y. Huang, M.Ash, and K. Chetty, "Determination of sweep linearity requirements in FMCW radar systems based on simple voltage-controlled oscillator sources," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 47, no. 3, pp. 1594–1604, July 2011.

[12] Y. Wang and G. Zhou, "On the use of high order ambiguity functions for multicomponent polynomial phase signals," *Signal Processing*, vol. 65, pp. 283–296, 1998.

[13] C. Ioana and A. Quinquis, "Time-frequency analysis using warped-based high-order phase modeling," *EURASIP Journal on Applied Signal Processing*, vol. 17, pp. 2856–2873, 2005.

[14] F. A. Marvasti, *Nonuniform Sampling:Theory and Practice*. New York: Kluwer Academic/Plenum Publishers, 2001.

[15] United States Department of Defense, "General specification for crystal controlled oscillator. Specification MIL-PRF-55310," Columbus, OH.

[16] E. Avignon-Meseldzija, W. Liu, H. Feng, S. Azarian, M. Lesturgie, and Y. Lu, "Compensation of analog imperfections in a Ka-band FMCW SAR," in *Proc. EuSAR'12*, Nuremberg, Germany, Apr. 2012, pp. 60–63.

[17] M. Soumekh, *Synthetic Aperture Radar Signal Processing with MATLAB Algorithms*. New York: Wiley-Interscience, 1999.